Time-Dependent Pricing for Revenue Maximization of Network Service Providers Considering Users Preference

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***Abstract*—Due to network users' different time-preference, network traffic load differs significantly at different time. In traffic-peak time, the quality of service provided to network users may deteriorate due to congestion. There are two ways to improve the quality of services: (1) Network service providers (NSPs) over-provision network capacity by investment; (2) NSPs use pricing to reduce the traffic at traffic-peak time by exploiting the elasticity of demand with respect to price. However, over-provisioning network capacity can be costly. Therefore, some researchers have proposed time-dependent pricing to control congestion as well as improve the revenue of NSP. To the best of our knowledge, all of the literature related to time-dependent pricing scheme only considers the monopoly NSP case. In this paper, a duopoly NSP case is studied. The NSPs try to maximize their overall revenue by setting time-dependent price, while users choose NSP by considering their own preference, congestion status in the networks, the price set by the NSPs and the switching cost set by NSPs. Analytical and experimental results show that the time-dependent pricing (TDP) benefits the NSPs, but the revenue improvement is limited due to the competition effect.**

I. Introduction

The huge growth in demand for broadband data is forcing Network Service Providers (NSPs) to use pricing as a congestion management tool. This trend is evidenced by the adoption of usage-based data pricing instead of the traditional flat-rate data plan by the major wired and wireless NSPs in US, Europe and so on [1], [2], [3], [4]. However, the usage-based pricing could not solve the congestion problem at a given time without giving network users time-dependent incentives when the congestion happens [4]. Therefore, some researchers have proposed time-dependent pricing to control congestion as well as improve the revenue of NSP.

Previous works have shown that time-dependent pricing (TDP) can give the network users right incentive to shift their traffic demands when the network get congested [5], [6]. Ha et al. proposed a time-dependent pricing scheme for mobile data communication, which gives users the monetary reward to delay traffic during traffic peak times [5]. Time is slotted in [5], such as 48 time slots for one day, 30 minutes per slot. They conducted surveys which revealed that users are indeed willing to wait 5 minutes (for YouTube videos) to 48 hours (for software updates). They concluded that the time-dependent pricing fattens temporal fluctuation of traffic usage and benefits both users and NSP. Jiang et al. [6] studied hourly time-dependent pricing offered by a monopoly selfish NSP, comparing the profit-maximizing time-dependent prices to the socially optimal ones in the case of complete information and incomplete information with users' utilities. Although the congestion effects were taken into account in [6], the competition between NSPs were not studied.

Different from above papers [5], [6], in this paper, both competition between NSPs and congestion effect are taken into account in the case of incomplete information of users' willingness to pay (WTP). A duopoly competition is studied in this paper, in which two NSPs set different pricing strategies to maximize their revenue.

In the network economics area, competition is also studied in many literatures with considering the characteristic of telecommunication networks. Jin et al. [7] studies the competition between incumbent and emerging network technologies with the consideration of positive network externality [8]. Gibbens Jin et al. [9] studies the duopoly competition between two NSPs with the consideration of negative network externality [8] when the NSPs differentiate their services. However, the prices set by the NSPs in [7], [9] are not time-dependent. D. Acemoglu and A. Ozdaglar in [10] studied the oligopoly competition between NSPs with considering the congestion costs that users imposed on others, and studied efficiency loss in terms of social welfare. However, an implicit assumption in [10] is that users are homogeneous in the sense that their valuations of Quality of Service (QoS) are the same.

Different from the aforementioned papers [7], [9], [10], in this paper, the prices set by the duopoly NSPs are time-dependent and the users valuations of QoS are heterogeneous, which means that different users may have different valuation on the same level of QoS.

The main contributions of this paper are as follows:

Firstly, the impact of NSP competition on the NSP revenue maximization is analyzed for duopoly case. In each time slot, we model the NSP duopoly competition as a Betrand competition (price competition) game, in which each NSP sets price to compete for market share (number of users) to maximize its revenue. The sufficient condition for the existence of Nash equilibrium is established. Unique Nash equilibrium is also established under the assumption that the users' valuation of QoS is uniformly distributed.

Secondly, heterogeneous users' valuations of QoS and the congestion effect are also modeled, which is much more realistic than the assumption that users's valuation of QoS is the same [10].

The rest of this paper is organized as follows. NSP model and user model are presented in Section II and Section III, respectively. In Section IV, the revenue maximization problem is formulated, then the Nash equilibrium of the Betrand competition game is established for NSPs to choose the time-dependent pricing strategy in each time slot. Numerical results are presented in Section V. Section VI concludes this paper.

