Proceedings of ISAP 2014, Kaohsiung, Taiwan, Dec. 2-5, 2014

Asymptotic Analysis of Scattering for Finite Periodic Reflectarray/Transmitarray Antennas in the Near-Zone Focused Radiations

Shih-Chung Tuan [1] and Hsi-Tseng Chou [2]
[1] Dept. of Communication Eng., Oriental Institute of Technology , Pan-Chiao ,Taiwan
[2] Dept. of Communication Eng., Yuan Ze University, Chung-Li , Taiwan

***Abstract* - This paper presents the asymptotic formulation of ray fields in the decomposition of electromagnetic (EM) scattering mechanisms from a one-dimensional, semi-infinite and periodic array when it is illuminated by a line source. This technique can be applied to analyze the passive FSS (frequency selective surface) type periodic structure with identical elements, or the reflectarray and transmitarray type antennas that are phased to radiate EM fields focused in the near zone of the array aperture. The solutions are built up based on the Floquet mode expansion of the scattering fields, and are obtained by asymptotically evaluating the resulted integrals to express the fields in terms of reflected/transmitted and edge diffracted fields as previously addressed in the framework of uniform geometrical theory of diffraction (UTD).**

***Index Terms* —Reflectarray Antennas, Transmitting Array Antennas, Floquet mode expansion.**

I. Introduction

The analysis of electromagnetic (EM) scattering from periodic array structures is very important in the designs of frequency selective surface (FSS), metamaterials and reflectarray/transmitarray antennas [1]-[4]. These types of applications employ periodic structures to enhance the performance of antenna radiation by enforcing the waves propagating through the structures. To well characterize these phenomena, an effective analysis approach with the capability to interpret the scattering mechanisms is very crucial to achieve the design goals. In the past development, most techniques suffer from the limitation of computational resources, and have to assume periodic arrays with identical elements. Plane wave illuminations were generally assumed in order to reduce the analysis to be over a unit cell of element. Numerical methods, such as method of moment (MoM), finite element method (FEM) and modal expansion method, are widely applied to analyze the scattering field within this unit cell. Array factors are afterward multiplied to account for the total contribution from every element of a finite array. In contrast to the time-cumbersome element-by-element computation for the scattering fields, which are also short of physical phenomenon interpretation, the effective formulation based on the asymptotic evaluation becomes more attractive because it may accelerate the computation by decomposing the scattering fields in terms of the components in the diffraction ray theory including reflected/transmitted and edge diffracted ray components. The computational efficiency is based on a fact that only a few rays are sufficient to provide accurate results. Nevertheless, this ray type solution provides good mechanisms to interpret the scattering phenomena.

II. A 2-D Finite Array Radiation Problem

(A) Composition of 2-D Radiation from a 1-D Array

The semi-infinite, linear array of line sources is illustrated in Figure 1, whose elements are indexed by $n = -\infty \sim N$ and located at $\bar{r}_n' = (nd_x, 0)$ on the x-axis with a period d_x. The line source is located at $\bar{\rho}_f = (x_f, z_f)$ and radiates fields by

$$u_f(\bar{\rho}) = \frac{1}{4j} I_0 H_0^{(2)}(k|\bar{\rho} - \bar{\rho}_f|) \cong \frac{e^{-j\pi/4} I_0}{4} \sqrt{\frac{2}{\pi k|\bar{\rho} - \bar{\rho}_f|}} e^{-jk|\bar{\rho} - \bar{\rho}_f|} \quad (1)$$

which will excite the reflecting or transmitting elements in Figure 1(a) and (b), respectively. In (1), $k = 2\pi/\lambda$ is the wave number with λ being the wavelength in free space, and I_0 is used as the reference amplitude of line source. The fields scattered from the array can be found from the radiation of equivalent current, $I(\bar{\rho}')$, on the closed surface enclosing the array structure as illustrated in Figure 1. To the degree of accuracy in the Kirchhoff approximation, the induced current is assumed to be found from the infinite array structure illuminated by the same incident field. Thus it can be approximately expressed as

$$I(\bar{\rho}') = u_f(\bar{\rho}_i) Q(\bar{\rho}') e^{j\phi(\bar{\rho}')} \quad (2)$$

where $\bar{\rho}_i$ and $\bar{\rho}'$ are the position vectors on the closed surface. The function, Q, is related to the reflection and transmission coefficients for the cases of Figure 1, with ϕ being the phase. In the current investigation, one is interested in the scattering field in the $+z$ space, thus only the current on S_t is considered with the others omitted for simplification. The scattering field can be expressed as where $x_a \to -\infty$ and $x_b = Nd_x$ in the current development.

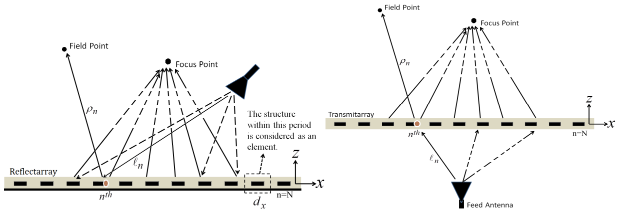

Figure 1: Illustration of 2-D reflectarray/ Transmitarray radiation problems, where the fields are focused in the near zone

(B) Composition of Uniform Asymptotic Formulations

The Floquet waves are obtained by using the following Poisson sum formula:

$$\sum_{n=-\infty}^{N} f(n) = -\frac{f(N^+)}{2} + \sum_{p=-\infty}^{\infty} \int_{-\infty}^{N^+} f(v) e^{-j2\pi pv} dv \qquad (3)$$

