A Modified Virtual Array ESPRIT Algorithm Based on Real Signal Subspace

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Abstract - **A new ESPRIT method based on virtual array in the presence of coherent signals is presented. Using the received dates and its conjugate information, we reconstructed array data matrix. To reduce the complexity of algorithm, a real signal subspace can be obtained by a unitary transformation. Using the modified U-VC-ESPRIT which has small amount of calculation, we estimated DOA of non-circular coherent signal with arbitrary phase without array aperture reduction and effective element loss. The simulation results verified the effectiveness and superiority of the algorithm.**

Index Terms — DOA, Virtual Array, Unitary ESPRIT, coherent signals

I. INTRODUCTION

Direction of arrival (DOA) estimation plays an important role in sonar, radar and communications. The non-circular incoming signals such as MASK, BPSK, and AM are widely used in the satellite systems and modern communication systems.

Ref.[1] proposed a C-ESPRIT method to estimate DOA in 2004. The method constructed the two sub-arrays which have the maximum elements N by increasing the virtual sensors to estimate DOA of N non-coherent signals using a uniform linear array composed of N sensors. Then based on the ideas of Ref.[1], the researchers proposed some MUSIC methods [2-7] to estimate the DOA of coherent signals . Ref.[8] formes a special data matrix using the conception of C-ESPRIT. The method not only improves the estimation accuracy, but also can detect N-1 coherent signals without aperture loss. However, in 2007 Ref.[9] queries Ref.[1]. He points out that the C-ESPRIT was derived from a rather unrealistic assumption that all the signals should have the same phase shifts. And also He points that this situation has little utility in practice.

This paper proposed a U-VC-Esprit method. By modify the virtual array, the proposed method realized DOA estimation of non circular signals under arbitrary phase shift without array aperture reduction. And in order to further reduce the workload, the method changes the complex filed into real
filed by utilizing a unitary transformation matrix.

II. PROPOSED ALGORITHM

Virtual array can be understood as a virtual extension of the actual array that the first sensor as reference, that is showed as Fig.1.

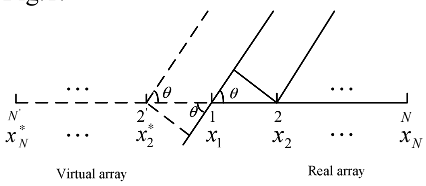

Fig. 1 Virtual array

Assuming the first sensor of array receives the K signal is $s_{1k} = e^{-jwt}$ at the t time, then the 2 sensor receives the K signal is $s_{2k} = e^{-jw(t-\frac{2\pi d\cos\theta}{\lambda})}$. So the virtual 2 sensor receives the K signal is $s'_{2k} = e^{-jw(t+\frac{2\pi d\cos\theta}{\lambda})}$. In the Ref. [1], the C-ESPRIT method should assume that $s'_{2k} = s^*_{2k}$, where

$e^{-jw(t+\frac{2\pi d\cos\theta}{\lambda})} = (e^{-jw(t-\frac{2\pi d\cos\theta}{\lambda})})^*$,

$e^{-jwt}e^{-jw(\frac{2\pi d\cos\theta}{\lambda})} = e^{jwt}e^{-jw(\frac{2\pi d\cos\theta}{\lambda})}, e^{jwt} = e^{-jwt}$.

It is clear that this assumption is completely unrealistic.

A. virtual array constructed

ESPRIT stands for Estimation of Signal Parameters via Rotational Invariance Techniques which required two subarrays. The two subarray must be same and the distance between the two subarrays must be known. When the signal does not satisfy the real time domain features, that is $s'_{2k} = s^*_{2k}$, a virtual array that is different from Fig.1 is constructed.

(example for 6 sensors of array)

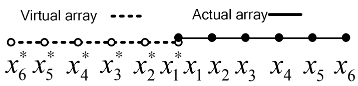

Fig.2 Modified Virtual Array

To estimate DOA of coherent signals, the two subarrays is constructed as Fig.3 . We can get from Fig.3 that the two subarrays have the same construction and the distance between the two subarrays is fixed that is distance of two sensors.

