Performance of Higher Order Correction of PO-MER in Diffraction

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***Abstract* - The Modified Edge Representation is an approximation method in surface-to-line integral reduction from Physical Optics to calculate the field of diffraction. The accuracy is good in most of cases but the errors become notable near reflection shadow boundary only if the scatterer is convex with big curvature. This paper focus on the correction with higher order of PO-MER EECs for enhancing the accuracy in this specific condition.**

***Index Terms* — Modified Edge Representation, Physical Optics, Reflection Shadow Boundary, Higher Order Correction.**

I. INTRODUCTION

Physical Optics (PO) [1] which is one of the high frequency asymptotic techniques and is widely used for the diffraction analysis. The Modified Edge Representation (MER) [2], [3] is an expression of line integration for computing the scattered and diffracted fields reduced from PO. It is important not only for reducing the computational load but also for extracting the mechanism of the field and relieving the difficulties in ray theories. According to the method of stationary phase, the surface radiation integral could be asymptotically decomposed into two leading components, the first, the GO reflection component is emanating from the stationary phase point (SPP) inside the integration area and the second the diffraction component expressed in terms of line integration of EECs along the edge of the scatterer.

This paper focuses on the second one, the agreement of diffraction component calculated by MER with the correction of higher order to PO has been investigated. Especially, for the observer near the reflection shadow boundary such as the incident and the reflected shadow boundary.

II. MODIFIED EDGE REPRESENTATION

MER line integration is an alternative methodology for the PO radiation pattern calculation of surfaces as shown in Fig. 1.

The diffracted field from periphery of the scatterer is defined by the MER line integration as [2], [3],

$$E_{MER}^{dif} = j\frac{k}{4\pi}\oint_{\Gamma} \hat{r}_o \times [\hat{r}_o \times \eta J_{MER} + M_{MER}]\frac{e^{-jkr_o}}{r_o}d\ell \qquad (1)$$

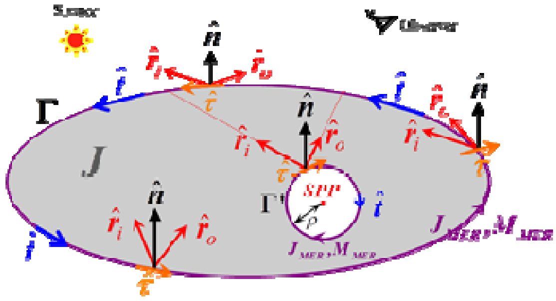

Fig 1 Parameters used in MER line integration.

equivalent electric and magnetic line currents along the actual edge $\hat{t}$ are defined using the modified edge vector $\hat{\tau}$ as,

$$J_{MER} = \frac{[\hat{r}_o \times (\hat{r}_o \times -J_{PO})]\cdot\hat{\tau}}{j\left(1-(\hat{r}_o\cdot\hat{\tau})^2\right)(\hat{r}_i+\hat{r}_o)\cdot(\hat{n}\times\hat{\tau})}\hat{t}$$

$$M_{MER} = \frac{\eta(\hat{r}_o \times J_{PO})\cdot\hat{\tau}}{j\left(1-(\hat{r}_o\cdot\hat{\tau})^2\right)(\hat{r}_i+\hat{r}_o)\cdot(\hat{n}\times\hat{\tau})}\hat{t} \qquad (3)$$

$$J_{PO} = 2\hat{n}\times H^i$$

where the unit vector $\hat{r}_i$'s direction is from integration point to source.

III. NUMERICAL DISCUSSIONS OF DIFFERENCES IN DIFFRACTION BETWEEN PO AND MER

This paper discusses the differences between MER line integration and PO for various geometrical parameters in diffraction, the scatterer is selected as the convex partial sphere shown as Fig. 2.

It has been checked that, the MER method gives the same contribution with PO in diffraction if the scatterer is concave or flat [4], [5], but a small error exists near reflection shadow boundary between MER and PO only in large curvature convex case as shown in Fig. 3 [5]. For solving this problem, we derived the correction with the higher order of line currents for MER method from PO, expressed as [6],

$$\Delta E_{MER}^{dif} = j\frac{k}{4\pi}\oint_{\Gamma} \hat{r}_o \times [\hat{r}_o \times \eta\Delta J_{MER} + \Delta M_{MER}]\frac{e^{-jkr_o}}{r_o}d\ell \qquad (4)$$

$$\Delta \mathrm{J}_{\mathrm{MER}} = \varphi_J\left(\hat{\tau}\cdot\hat{\mathrm{t}}\right) - \phi_J\left(\hat{\tau}\cdot\hat{\mathrm{t}}\right)$$
$$\Delta \mathrm{M}_{\mathrm{MER}} = \varphi_M\left(\hat{\tau}\cdot\hat{\mathrm{t}}\right) - \phi_M\left(\hat{\tau}\cdot\hat{\mathrm{t}}\right) \tag{5}$$

φ_J, ϕ_J, φ_M and ϕ_M are expressed in (6), (7). In these expressions, the new parameter $\hat{\sigma}$ is a unit vector perpendicular to the modified edge vector $\hat{\tau}$ and $\hat{\mathrm{n}}$ on the scatterer. The vector $\hat{\tau}$ of the point on the edge changes its direction smoothly along the periphery. Two vectors $\hat{\sigma}_{\max}$ and $\hat{\sigma}_{\min}$ are indicating for those which maximize and minimize the value of $\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}$.

