Scattering of Light by Gratings of Metal-Coated Nanocylinders on Dielectric Substrate

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Abstract - Scattering of TE polarized plane wave by a grating of metal-coated dielectric nanocylinders is investigated with a particular emphasis on the enhancement of the near fields. Two types of resonances such as surface plasmon resonance associated with the metal-dielectric nanocylinder and Rayleigh resonance associated with the periodic nature of the grating are considered.

Index Terms **— Surface plasmon, planar grating, scattering.**

I. INTRODUCTION

Surface plasmons are accompanied by localized enhancement of the electromagnetic field. They have ability to guide electromagnetic energy on length scales below the diffraction limit. This leads to developing metallic nanostructures that can control light at the nanoscale in direct analogy to traditional optical components such as lens, mirrors and waveguides. Many of these applications rely on a planar geometry in practice. In this regard, scattering of TE polarized plane wave by planar periodic grating of the metal-coated dielectric nanocylinders supported on the dielectric substrate is rigorously investigated utilizing the recursive algorithm method combined with the Lattice Sums technique. For metal-dielectric gratings, there exist two type of resonances. One is the resonance associated with the Rayleigh wavelength (due to periodicity of the structure) and the other is the surface plasmon resonance. Enhancement of the near fields at the wavelengths associated with the resonances of the structure is studied.

II. FORMULATION OF THE PROBLEM

The cross section of the planar periodic grating with a period h composed of the metal-dielectric coaxial cylinders per unit cell and supported on a dielectric substrate is illustrated in Fig. 1. The coaxial cylinder with outer radius r_1 consists of a circular dielectric core with radius r_2 and a metal coating layer of thickness $r_1 - r_2$. The material constants of the dielectric substrate, outer free space, coating metal, and dielectric core are denoted by (ε_s, μ_0), (ε_0, μ_0), (ε_M, μ_0) and (ε, μ_0), respectively. The structure is illuminated by a plane wave of unit amplitude. The angle of incidence of the plane wave is φ^i with respect to the x axis. Since we are interested in the scattering problem related to the plasmon resonances, we focus our investigation on the scattering of

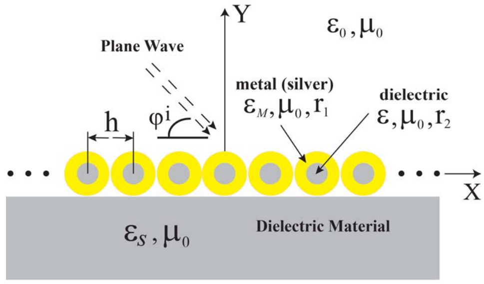

Fig. 1. Planar periodic structure composed of the metal-dielectric circular cylinders supported on a dielectric substrate.

TE wave with (H_z, E_x, E_y) component.

The solutions to the H_z field in the region $|y| > r_1$ (Fig. 1) are obtained based on the superposition of the plane waves:

$$H_z(x,y) = \boldsymbol{e}^-(x,y)\cdot\boldsymbol{a}^i + \boldsymbol{e}^+(x,y)\cdot\mathbf{R}\cdot\boldsymbol{a}^i \qquad y > r_1 \tag{1}$$

$$H_z(x,y) = \tilde{\boldsymbol{e}}^-(x,y)\cdot\mathbf{F}\cdot\boldsymbol{a}^i \qquad y < -r_1 \tag{2}$$

with

$$\boldsymbol{a}^i = [a_m^i] = [\cdots 0 \cdots 0\,1\,0 \cdots 0 \cdots]^T \tag{3}$$

$$\boldsymbol{e}^\pm(x,y) = [\mathrm{e}^{i(k_{xm}x \pm k_{ym}y)}], \quad \tilde{\boldsymbol{e}}^-(x,y) = [\mathrm{e}^{i(k_{xm}x - \tilde{k}_{ym}y)}] \tag{4}$$

$$k_{xm} = k_0\left(\cos\varphi^i + \frac{2\pi m}{k_0 h}\right) \tag{5}$$

$$k_{ym} = \sqrt{k_0^2 - k_{xm}^2}, \qquad \tilde{k}_{ym} = \sqrt{k_s^2 - k_{xm}^2} \tag{6}$$

$$\mathbf{R} = \mathbf{R}_{gr} + \mathbf{F}_{gr}\mathbf{U}\mathbf{R}_{fr}(\mathbf{I} - \mathbf{U}\mathbf{R}_{gr}\mathbf{U}\mathbf{R}_{fr})^{-1}\mathbf{U}\mathbf{F}_{gr} \tag{7}$$