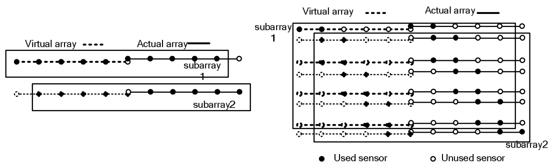

Fig.3 Modified Virtual Sub Array

B. DOA estimation of real signal subspace

The ESPRIT decomposes the covariance matrix to get the signal subspace or noise subspace. But those processing are all in complex domain. We can reduce the workload by utilizing a unitary transformation transform complex domain to real domain.

A $N \times N$ unitary transformation U is defined which can transform covariance matrix of complex domain R_w to covariance matrix of real domain PR.

$$PR = U^H R_w U \qquad (1)$$

when the dimension of R_w is even, the unitary matrix U can be written as

$$U = \frac{1}{\sqrt{2}} \begin{bmatrix} I & jI \\ J & -jI \end{bmatrix} \qquad (2).$$

when the dimension of R_w is odd, the unitary matrix U can be written as

$$U = \frac{1}{\sqrt{2}} \begin{bmatrix} I & 0 & jI \\ \overline{0} & \sqrt{2} & \overline{0} \\ J & 0 & -jJ \end{bmatrix} \qquad (3)$$

where J represent the exchange matrix with one's in its anti-diagonal and zeros elsewhere, I represent the exchange matrix with one's in its diagonal and zeros elsewhere.

In Eq.(2), J and I are exchange and identity matrix with dimension $N/2 \times N/2$ .In Eq. (3), J and I are exchange and identity matrix with dimension $((N-1)/2) \times ((N-1)/2)$.

then we can use TLS-ESPRIT to estimate DOA of signal.

III. Simulation Result

In this section, the performance of the proposed method is compared with C-ESPRIT proposed in Ref.[8]. The uniform linear array consists of 4 sensors. We assume that two coherent signals are received with elevation DOAs at [30º, -45º]. The initial phase of the signal with 30º is 0, and the initial phase of the signal with-45º is not 0.

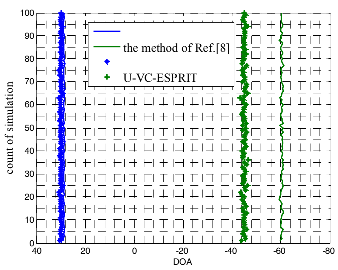

Fig.4 DOA estimation

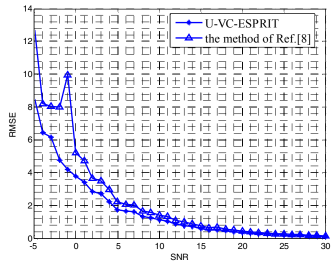

Fig.5 RMSE of DOA estimation

For Fig.4, SNR value is 20dB.We calculate 100 Monte Carlo with only one snapshot. From Fig.4,It is clear that even with a single snapshot, U-VC-ESPRIT can detect precisely the signals direction of arrival whether the initial phase is 0 or not. The method of Ref.[8] can only detect the signal with the initial phase is 0.

For Fig.5 the same parameters as Fig. 4 are assumed. The only difference is that the SNR value changes from -5dB to 30dB. From Fig.5, it is clear that under lower SNR, the RMSE of U-VC-ESPRIT is better than the Ref.[8] method, and with the SNR increasing , the result of RMSE tend to smooth, but the performance of U-VC-ESPRIT is better.

IV. Conclusion

This paper proposed a new de-coherent ESPRIT method based on reconstruction virtual array. The proposed method can effectively estimate DOA of non circular signals with arbitrary initial phase. And this method utilizes the unitary transformation to reduce the workload without scarifying the accuracy of DOA estimation.

Acknowledgment

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