The evaluation of (3) can be performed asymptotically by employing the spectrum representation of Hankel functions to decompose the radiating fields into components of diffraction mechanisms. Within the UTD framework, the asymptotic evaluation of (3) can be formulated into the following format:

$$A^p_{net}(\vec{r}) = A^p_{dir}(\vec{r}) \cdot U(\mathrm{Re}(x_e - x_s)) + A^p_{end}(\vec{r}) F(ka) \qquad (4)$$

where $U(\cdot)$ is the Heaviside step function and $F(\cdot)$ is a transition function to assure the uniform field distribution. In (4), $A^p_{dir}(\vec{r})$ is the asymptotic solution of radiation when the array is extended to a infinity while $A^p_{end}(\vec{r})$ accounts for the effects of truncation. Both components arise from a radiation point, x_s, on the array aperture, and a diffraction point, x_e, on the edge. This formulation remains valid as the radiation point is close to the diffraction point, i.e. $x_s \to x_e$. However, as mentioned , it exists two radiation points in a general NSA problem. The solution in (4) becomes singular as these two radiation points coincide near the diffraction point at the edge. The characteristics of these terms are addressed in the following subsections.

(C) The Asymptotic Solution of Direct Radiation from a Non-truncated Array, $A^p_{dir}(\vec{r})$

In (4), $A^p_{dir}(\vec{r})$ is identical to the solution of radiation for an infinite array, whose characteristics have been investigated .The formulation is summarized in the following by

$$A^p_{dir}(\vec{r}) = \frac{jI_oQ}{2dxk^s_z} \sqrt{\frac{-\rho_c}{\rho^s_v - \rho_c}} \frac{1}{\sqrt{\ell^s_v}} e^{-jk(\rho^s_v - \rho^s_{vo})} e^{-j\beta_p x_i} \qquad (5)$$

where $x_s = v_s d_x$ with v_s being the contributing saddle point. Here the sadd le point, $x_i = x_s$,satisfies the following relation by

$$\frac{k^s_x}{k} = \frac{k^i_x}{k} + \frac{p\lambda}{d_x}\ ;\ (k^s_x, k^i_x) = k\left(\frac{x - x_s}{\rho^s_v}, \frac{x_s - x_f}{\ell^s_v}\right) \qquad (6)$$

(D) The Asymptotic Solution of Edge Point Contribution from a Truncated Array, $A^p_{end}(\vec{r})$

The asymptotic edge point contribution in (4) is developed and summarized in the following formulation:

$$A^p_{end}(\vec{r}) \cong \frac{jI_oQ}{2dx\sqrt{\ell_e}\sqrt{\rho_e}} \left[\frac{-e^{-j\frac{\pi}{4}}}{\sqrt{2\pi k}[(k^e_{xo} - k^e_x) + \beta_p]}\right] e^{-jk(\rho_e - \rho_{eo})} e^{-j\beta_p x_e} \qquad (7)$$

III. NUMERICAL EXAMPLES

One considers the scattering problem of reflectarray with the impression phases assumed to produce a near-field focused scattering field at $(0, 50\lambda)$. Again the fields are observed at $z = 100\lambda$. Figure 2. shows the patterns compared with the reference solutions obtained by element-by-element summation. It is observed that the near-field focus reflect array results in a narrower main beam in the near zone. The comparison with the reference is observed to be very well.

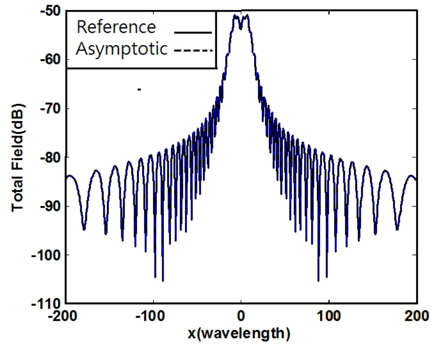

Figure 2.: Numerical examples to validate the accuracy of the developed asymptotic solutions for the reflectarray problems with elements phased to focus scattering fields in the near-zone.

IV. CONCLUSION

The solutions are built up based on the Floquet mode expansion of the scattering fields, and are obtained by asymptotically evaluating the resulted integrals to express the fields in terms of reflected/transmitted and edge diffracted fields as previously addressed in the framework of uniform geometrical theory of diffraction (UTD).

REFERENCES

[1] Rajagopalan, H.; Shenheng Xu; Rahmat-Samii, Y., "On Understanding the Radiation Mechanism of Reflectarray Antennas: An Insightful and Illustrative Approach," *Antennas and Propagation Magazine, IEEE* , vol.54, no.5, pp.14,38, Oct. 2012

[2] Lau, J.Y.; Hum, S.V., "Analysis and Characterization of a Multipole Reconfigurable Transmitarray Element," *Antennas and Propagation, IEEE Transactions on* , vol.59, no.1, pp.70,79, Jan. 2011

[3] Lau, J.Y.; Hum, S.V., "Reconfigurable Transmitarray Design Approaches for Beamforming Applications," *Antennas and Propagation, IEEE Transactions on* , vol.60, no.12, pp.5679,5689, Dec. 2012

[4] Sazegar, M.; Yuliang Zheng; Kohler, C.; Maune, H.; Nikfalazar, M.; Binder, J.R.; Jakoby, R., "Beam Steering Transmitarray Using Tunable Frequency Selective Surface With Integrated Ferroelectric Varactors," *Antennas and Propagation, IEEE Transactions on* , vol.60, no.12, pp.5690,5699, Dec. 2012