Wideband DOA Estimation Technique for Correlated Sources

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***Abstract* - Recently, researches on the wideband wireless system becomes important increasingly due to growth of the requirements on the high-speed broadband communications. Direction of arrival estimation of such a wideband sources with array also important for radio propagation estimation and/or surveillance. DOA estimation of wideband sources is one of the difficult problem because suitable element separation of the array essentially determined by the wavelength of the wave(s). In this report, we propose a new wideband DOA estimation technique that is available depending on the signal correlations.**

***Index Terms* — DOA estimation, Wideband signals, EM algorithm, Virtual array.**

I. Introduction

Broadband wireless era has come. There have been many reports on the wideband applications of the array antenna. Direction of arrival (DOA) estimation is one of the important applications. Suitable element separation of the array is essentially determined by the operational frequency, then the difficulty arises for the wideband sources. We have found that the wideband sources can be effectively modeled by unified sub-frequency band vectors [1]. It is a kind of virtual array transform by using Khatri-Rao matrix product [2]-[4]. However, the method in [1] has a problem for correlated waves. In this report, we propose a new algorithm which can be applied to the correlated waves. The proposed algorithm based on the Expectation-Maximization (EM) Algorithm estimates correlation terms as well as the original signals, hence robust estimation can be realized regardless of correlations. In this report, Computer simulation results are provided to show the validity of the algorithm.

II. Received Data Model and KR Transform

Let us assume that we observe K broadband incident waves having frequency components from f_1 to f_N with L element array. The received data can be written by

$$\boldsymbol{x}(t)=\begin{bmatrix}\boldsymbol{x}_1(t)\\ \boldsymbol{x}_2(t)\\ \vdots\\ \boldsymbol{x}_N(t)\end{bmatrix}=\sum_{k=1}^{K}\begin{bmatrix}\boldsymbol{a}(\theta_k,f_1)s_{1k}(t)\\ \boldsymbol{a}(\theta_k,f_2)s_{2k}(t)\\ \vdots\\ \boldsymbol{a}(\theta_k,f_N)s_{Nk}(t)\end{bmatrix}+\begin{bmatrix}\boldsymbol{n}_1(t)\\ \boldsymbol{n}_2(t)\\ \vdots\\ \boldsymbol{n}_N(t)\end{bmatrix}, \qquad (1)$$

where $\boldsymbol{x}_n(t)$ and $\boldsymbol{n}_n(t)$ denote L dimensional received data vector and noise vector at frequency of f_n, respectively. $s_{nk}(t)$ and $\boldsymbol{a}(\theta_k, f_n)$ denote complex amplitude of the k-th incident waves and its modevector at the frequency of f_n.

In the proposed wideband DOA estimation technique, we use a transformed data vector which can be derived by the matrix-vector transform for the correlation matrix of (1). Correlation of the difference frequency data becomes zero with sufficient snapshots. These zeros can be omitted. Furthermore, assuming that power at each frequency band of the wave is the same for every wave, that is $|s_{nk}(t)|^2=|s_{mk}(t)|^2$. In this case, we can simplify the transformed data vector into the extended data vector given by

$$\begin{aligned}\boldsymbol{x}_e&=\sum_{k=1}^{K}\left[\boldsymbol{a}_e(\theta_k,f_1)^T \quad \cdots \quad \boldsymbol{a}_e(\theta_k,f_N)^T\right]P_k+\boldsymbol{n}_e\\ &=\sum_{k=1}^{K}\boldsymbol{a}_e(\theta_k)P_k+\boldsymbol{n}_e,\end{aligned} \qquad (2)$$

where $\boldsymbol{a}_e(\theta_k, f_n)$ is the extended modevector of the k-th waves at frequency of f_n, and P_k is the power of the waves, and $\boldsymbol{n}_e$ is the additive noise vector. Note that the overall modevector $\boldsymbol{a}_e(\theta_k)$ in (2) is the NL dimensional vector, hence degrees-of-freedom of the array can be enhanced by the transformation. In this transformation, we also assume that all of the incident waves are uncorrelated. See the reference [2] for the details.

However, when correlated and/or coherent waves are included in (1), their correlation terms appear in (2), hence we can hardly apply the conventional DOA estimation technique such as the Beamformer, Capon, MUSIC, EM algorithm and so forth.

