A Fast Calculation Method of 2-Dimensional MUSIC for Simultaneous Estimation of DOA and Frequency

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***Abstract* – A fast calculation method of 2-dimensional MUSIC, which can estimate two parameters such as DOA and frequency, is proposed. In the proposed method, 2-dimensional search calculation is substituted with FFT (Fast Fourier Transformation) and convergent calculation. Simulation results showed the proposed method is approximately twenty times faster than the conventional method.**

***Index Terms* — MUSIC, DOA, frequency, FFT, 2-dimension.**

I. Introduction

MUSIC[1] is an useful method to enhance resolution in the estimation of DOA, frequency, time delay and so on. However, its computational load is slightly large, especially this weak point becomes a quite serious problem when MUSIC method is extended to 2-dimensional processing to apply such as simultaneous estimation of DOA and frequency[2].

In this report, we propose a fast calculation method of 2-dimensional MUSIC in which heavy search calculation is substituted with rough search calculation using FFT and precise convergent calculation using Newton's iterative method.

II. Proposed Fast 2D-MUSIC method

The estimate function of conventional 2D-MUSIC is expressed as

$$P_{2DMU}(\theta,\omega) = \frac{1}{\mathbf{d}^H(\theta,\omega)\mathbf{E}_N\mathbf{E}_N^H\mathbf{d}(\theta,\omega)}, \qquad (1)$$

where θ and ω are direction angle and normalized angle frequency of the incident signals, $\mathbf{d}(\theta,\omega)$ is the mode vector, $\mathbf{E}_N$ is the noise eigen vector matrix. The noise eigen vector matrix is a set of noise eigen vectors calculated from the received signal covariance matrix.

$$\mathbf{E}_N = [\mathbf{e}_{R+1}, \mathbf{e}_{R+2}, \cdots, \mathbf{e}_{QL}]. \qquad (2)$$

In the above equation, R is number of incident signals, L is number of elements of array antenna, and Q is maximum time shift in the received signal covariance matrix for frequency estimation.

The mode vector for simultaneous estimation of DOA and frequency is expressed as

$$\mathbf{d}(\theta,\omega) = \begin{bmatrix} \mathbf{a}(\theta) \\ \mathbf{a}(\theta)e^{-j\omega} \\ \mathbf{a}(\theta)e^{-2j\omega} \\ \vdots \\ \mathbf{a}(\theta)e^{-(Q-1)j\omega} \end{bmatrix}, \qquad (3)$$

where $\mathbf{a}(\theta)$ is the steering vector. Exponential terms multiplying to the steering vectors are required in order to estimate frequency.

In the conventional MUSIC, the estimation of DOA and frequency is executed by calculating Eq. (1) with desired frequency step resolution and desired DOA step resolution and finding peaks of the function. However, this 2-dimensional search calculation requires much calculation power. This fact has regulated practical use of multi-dimensional MUSIC.

In the proposed method, steering vector is expanded into Fourier series.

$$a_l(\theta) = \sum_{m=-M}^{M} b_{l,m} z^m, \qquad (4)$$

or matrix form is written as

$$\mathbf{a}(\theta) = \mathbf{B}^H \mathbf{z} \qquad (5)$$

where $\mathbf{B}$ and $\mathbf{z}$ are the matrix and the vector shown as below.

$$\mathbf{B}^H = \begin{bmatrix} b_{1,-M} & b_{1,-M+1} & \cdots & b_{1,M-1} & b_{1,M} \\ b_{2,-M} & b_{2,-M+1} & \cdots & b_{2,M-1} & b_{2,M} \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ b_{L,-M} & b_{L,-M+1} & \cdots & b_{L,M-1} & b_{L,M} \end{bmatrix} \qquad (6)$$

$$\mathbf{z} = [z^{-M}, z^{-M+1}, \cdots, z^{M-1}, z^{M}]^T \qquad (7)$$

where $z \equiv e^{-j\theta}$. In the above equation, M is number of terms of Fourier series.

