Analysis of Reflection Characteristics of Periodic Structures with Oblique Incidence by using ARMA-FDTD Method

Takuji Arima, Kazuki Ogawa and Toru Uno
Faculty of Engineering, TOKYO UNIVERSITY OF AGRICULTURE AND TECHNOLOGY, NAKAMACHI 2-24-16, KOGANEI-SHI, TOKYO, 184-8588, JAPAN

***Abstract* – In recent years, metamaterials are widely developed to enhance wireless applications. Meta-surfaces such as EBG (Electromagnetic Band Gap) structure and FSS (Frequency Selective surface), are one of the metamaterials. In order to use the meta-surfaces near the antenna, reflection characteristics for oblique incidence is important. On the other hand, the metamaterials are realized by using periodic structures. In this paper, reflection characteristics of Periodic Structures with oblique incidence are investigated by using ARMA-FDTD method. The results show the resonant frequency of structures is becoming higher with incident angles.**

***Index Terms* — Metamaterials, Periodic structures, FDTD method, ARMA.**

I. Introduction

In recent years, new materials for EM fields are widely developed. These materials are called as "Metamaterial" or "Meta-surface"[1][2]. Meta-surfaces such as EBG structures and AMC (Artificial Magnetic Conductor) are widely applied for antenna engineering to enhance characteristics. These materials are realized by periodic structures. On the other hand, in order to use the meta-surfaces near the antenna, reflection characteristics for oblique incidence is important characteristics. Therefore, analyzing techniques including oblique incidence characteristics for periodic structures are required to develop high performance meta-surface. The FDTD method and the Method of Moment (MoM) are widely used to analyze periodic structures. The FDTD method has high capability of modeling due to simple algorithm. Therefore, the FDTD method can be analyze complex structures easily. In order to analyze periodic structures by the FDTD method, periodic boundary conditions are applied. A lot of periodic conditions are proposed[3]. The constant-*k* method is one of the effective methods to realize periodic conditions in FDTD method. However, the electric and magnetic fields are not converged easily in FDTD method with periodic conditions for oblique incident case. Therefore, long calculation time is required the oblique incident case by the FDTD method with periodic condition. In order to analyze periodic structures with oblique incident by FDTD method effectively, ARMA (Autoregressive Moving Average Model) is utilized into FDTD method[4][5]. In this paper, reflection characteristics of periodic structures with oblique incidence are analyzed by using ARMA-FDTD method. The accuracy of this method is also confirmed by comparing wis MoM for oblique incident case. The results show resonant frequency of structures is becoming higher with incident angles. The ARMA-FDTD method will be helpful to develop wide angle meta-surface.

II. Oblique Incidence Analysis in ARMA-FDTD Method

In this section, oblique incidence analysis of periodic structures by ARMA-FDTD method is mentioned briefly.

Figure 1 shows analysis model. In this model, oblique incident wave plane is placed above the structures as shown in Fig.1. In order to obtain reflection coefficient of the structures, periodic, observation plane is placed as shown in Fig.1. The structure surface is used as a reference plane of reflection coefficient. On the other hand, observation plane is placed at small distance away from the structure surface in the FDTD method. Therefore, phase shift due to small distance of the structures surface should be corrected.

Next, the ARMA technique in the FDTD method is mentioned. In the oblique incident case, the electric and magnetic fields are not converged in time domain. ARMA technique is useful method to estimate converged electric and magnetic field value. In ARMA analysis, function $H(z)$ that is linear model [6] is used. The transfer function $H(z)$ is expressed by Eq.(1).

$$H(z)=\frac{a_0+a_1Z^{-1}+a_2Z^{-2}+\cdots+a_qZ^{-q}}{1+b_1Z^{-1}+b_2Z^{-2}+\cdots+b_pZ^{-p}} \quad (1)$$

Where z is complex variable, a and b are unknown coefficients. To obtain transfer function $H(z)$, unknown coefficients a and b should be obtained. The transfer function $H(z)$ can also be expressed in the time domain as

$$y(n)=-\sum_{i=1}^{p}b_i y(n-i)+\sum_{j=0}^{q}a_j x(n-j) \quad (2)$$

Where $x(n)$ and $y(n)$ are input and output signals of the linear system. b_i and a_j are ARMA coefficients to be determined. In the reflection coefficient analysis, y(n) should be reflection electric fields and x(n) should be electric fields without structures. In order to obtain b_i and a_j, N th samples are required in the FDTD calculation. Eq.(2) can be expressed linear equation system as follows.