The effectiveness of this correction has been checked for the problem with the same parameters' condition as Fig. 3, and the result is drawn in Fig. 4.

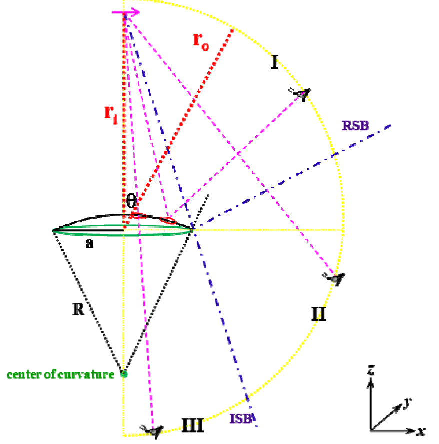

Fig. 2. Scatterer of concave and convex models.

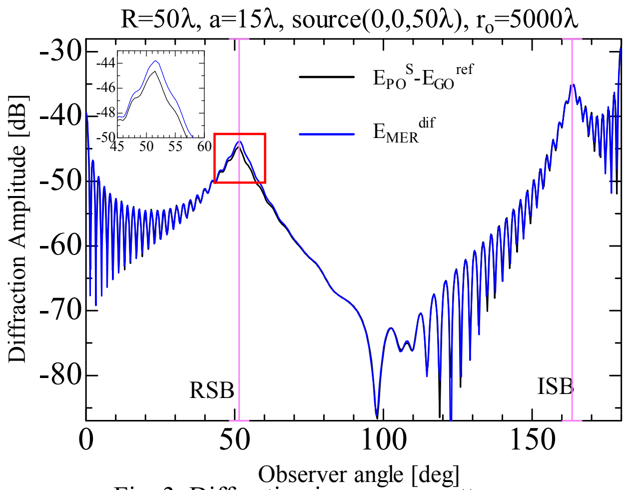

Fig. 3. Diffraction in convex scatterer case.

IV. Conclusion

The diffraction components calculated by PO and MER have been compared for convex scatterer illuminated by a point source. The errors of MER surface-to-line integral reduction in diffraction patterns, are localized in the vicinity of RSB. They vanish for flat scatterers and appear only for the curved surfaces; it has been demonstrated that the errors are generally smaller for the concave surfaces than those for the convex ones. This paper provides the correction with higher order for MER and succeeds in enhancing the accuracy of surface-to-line integral reduction by MER.

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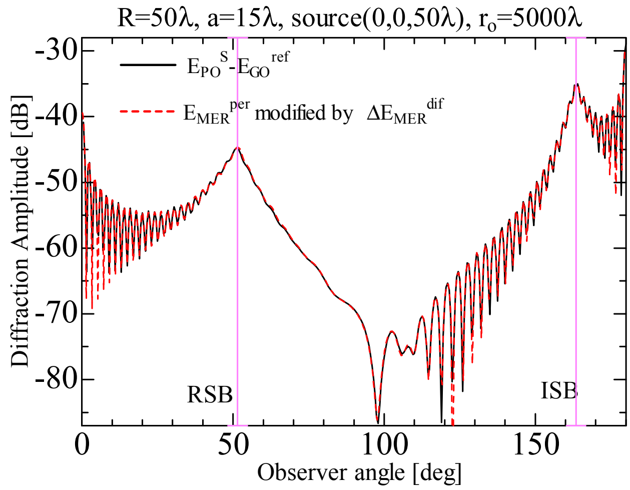

Fig. 4. Diffraction in convex scatterer case with higher order correction.

