# Permittivity Estimation of Multilayered Dielectrics by Wall-Thru Radar Image

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Abstract - Using wall-through radar, we obtain image of wall and target by means of scanning an antenna mechanically or electrically. In this case, the wall information, permittivity and thickness, are necessary in order to compensate position shift of the image caused by the delay phase of the wall. So, we can inversely estimate the permittivity by comparing with same target image position with no wall image. In this paper, we propose novel method to estimate equivalent permittivity of multilayered dielectric flat plate.

Index Terms — Wall-thru radar, Multilayered dielectric, Radar image, Equivalent permittivity.

#### I. Introduction

For detection of invisible objects in underground or behind wall, if the image of the target has high quality, accuracy of identification processing becomes to be improved drastically by using such surface-penetrating or wall-trough wall radar. In the field of imaging processing by micro/millimeter wave, we widely use the method of synthetic aperture radar (SAR) moving it spatially to equivalently obtain a large antenna aperture since position-fixed small antenna may be low resolving of the image [1-3]. We adopt simple imaging method to obtain radar image by *array-factor* (AF) theory as described in [4], not by SAR processing. The AF is also improved by considering *focusing* between radar and target.

In this paper, we propose new measurement method to estimate equivalent permittivity of multilayered dielectric plate by means of the wall-thru radar. When obstacle such as wall exists between target and radar, target image position moves along line of the target and the radar due to the electromagnetic wave delay in the dielectric obstacle. The each permittivity and thickness of the multilayered wall is necessary in order to compensate this error of shift. Then, inversely, we can also estimate the permittivity of the wall by comparison with no obstacle image or by attaching extra outer dielectric plate with known permittivity and thickness.

## II. IMAGING PROCEDURE

We assume plural point sources with isotropic spatial pattern for the electromagnetic field formed by these point sources. Regarding each wave source operating as antenna has same phase each other at their feeding point, and then its far-field can be expressed by superposition each EM field:  $f(\theta,\phi) = \sum_{n} a_n \exp(jk(x_n u + y_n v + z_n \cos \theta))$ . This is called

array-factor and  $(x_n, y_n, z_n)$  is the  $n^{\text{th}}$  element in spherical coordinates system  $(u = \sin \theta \cos \phi, v = \sin \theta \sin \phi)$ , and  $a_n$  is its complex amplitude.

In order to consider *focusing* procedure, we employ the rectangular coordinate system (x, y, z) to describe azimuth, height and range from point of radar view. Coordinates of  $m^{\text{th}}$  Tx antenna are expressed to  $\mathbf{r}_m = (x_m', y_m', z_m')$  and  $n^{\text{th}}$  Rx antenna to  $\mathbf{r}_n = (x_n', y_n', z_n')$ . The target position is described as  $\mathbf{r}_p = (x_p, y_p, z_p)$  for image data variable. Here  $\mathbf{r}$  shows position vector. The transmitted wave from  $m^{\text{th}}$  Tx antenna is reflected on the target which is assumed to be focused at position  $\mathbf{r}_p$ , and then returns to  $n^{\text{th}}$  Rx antenna. Its optical path length from Tx to Rx via the target is given as  $r_m(x_p, y_p, z_p) = |\mathbf{r}_p - \mathbf{r}_m| + |\mathbf{r}_p - \mathbf{r}_n|$ .

Adopting above procedure, we can obtain target image. Denoting receiving power to  $P_{\ell mm}(f_\ell,r_{mn})$  by  $m^{\text{th}}$  Tx antenna,  $n^{\text{th}}$  Rx antenna, and  $\ell^{\text{th}}$  step frequency  $f_\ell$ , returned signal from target is be expressed as follows:

$$Q_{0}(\mathbf{r}) = \sum_{\ell=1}^{L} \sum_{m=1}^{M} \sum_{n=1}^{N} P_{\ell m n}(f_{\ell}, r_{m n}) \cdot e^{ikr_{m n}(x, y, z)} \cdot e^{j\phi_{\ell}x},$$
(1)

where the target coordinates  $\mathbf{r}_p$  is replaced with imaging area variables  $\mathbf{r}(x,y,z)$ , L is number of frequency step, and M/N is  $\mathrm{Tx/Rx}$  antenna position (or antenna number) respectively. The receiving power is desired to be calibrated by frequency characteristic of a standard target like sphere, and  $\phi_\ell$  is inner delayed phase quantity of measurement instrument. Lastly we can obtain AF focusing image by direct drawing (1) with variables  $\mathbf{r}(x,y,z)$ .