Substituting Eq. (5) into Eq. (3), the mode vector is written as

$$\mathbf{d}(\theta,\omega) = \begin{bmatrix} \mathbf{B}^H \mathbf{z} w^0 \\ \mathbf{B}^H \mathbf{z} w^1 \\ \mathbf{B}^H \mathbf{z} w^2 \\ \vdots \\ \mathbf{B}^H \mathbf{z} w^{Q-1} \end{bmatrix}, \qquad (8)$$

$$= \mathbf{B}'^H \mathbf{Z} \mathbf{w}$$

where $\mathbf{B}'$ and $\mathbf{Z}$ are matrixes shown as below,

$$\mathbf{B}' = \begin{bmatrix} \mathbf{B} & & & \mathbf{0} \\ & \mathbf{B} & & \\ & & \ddots & \\ \mathbf{0} & & & \mathbf{B} \end{bmatrix} \qquad (9)$$

$$\mathbf{Z} = \begin{bmatrix} \mathbf{z} & & & \mathbf{0} \\ & \mathbf{z} & & \\ & & \ddots & \\ \mathbf{0} & & & \mathbf{z} \end{bmatrix} \qquad (10)$$

and $\mathbf{w}$ is a vector shown as

$$\mathbf{w} = [w^0, w^1, \cdots, w^{Q-1}]^T \qquad (11)$$

where $w \equiv e^{-j\omega}$.

Substituting Eq.(8) into the denominator of Eq.(1), the new estimate function has been obtained.

$$P(z,w) = \mathbf{w}^H \mathbf{Z}^H \mathbf{B}' \mathbf{E}_N \mathbf{E}_N^H \mathbf{B}'^H \mathbf{Z} \mathbf{w} \qquad (12)$$

The above equation is definitely the polynomial function of z and w, so that it can be rewritten as the below.

$$P(z,w) = \tilde{\mathbf{z}}^T \mathbf{G} \tilde{\mathbf{w}} \qquad (13)$$

where $\tilde{\mathbf{w}}$ and $\tilde{\mathbf{z}}$ are the

$$\tilde{\mathbf{w}} = [w^{-Q+1}, w^{-Q+2}, \cdots, w^{Q-2}, w^{Q-1}]^T \qquad (14)$$

$$\tilde{\mathbf{z}} = [z^{-2M}, z^{-2M+1}, \cdots, z^{2M-1}, z^{2M}]^T \qquad (15)$$

and $\mathbf{G}$ is the coefficient matrix calculated from $\mathbf{B}'$ and $\mathbf{E}_N$.

Eq.(13) becomes minimal value at the correct DOA and frequency to estimate. The form of Eq.(13) enables use of FFT in calculating Eq.(13) to find minima of the function.

The form of Eq.(13) provides another merit that the convergent calculation method, such as Newton's method, can be used to calculate precise minima, since Eq.(13) is continuous and differentiable function of z and w. The use of convergent calculation minimizes the number of FFT points and contributes to more reduction of calculation.

The practical convergent calculation should be executed in the θ - ω space instead of z-w space, since z and w are complex number and search dimension becomes larger than the θ - ω space.

III. Simulation Results

The estimation error and calculation time was evaluated by Monte Carlo simulation with 100 times iteration. The summary of simulation condition is shown in Table I.

The results are shown in Table II. It shows estimation errors of proposed method are equal or less than conventional method.

Calculation time of 100 times iteration of the proposed method was 17.8 sec, whereas that of conventional method was 298.8 sec.

TABLE I
Summary of Simulation Condition

Antenna	5-element uniform circular array, the diameter is 2 times of wave length.
Incident signals	Number of signals is 3, SNR of each signal is 10dB.

TABLE II
Estimation Errors of DOA and Frequency

		Conventional method	Proposed method
DOA error (rms)	First wave	1.681 deg	1.668 deg
	Second wave	1.770 deg	1.777 deg
	Third wave	0.617 deg	0.586 deg
Frequency error (rms)	First wave	1.881 Hz	1.824 Hz
	Second wave	2.089 Hz	2.088 Hz
	Third wave	0.583 Hz	0.441 Hz

IV. Conclusion

A fast calculation method of 2D-MUSIC is proposed. In the proposed method, 2-dimensional search calculation is substituted with FFT and convergent calculation. Simulation results showed the proposed method is approximately twenty times faster than the conventional method.

References

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