III. The Proposed Wideband DOA Estimation Technique

When there exist correlated waves, correlation coefficients among the waves appear in addition to the original unknowns in (2), that is DOAs and signal powers. The following is the simple example for two wave incidence.

$$\boldsymbol{x}_e=\boldsymbol{a}_e(\theta_1)P_1+\boldsymbol{a}_e(\theta_2)P_2+|\rho_{12}|\sqrt{P_1P_2}\,\boldsymbol{b}_e(\theta_1,\theta_2,\phi_{12})+\boldsymbol{n}_e, \qquad (3)$$

where ρ_{12} is the complex correlation between wave #1 and #2, and ϕ_{12} is the phase of the correlation. The vector $\boldsymbol{b}_e$ is the modevector of the correlation term, hence the (3) can be regarded by the 3 signal model as follows:

$$\boldsymbol{x}_e = \boldsymbol{a}_e(\theta_1)P_1 + \boldsymbol{a}_e(\theta_2)P_2 + \boldsymbol{b}_e(\theta_1,\theta_2,\phi_{12})P_{12} + \boldsymbol{n}_e, \quad (4)$$

The EM algorithm can be applied to this extended data vector assuming that the third correlation term is the third wave term having amplitude of P_{12} and modevector of $\boldsymbol{b}_e$. The number of signals to be estimated is increased in this case. However, we can still estimate all the unknowns with sufficient number of frequency bands. Since all the signal components as well as correlation terms can be estimated by this extended EM algorithm, DOA estimation accuracy with the extended data vector can be improved. Concept of this algorithm can also be reported in [3].

IV. Simulation Results

In this section, we evaluate performance of the proposed method by computer simulations. The simulation parameters used here are listed in Table.1. In this example, we adopt 4-elemet uniform linear array. Ten frequency band is used in this simulation. Figure 1 shows the Beamformer spectrums. The spectrums of dashed blue and green lines shows the spectrums for narrow band, or single frequency (single) frequency, signal at the frequency of 300 MHz and 1300 MHz, respectively. Since the array length is relative small at 300 MHz, the incident waves cannot resolved at all. The signal peaks can be resolved at 1300 MHz, but we cannot distinguish the real and spurious peaks among them. On the other hand, the signal peaks can be correctly resolved by the proposed technique as shown by the red line in Fig. 1. Figure 2 shows the estimated RMSE for the waves with the 100 trials. In this simulation, all of the waves have the same correlations with each other. The blue-line for the "EM algorithm" shows the results of the direct application of the EM algorithm to the data. As discussed in the previous section, error becomes larger as the correlation increases. The green line for the "EM algorithm w/ *N*-th root preprocessing" is the results with correlation term suppression algorithm in [2]. This preprocessing is effective to a certain extent. However DOA estimation bias still remains. On the other hand, as shown in the red line in Fig.1, bias due to the wave correlation can be almost suppressed by the proposed algorithm [5].

V. Conclusion

In this report, we propose a new DOA estimation technique for the wideband correlated sources by the EM algorithm and verify performance of the algorithm by computer simulations. Experimental study for the actual wideband sources such as OFDM signals will be done in near future.

Acknowledgment

This work was partly supported by JSPS KAKENHI Grant Number 26420346, and research grant by SCAT, Japan.

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Table.1 Simulation Parameters

Array arrangement	Uniform Linear Array (ULA)
Number of elements	4
Element separation	50 cm (λ /2 at 300 MHz)
Frequencies	300-1300 MHz in every 100 MHz(N=10)
SNR	20 dB
Number of snapshots	1000
Number of trials	100

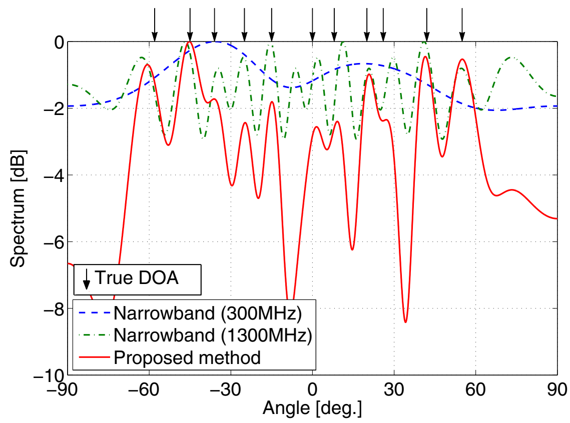

Fig.1 DOA estimation results by Beamformer with conventional narrowband arrays and proposed wideband array. K=11, (θ_1, θ_2, θ_3, θ_4, θ_5, θ_6, θ_7, θ_8, θ_9, θ_{10}, θ_{11})=(-58, -45, -36, -25, -15, 0, 8, 20, 26, 42, 55 deg.), no signal correlation.

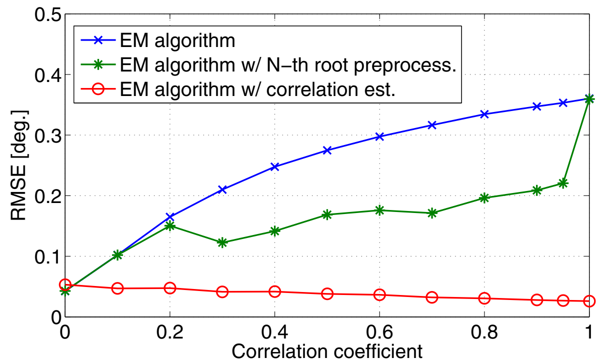

Fig. 2 DOA estimation error in each wave correlation. K=5, (θ_1, θ_2, θ_3, θ_4, θ_5)=(-54, -32, 0, 23, 40 deg)