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Asymptotic Formulations for the Effective Analysis of EM Scattering from Array Structures for the Propagation Mechanism Interpretation

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Abstract – **This paper presents the asymptotic formulation to analyze the EM scattering from periodic array structure. The focus is to provide the interpretation mechanism for the wave propagation as previously performed by the conventional UTD. Numerical examinations will be presented to demonstrate the characteristics and accuracy of the solution.**

Index Terms — **Periodic array structure, Asymptotic Analysis, EM scattering, Propagation mechanisms.**

I. Introduction

In this paper we present the analysis of electromagnetic (EM) scattering from periodic array structures, such as selective surface (FSS) [1] and reflectarray/transmitarray antennas [2],[3]. In particular, we focus on creating an UTD-type formulation [4],[5] such that the scattering mechanisms can be interpreted by the diffraction mechanisms, which allows one to better understand the physical characteristics.

In the investigation, the scattering fields are first assumed to be from the radiation of equivalent currents, and result in a summation of radiations from the array elements. This summation is then transferred into Floquet modes via the utilization of Poisson sum formula, where each Floquet mode is a form of radiation integral. The asymptotic techniques are then applied to decompose the integrations into the components of direct reflections from a corresponding infinite array and diffraction effects due to the truncations. Physical interpretation is then performed based on these reflected and diffracted field components. Numerical examples will be presented to demonstrate the characteristics.

II. Problem Description

Fig. 1 illustrates the EM scattering problem from a periodic array structure, where the array element is assumed to be within the unit cell on the figure. The structure within the unit cell may be different, and allows one to treat the scattering problems of reflectarray and transmitarray antennas. It is assumed that the EM mutual coupling within the unit cell can be analyzed by low-frequency numerical techniques over an infinite array of same elements, and results in the equivalent currents (both magnetic and electric currents) on the boundary of this array with free space, where the incident field is assumed to be plane wave approximated by the local wave phenomena over the selected element.

Thus for a cylindrical wave illumination in 2-D scattering problems, the localized equivalent currents can be used as an alternative to equivalently radiate the scattering fields. The fields can be expressed

$$u_s(\bar{\rho}) = \left(\frac{I_0 d_x}{8j\pi k}\right)\sum_{n=-\infty}^{N}\frac{e^{-jk\ell_n}e^{-jk\rho_n}e^{j\phi_n^c}}{\sqrt{\ell_n}\sqrt{\rho_n}}G_n(k_{r,x}^n - k_{f,x}^n) \qquad (1)$$

where the parameters of distances are shown on Fig. 1, and I_0 is used as the reference amplitude of line source. The $G_n(\xi)$ represents the radiation pattern of the n^{th} element, and the $\phi_n(x') = \phi_n^c + \phi_n^\ell(x')$ with ϕ_n^c and $\phi_n^\ell(x')$ being the phase at the element's center and local phase distribution along the element, respectively.

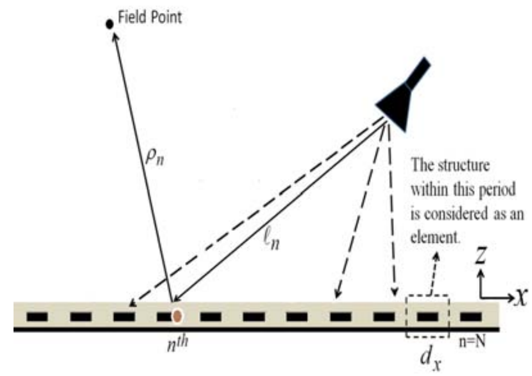

Fig. 1: Illustration of scattering from periodic array structures.

This formulation builds up the fundamentals of the scattering from a semi-infinite array, which allows one to consider the truncation effects. The asymptotic technique analysis based on the Floquet mode decomposition will be able to provide the scattering mechanism.

III. The Asymptotic Analysis of the Floquent Modes

After applying the Poisson sum formula over (1) will produce an infinite series of Floquet modes, where each mode is a form of radiation integral. The asymptotic technique is applied to formulate the solution in the following form:

$$u_s^p(\bar{\rho}) = u_{dir}^p(\bar{\rho})\cdot U(\mathrm{Re}(x_e - x_s)) + u_{end}^p(\bar{\rho})F(ka), \qquad (2)$$

where $U(\cdot)$ is the Heaviside step function and $F(\cdot)$ [4],[5] is the UTD Fresnel transition function to assure the uniform field distribution when the field point is across the shadow boundary of incident/reflected fields. In (2), $u_{dir}^{p}(\overline{\rho})$ is the asymptotic solution of radiation when the size of array is extended to infinity while $u_{end}^{p}(\overline{\rho})$ accounts for the effect of truncation. Both components arise from a radiation point, x_s on the array aperture, and a diffraction point, x_e on the edge, respectively. This formulation remains valid as the radiation point is close to the diffraction point, i.e. $x_s \to x_e$.