$$\mathbf{F} = \mathbf{F}_{fr}(\mathbf{I} - \mathbf{U}\mathbf{R}_{gr}\mathbf{U}\mathbf{R}_{fr})^{-1}\mathbf{U}\mathbf{F}_{gr} \tag{8}$$

$$\mathbf{U} = \left[\mathrm{e}^{ik_0 r_1 \sin\varphi_m}\,\delta_{mm'}\right] \tag{9}$$

where $k_s = k_0\sqrt{\varepsilon_s \mu_0}$, $\mathbf{R}_{gr}$ and $\mathbf{F}_{gr}$ are the reflection and transmission matrices of the planar periodic grating, which are accurately calculated using the Lattice Sums technique combined with the T-matrix approach [1], $\mathbf{R}_{fr}$ and $\mathbf{F}_{fr}$ are Fresnel matrices, $\mathbf{I}$ is the unit matrix and $\delta_{mm'}$ is Kronecker's delta, and the superscript T indicates transpose of the vector.

Total field in region $|y| < r_1$ inside the unit cell around the global origin is calculated using cylindrical wave expansion:

$$H_{z,\,\rho_0 > r_1} = (\mathbf{\Phi}_0^T + \mathbf{\Psi}_0^T\cdot\mathbf{T})\cdot(\mathbf{I} + \mathbf{L}\bar{\bar{\mathbf{T}}})(\mathbf{P}^{(-)} + \mathbf{P}^{(+)}\mathbf{A})\cdot\boldsymbol{a}^i \tag{10}$$

$$H_{z,\,r_1 > \rho_0 > r_2} = (\bar{\mathbf{\Phi}}_0^T\cdot\mathbf{B} + \bar{\mathbf{\Psi}}_0^T\cdot\mathbf{C})\cdot(\mathbf{I} + \mathbf{L}\bar{\bar{\mathbf{T}}})(\mathbf{P}^{(-)} + \mathbf{P}^{(+)}\mathbf{A})\cdot\boldsymbol{a}^i \tag{11}$$

$$H_{z,\,\rho_0 < r_2} = \bar{\bar{\mathbf{\Phi}}}_0^T\cdot\mathbf{D}(\mathbf{I} + \mathbf{L}\bar{\bar{\mathbf{T}}})(\mathbf{P}^{(-)} + \mathbf{P}^{(+)}\mathbf{A})\cdot\boldsymbol{a}^i \tag{12}$$

with

$$\mathbf{\Phi}_0 = [J_m(k_0\rho_0)\mathrm{e}^{im\varphi_0}],\ \mathbf{\Psi}_0 = [H_m^{(1)}(k_0\rho_0)\mathrm{e}^{im\varphi_0}] \tag{13}$$

$$\overline{\mathbf{\Phi}}_0 = [J_m(k_M\rho_0)\mathrm{e}^{im\varphi_0}],\ \overline{\mathbf{\Psi}}_0 = [H_m^{(1)}(k_M\rho_0)\mathrm{e}^{im\varphi_0}] \tag{14}$$

$$\overline{\overline{\mathbf{\Phi}}}_0 = [J_m(k\rho_0)\mathrm{e}^{im\varphi_0}] \tag{15}$$

$$\mathbf{A} = \mathbf{UR}_{fr}(\mathbf{I} - \mathbf{UR}_{gr}\mathbf{UR}_{fr})^{-1}\mathbf{UF}_{gr} \tag{16}$$

$$\overline{\overline{\mathbf{T}}} = (\mathbf{I} - \mathbf{TL})^{-1}\mathbf{T} \tag{17}$$

$$\mathbf{P}^{(-)} = [i^m \mathrm{e}^{im\varphi_n}],\ \mathbf{P}^{(+)} = [i^m \mathrm{e}^{-im\varphi_n}] \tag{18}$$

$$\rho_0 = \sqrt{x^2 + y^2},\quad \cos\varphi_0 = \frac{x}{\rho_0} \tag{19}$$

where $k_M = k_0\sqrt{\varepsilon_M \mu_0}$, $k = k_0\sqrt{\varepsilon\mu_0}$, J_m and $H_m^{(1)}$ are Bessel and Hankel functions, respectively, $\mathbf{L}$ is a matrix characterizing the Lattice Sums and $\mathbf{T}$ is the T-matrix of the circular cylinder per unit cell. Due to the limited space, the detailed expressions for $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ matrices, which relate the incident amplitude to the scattering amplitudes inside the metal-dielectric cylinder, are omitted in the paper. The interested readers can refer to our recent papers [2,3]. Since the field inside the unit cell is defined, the field in the other regions of the planar periodic grating is easily calculated using Floquet theorem.