Furthermore we can improve (1) to calibrate the defocus image, as follows:

$$Q(\mathbf{r}) = Q_0(\mathbf{r}) \cdot \exp\{jkD'(\mathbf{r}_m, \mathbf{r}, I)\} \cdot \exp\{jkD'(\mathbf{r}_n, \mathbf{r}, I)\}$$
(2)

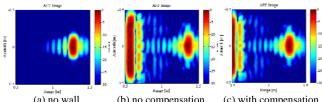
where  $D^t(\mathbf{r}_m, \mathbf{r}, I)$  and  $D^r(\mathbf{r}_n, \mathbf{r}, I)$  are inserted path-length by I-layered dielectric (wall) plate from Tx and the target side, respectively, and from Snell's law, given as follows:

$$D = \sum_{i=1}^{I} k_i z_i \cdot (k_i^2 - k_i^2 \sin^2 \theta)^{-1/2}.$$
 (3)

Next, when we perform simulation for  $P_{\ell mn}(f_{\ell}, r_{mn})$  in (1), it is necessary to express to

$$P_{\ell mn}(f_{\ell}, r_{mn}) = G_{\ell mn}^{t} \cdot \left| T_{\ell mn}^{t} \right|^{2} \cdot \sigma \cdot \left| T_{\ell mn}^{r} \right|^{2} \cdot G_{\ell mn}^{r} \tag{4}$$

where  $G^t_{\ell mn}(f_\ell,r_{mn})/G^r_{\ell mn}(f_\ell,r_{mn})$  are Tx/Rx antenna beam pattern,  $T^t_{\ell mn}(f_\ell,r_{mn})/T^r_{\ell mn}(f_\ell,r_{mn})$  are refraction coefficient of *I*-layered plate from view of Tx/Rx which is indicated in [5], respectively, and  $\sigma(\text{pol.},f_\ell,\mathbf{r}_{in};\mathbf{r}_{out})$  is radar cross-



(a) no wall (b) no compensation (c) with compensation Fig. 1. Measured radar images of 1-layered wall w/o path-length and position shift compensation (*L*=8cm).

section of the target. It is better for this  $\sigma$  that its near-field is taken into account as described in [4].

#### III. PERMITTIVITY ESTIMATION

Next, we discuss the method of permittivity estimation. Introducing relative permittivity and permeability as  $\varepsilon_r = \varepsilon/\varepsilon_0$  and  $\mu_r = \mu/\mu_0$ , respectively, then the wave speed in the material is expressed as  $v = c/\sqrt{\varepsilon}_r$  where we assume non-magnetic material ( c: speed of light). In this case wave-number is given as  $k = \sqrt{\varepsilon_r}\omega/c$  (  $\omega = 2\pi f$  ). While wave toward +z is expressed using transmission model as  $E_z = E_0 e^{-\alpha z} e^{j(\omega t - \beta z)}$ ,  $jk = \alpha + j\beta$ . Obviously, path-length of the wave in the material  $\varepsilon_r$  is extended to follows:

$$\Delta \phi = k_0 \sqrt{\varepsilon_r} z_0 - k_0 z_0 = k_0 \left( \sqrt{\varepsilon_r} - 1 \right) z_0. \tag{5}$$

This inserted phase, caused by the plate, corresponds to the image shifting value  $\Delta\phi/k_0=L$ , where  $z_0$  is thickness of the dielectric plates under test. Thus we obtain a relation of the 1-way inserted phase,  $\Delta\phi=k_0L=k_0(\sqrt{\varepsilon_r}-1)z_0$ , with respect to the image position shift L. Because radar image is constructed by reflected and scattered wave to Rx not incident wave from Tx, we can assume the validity of 1-way propagation delay. From this relation, following simple but novel expression is derived as:

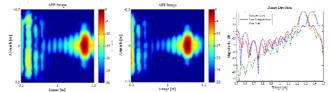
$$\varepsilon_r = (L/z_0 + 1)^2. \tag{6}$$

When we analyze shifted length from the obtained image with reading error  $\pm \Delta L$ , estimation of permittivity changes with respect to the first order of  $\Delta L$  as  $\varepsilon_r \to \varepsilon_r \pm \sqrt{\varepsilon_r} \Delta L/z_0$ . Incidentally, regarding insertion loss of the plate to IL, the attenuation constant  $\alpha$  is calculated by  $\alpha = \ln(IL)/z_0$ , and then complex permittivity under test will be estimated.

We show measured results of 1-lyayered concrete plate with 6 cm thickness. Distance between antenna and target is 100 cm, length of array aperture is 90 cm and width of metal plate target is 22 cm, respectively. Operating center frequency is 4.1 GHz with 48 % band-width. AFF image are shown in Fig.1. Since thickness of the concrete is  $z_0 = 6$  cm and position shift on the image is L = 8.0 cm, we can calculate the permittivity  $\varepsilon_r = 5.4$  by using (6). Fig.1(c) is already reflected this image shift.

## IV. EXTENDED DISCUSSION FOR MULTILAYERED PLATES

Now, a wall consists of I-layered plates without air-layer. At this point, inserted phase of each layered plate is approximately given by  $k_0\sqrt{\varepsilon_i}\,z_i$  for  $i^{\text{th}}$  layer, respectively. Taking incident angle  $\theta_0$  measured from the normal face into account, the inserted phase is modified to  $k_0\sqrt{\varepsilon_i}\,z_i$   $\cdot \sec\theta_0$ . Therefore, simply adding inserted phase from each layer, its relation is extended to follows:



(a) no compensation (b) with compensation (c) cross-section in range Fig. 2. Measured AFF images of 3-layered wall w/o compensation.

$$\sum_{i=1}^{I} \left( \sqrt{\varepsilon_i} - 1 \right) z_i \sec \theta_0 = L. \tag{7}$$

By attaching a few numbers of other dielectric plates with known information, therefore, we will be able to estimate equivalent permittivity of multilayered dielectrics without comparison of no wall, which will be our future task. From (7), we can derive equivalent permittivity of multilayered dielectric plates as follows:

$$\sqrt{\varepsilon_{eq}} = \sum_{i=1}^{I} \left( z_i \sqrt{\varepsilon_i} \right) / \sum_{i=1}^{I} z_i. \tag{8}$$

For example I=2 ( $\theta_0=0$ ), from (7) we get permittivity of wall (unknown),  $\varepsilon_{eq}$  is estimated by  $z_{eq}\sqrt{\varepsilon_{eq}}-z_{eq}=L_{eq-ex}$ , where  $L_{eq-ex}=L_{eq}-(z_{ex}\sqrt{\varepsilon_{ex}}-z_{ex})$  measurement image position data, <with extra plate> minus < no extra plate>.

Fig.2 is a measured case, 3-layered wall plate, which is consisted of 1 core concrete with 6 cm thickness and 2 plaster-boards with 0.96 cm thickness on both sides of the concrete. The plaster-board permittivity is unknown, while, permittivity and thickness of the concrete is evaluated to  $\varepsilon_r = 5.4$  and 6 cm which is same to Fig.1. Using these information and (7), we calculate the permittivity of the plaster-board  $\varepsilon_r = 4.0$ .

### V. CONCLUSION

By using the proposed image shifting method, we find to obtain permittivity of multilayered dielectric plate as wall with known thickness, which is located between radar and target. There are typically coaxial cable, waveguide, and cavity method for permittivity measurement. Comparing with these traditional methods, this image shifting method has stable S/N ratio due to radar image and will be applicable to measure for spreading material in wide area such as forest.

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