$$\varphi_J = \frac{1}{\mathrm{jk}}\frac{\hat{\sigma}_{\min(\tau)}\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}_{\max(\tau)}\right]^2 - \hat{\sigma}_{\max(\tau)}\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}_{\min(\tau)}\right]^2}{\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}_{\min(\tau)}\right]^2\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}_{\max(\tau)}\right]^2}$$
$$\left\{\frac{\left(\hat{\mathrm{r}}_{\mathrm{o}}\cdot\text{-}\mathrm{J}_{\mathrm{PO}}\right)\left(\hat{\mathrm{r}}_{\mathrm{o}}\cdot\hat{\tau}\right)-\left(\hat{\mathrm{r}}_{\mathrm{o}}\cdot\hat{\tau}\right)^2\left(\text{-}\mathrm{J}_{\mathrm{PO}}\cdot\hat{\tau}\right)}{\left[1-\left(\hat{\mathrm{r}}_{\mathrm{o}}\times\hat{\tau}\right)^2\right]}+\frac{\left[\hat{\mathrm{r}}_{\mathrm{o}}\times\left(\hat{\mathrm{r}}_{\mathrm{o}}\times\text{-}\mathrm{J}_{\mathrm{PO}}\right)\right]\cdot\hat{\tau}}{\left[1-\left(\hat{\mathrm{r}}_{\mathrm{o}}\times\hat{\tau}\right)^2\right]}+\frac{4\left(\hat{\mathrm{r}}_{\mathrm{o}}\cdot\hat{\tau}\right)\left[\hat{\mathrm{r}}_{\mathrm{o}}\times\left(\hat{\mathrm{r}}_{\mathrm{o}}\times\text{-}\mathrm{J}_{\mathrm{PO}}\right)\right]\cdot\hat{\tau}}{\left[1-\left(\hat{\mathrm{r}}_{\mathrm{o}}\times\hat{\tau}\right)^2\right]^2}\right\}$$
$$\phi_J = -\frac{1}{\mathrm{jk}}\frac{\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}_{\max(\tau)}\right]^3-\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}_{\min(\tau)}\right]^3}{\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}_{\min(\tau)}\right]^3\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}_{\max(\tau)}\right]^3}\left\{\frac{\text{-}\left(\hat{\mathrm{r}}_{\mathrm{o}}\cdot\hat{\sigma}\right)^2\mathrm{J}_{\mathrm{PO}}\cdot\hat{\tau}}{\left[1-\left(\hat{\mathrm{r}}_{\mathrm{o}}\cdot\hat{\tau}\right)^2\right]}+\frac{2\left[\hat{\mathrm{r}}_{\mathrm{o}}\times\left(\hat{\mathrm{r}}_{\mathrm{o}}\times\text{-}\mathrm{J}_{\mathrm{PO}}\right)\right]\cdot\hat{\tau}}{\left[1-\left(\hat{\mathrm{r}}_{\mathrm{o}}\cdot\hat{\tau}\right)^2\right]}\right\}\hat{\tau} \tag{6}$$

$$\varphi_M = \frac{1}{\mathrm{jk}}\frac{\hat{\sigma}_{\min(\tau)}\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}_{\max(\tau)}\right]^2-\hat{\sigma}_{\max(\tau)}\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}_{\min(\tau)}\right]^2}{\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\times\hat{\sigma}_{\min(\tau)}\right]^2\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}_{\max(\tau)}\right]^2}\left\{\frac{\left(\hat{\mathrm{r}}_{\mathrm{o}}\times\mathrm{J}_{\mathrm{PO}}\right)\cdot\hat{\tau}}{\left[1-\left(\hat{\mathrm{r}}_{\mathrm{o}}\cdot\hat{\tau}\right)^2\right]}+\frac{4\left(\hat{\mathrm{r}}_{\mathrm{o}}\cdot\hat{\tau}\right)\left(\hat{\mathrm{r}}_{\mathrm{o}}\times\mathrm{J}_{\mathrm{PO}}\right)\cdot\hat{\tau}}{\left[1-\left(\hat{\mathrm{r}}_{\mathrm{o}}\cdot\hat{\tau}\right)^2\right]^2}\right\}$$
$$\phi_M = -\frac{1}{\mathrm{jk}}\frac{\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}_{\max(\tau)}\right]^2-\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}_{\min(\tau)}\right]^2}{\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}_{\min(\tau)}\right]^2\left[\left(\hat{\mathrm{r}}_{\mathrm{i}}+\hat{\mathrm{r}}_{\mathrm{o}}\right)\cdot\hat{\sigma}_{\max(\tau)}\right]^2}\left\{\frac{\left(\mathrm{J}_{\mathrm{PO}}\cdot\hat{\sigma}\right)\left(\hat{\mathrm{r}}_{\mathrm{o}}\cdot\hat{\mathrm{n}}\right)}{\left[1-\left(\hat{\mathrm{r}}_{\mathrm{o}}\cdot\hat{\tau}\right)^2\right]}+\frac{2\left(\hat{\mathrm{r}}_{\mathrm{o}}\times\mathrm{J}_{\mathrm{PO}}\right)\cdot\hat{\tau}}{\left[1-\left(\hat{\mathrm{r}}_{\mathrm{o}}\cdot\hat{\tau}\right)^2\right]}\right\}\hat{\tau} \tag{7}$$

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