III. Numerical Results and Discussions

Equations (1)-(19) are used to calculate the near field distributions of H_z for several configurations of planar periodic grating composed of metal-coated nanocylinders. In the numerical examples in what follow, we assume Ag for the metal employing the Drude-Lorentz model. We study the normal incidence of the plane wave $\varphi^i = 90^0$ at the following parameters of the structure: $r_1 = 60nm$, $r_2 = 30nm$, $\varepsilon_s = 2.25\varepsilon_0$ and $\varepsilon = 2.0\varepsilon_0$. Scattering and absorption cross sections of a single Ag-coated nanocylinder are plotted in Fig. 2 for the wavelength range from 250nm to 1000nm. It is seen that the scattering cross section (the red line) has two peaks in the spectral response at $\lambda = 315nm$ and $\lambda = 450nm$, respectively. Shorter wavelength peak corresponds to the resonance to the surface plasmon mode on the outer interface of the metal and free space, whereas the longer wavelength peak corresponds to the resonance to the surface plasmon mode on the inner metal-dielectric interface [3]. Another type of resonance is Rayleigh resonance associated with the periodic geometry of the structure. At the normal incidence, the Rayleigh resonance wavelength is equal to the period of the grating h.

The near field distributions of $|H_z|$ at $h = 314nm$, $\lambda = 315nm$ and $h = 449$nm, $\lambda = 450nm$ are illustrated in Fig.3(a) and Fig.3(b), respectively. The incident wave resonates to the surface plasmon modes and enhances the near field at the outer (Fig.3(a)) and inner (Fig.3(b)) interfaces of the metal. At the same time, a strong reflection by the planar grating is observed in the upper region $y > r_1$ due to the Rayleigh resonance associated with the periodic nature of the structure.

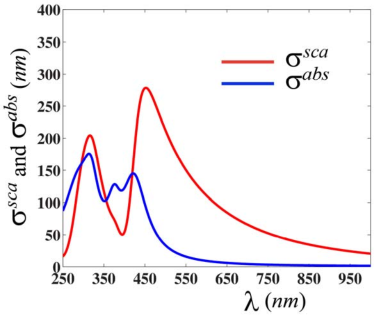

Fig. 2. Scattering cross section σ^{sca} and absorption cross section σ^{abs} of a single metal-coated nanocylinder per unit cell.

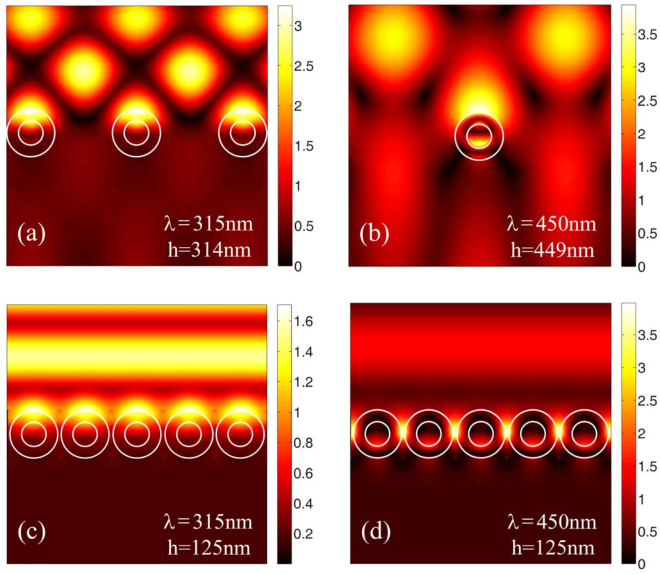

Fig. 3. Near field distribution of $|H_z|$ for the planar period grating composed of the metal-dielectric nanocylinder per unit cell.

The near field distributions of $|H_z|$ at $\lambda = 315nm$ and $\lambda = 450nm$ are shown in Fig.3(c) and Fig.3(d), when the period of the grating is fixed at $h = 125nm$ (two nearest nanocylinders are located very close to each other). The resonated field to the surface plasmon at the outer interface occurs at $\lambda = 315nm$ in Fig.3(c). Interestingly, at $\lambda = 450nm$ the surface plasmons on the inner metal interfaces are strongly coupled and a new surface plasmon mode is formed in the gap region between the nanocylinders.

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