A. *Direct Scattering Fields of pth Floquet Mode*

$u_{dir}^{p}(\overline{\rho})$ in (2) is the scattering from an infinite, non-truncated array with $x_e \to \infty$. The asymptotic solution is given by

$$u_{dir}^{p}(\overline{\rho}) = u_f(\overline{\rho}_s^{p}) G_{v_s}(k_x^s - k_x^i) \frac{je^{-j\beta_p x_s}}{2k_z^s} \sqrt{\frac{\pm\rho_c}{\rho_v^s \pm \rho_c}} e^{-jk\rho_v^s} \qquad (2)$$

where the sub-/superscript "s" indicates that all parameters are found at a saddle point, $x_i = x_s$, on the array aperture. One may compare (2) with (1) to see the parameter changes. Here $u_f(\overline{\rho}_s^p)$ represents the incident field at the saddle point, (k_x^s, k_x^i) as well as k_z^s represents the scattering and incident directions in this mode. In (2), $\beta_p = 2\pi p / d_x$ is resulted from Floquet mode. The ray caustic distance ρ_c is obtained by the equivalent focus point, which will determine the selection of sign in (2) depending on whether it is focused in the real or virtual space.

B. *The Truncation Effects*

The general truncation diffraction effects in (2), $u_{end}^{p}(\overline{\rho})$, is represented by the following form:

$$u_{end}^{p}(\overline{\rho}) = u_f(\overline{\rho}_e^{p}) G_N(k_x^e - k_x^{ie}) \frac{je^{-j\beta_p x_e}}{2} \frac{e^{-jk\rho_e}}{\sqrt{\rho_e}} \cdot \left(\frac{e^{-j\pi/4}}{\sqrt{2k\pi}\left((k_x^{ie} - k_x^e) + \beta_p\right)} \right) \qquad (3)$$

where the sub-/superscript "e" indicates that all parameters are computed at the truncation point at x_e with $\overline{\rho}_e^p = (x_e, z_i)$. Similar to the definitions of parameters in (2) to interpret (k_x^e, k_x^{ie}) and other terms, it is recognized that the term inside the bracket is the diffraction coefficient for this truncation. The form of this formulation is similar to that of conventional UTD formulation. One is thus able to interpret the diffracted ray at the truncation.

IV. Numerical Example

One considers the scattering problem of reflectarray with the impression phases assumed to produce a near-field focused scattering field at $(0, 50\lambda)$. The fields are observed at $z = 100\lambda$. Fig. 2 shows the patterns compared with the reference solutions obtained by element-by-element summation. It is observed that the near-field focus reflect array results in a narrower main beam in the near zone. The comparison with the reference is observed to be very well.

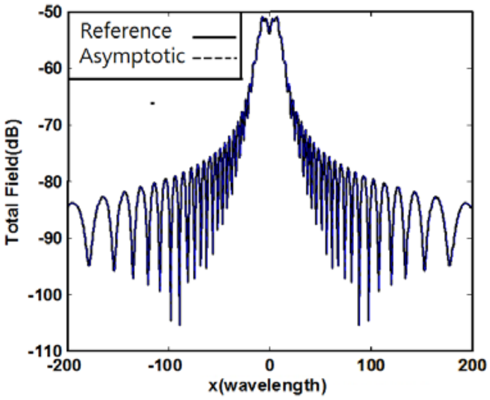

Fig. 2: Numerical examples to validate the accuracy of the developed asymptotic solutions for the reflectarray problems with elements phased to focus scattering fields in the near-zone.

V. Conclusion

The solutions presented in this paper are effectively capable of well interpreting the scattering mechanisms in the design of FSS, and reflectarray/transmitarray problems in terms of UTD mechanism. The future work will extend the concept to treat the 3-D scattering problem.

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References

[1]. B. A. Munk, *Frequency Selective Surfaces: Theory and Design*. New Yor: Wiley, 2000.

[2]. J. Huang and J. A. Encinar, *Reflectarray antennas*, 2007, Wiley-IEEE Press

[3]. Lau, J.Y.; Hum, S.V., "Analysis and Characterization of a Multipole Reconfigurable Transmitarray Element," *Antennas and Propagation, IEEE Transactions on* , vol.59, no.1, pp.70,79, Jan. 2011

[4]. Kouyoumjian, R.G.; Pathak, P.H., "A uniform geometrical theory of diffraction for an edge in a perfectly conducting surface," *Proceedings of the IEEE* , vol.62, no.11, pp.1448,1461, Nov. 1974

[5]. P. C. C1emmow, "Some extensions to the method of integration by steepest descents," *Quart. J Meek. Appl. Math.*. vol. 3, pp. 241-256,1950.

[6]. O. A. Civi, P. H. Pathak, and H. Chou, "On the poisson sum formula for the analysis of wave radiation and scattering from large finite arrays", *IEEE Trans. Antennas Propagat.*, vol. 47, pp.958 - 959 , 1999.