$$[Y]_{N\times 1}=[D]_{N\times(p+q+1)}[C]_{(p+q+1)\times 1} \quad (3)$$

Where [D] is indicated as

$$[D]=\begin{bmatrix} 0 & 0 & \cdots & 0 & x(1) & 0 & \cdots & 0 \\ -y(1) & 0 & \cdots & 0 & x(2) & x(1) & \cdots & 0 \\ \vdots & \ddots & \ddots & 0 & \vdots & \ddots & \ddots & 0 \\ -y(p) & -y(p-1) & \cdots & -y(1) & x(p+1) & x(p) & \cdots & x(p+1-q) \\ -y(p+1) & -y(p) & \cdots & -y(2) & x(p+2) & x(p+1) & \cdots & x(p+2-q) \\ \vdots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \ddots \\ -y(N-1) & -y(N-2) & \cdots & -y(N-p) & x(N) & x(N-1) & \cdots & x(N-q) \end{bmatrix} \in R^{N\times(p+q+1)} \quad (4)$$

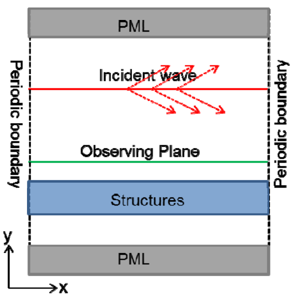
$[C]$ is the unknown coefficient to be determined and the number of unknowns is $p+q+1$. The coefficients can be determined using the least mean square error estimation.

$$[C] = \left([D]^T [D]\right)^{-1} [D]^T [Y] \qquad (5)$$

Once $[C]$ is determined, the frequency domain transfer function $H(j\omega)$ can be obtained from Eq.(1) by replacing Z with $\exp(j\omega)$.

III. Results

In this section, in order to investigate reflection characteristics of periodic structures with oblique incident, two periodic structures as shown in Fig.2(a)(b) are calculated. Fig.2(a) is FSS structure and Fig.2(b) is EBG structured.



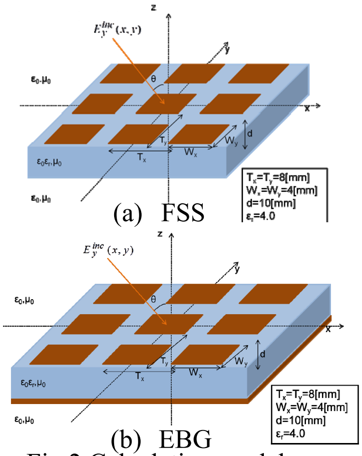


Fig.1 Calculation model Fig.2 Calculation models

Parameters of the structures are indicated in the figures. The FDTD cell size is set as 0.3 mm for FSS calculation, 0.2 mm for EBG calculation for each direction. The observing plane is set at 15 cells away from the structure and incident wave is put at 60 cells away from the structures.

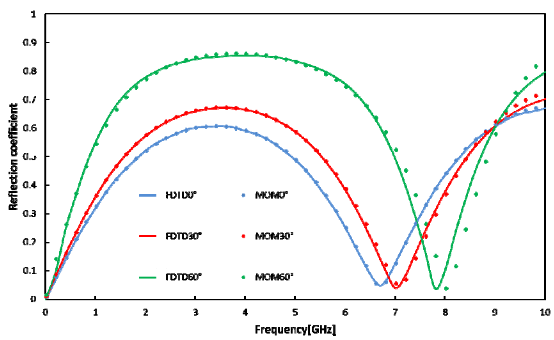


Fig.3 Reflection coefficient of FSS structures

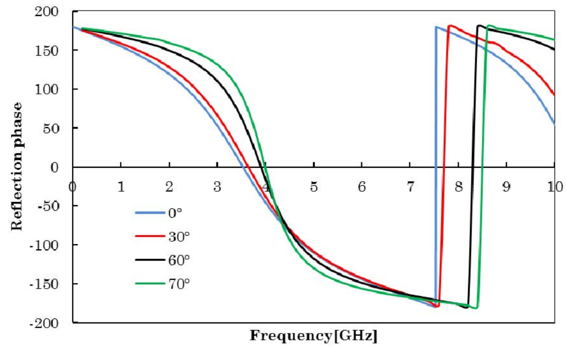


Fig.4 Reflection phase of EBG structures

Figures 3 and 4 show calculated reflection characteristics of FSS and EBG structures. In these calculation, number of p and p of eq.(1) is set as p=q=40. From Fig.3, resonant frequency is shifted to high frequency with increasing incident angle. Furthermore, complex characteristics are observed at high incident angle case after first resonant. Furthermore, ARMA-FDTD method has high accuracy comparing with MoM. Figure 4 shows reflection phase of EBG structure. This result shows same future as FSS case, because, AMC frequency (this means phase=0) is shifted to high frequency with increasing incident angle.

IV. Conclusion

In this research, reflection coefficient of FSS structures with oblique incident are investigate by using ARMA-FDTD method. From results, resonant frequency is shifted to high frequency with increasing incident angle in both FSS and EBG calculation. Furthermore, complex characteristics are observed at high incident angle case after first resonant. We will develop wide angle structures by using our developed method.

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