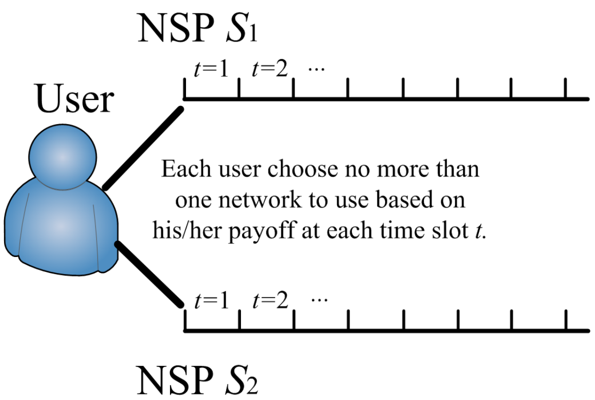


Fig. 1. Network model: network users choose NSP S_1 or NSP S_2.

II. NSP Model

Consider a communication market with two NSPs, denoted by S_1 and S_2, respectively. NSP S_1 and S_2 provide the substitute network services to network users. There exists a sequence of time, i.e., $t = 1, 2, ..., T$, at which each NSP sets time-dependent price p_i^t (where i = 1 or 2). It is assumed that the population of users denoted by N is fixed, with N_i^t as the number of users choosing NSP S_i for i=1 or 2 at time slot t. The proportion of users who choose NSP S_i at time t is denoted by Eq.(1) as presented in [7].

$$x_i^t = \frac{N_i^t}{N}, \text{ where } i = 1 \text{ or } 2 \tag{1}$$

It is assumed that the value of N is very large, either of the NSP cannot accommodate all the N users.

The following set D^t defined in Eq. (2) is the domain for x_1^t and x_2^t.

$$D^t = \{(x_1^t, x_2^t) | x_1^t + x_2^t \leq 1, 0 \leq x_1^t \leq 1, 0 \leq x_2^t \leq 1\} \tag{2}$$

The quality of service (QoS) provided by the NSP S_i for $i = 1$ or 2, denoted as q_i^t, is assumed decreased with the number of its subscribers due to the congestion. We employ a function $h_i(\cdot)$ defined on $[0, 1]$ to express the QoS provided by NSP S_i at time slot t as $q_i^t = h_i(x_i^{t-1})$. The following assumption is for the QoS function $h_i(\cdot)$,

Assumption 1: $h_i(\cdot)$ is a non-increasing and continuous differentiable positive function of the number of users in network S_i for $i = 1$,or 2. Without loss of generality, the QoS provided by NSP S_1 is greater than that provided by NSP S_2.

Remark 1: The assumption of function $h_i(\cdot)$ captures the congestion effects that users experience when choosing NSP S_i with limited resources. Ren et al. adopted the same QoS assumption when consider the QoS formulation of an entrant NSP in a Femtocell communication market [11].However, our analysis differs from [11] in that (i) two incumbent NSPs are considered, (ii) the QoS of both NSPs is decreased with the number of the users in their respective network, and (iii) the prices set by NSPs are time-dependent.

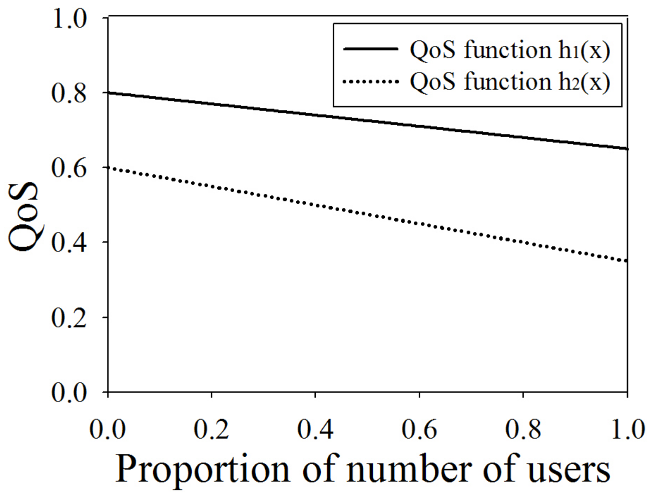


Fig. 2. QoS functions of NSP S_1 and S_1 used in the simulation.

III. User Model

A continuum model of users is employed in this paper. If there are a large number of users in the communication market and each individual user is negligible, the continuum model approximates well the real user population [6]. The payoff of user k at time t is denoted as Eq.(3)

$$u_{k,i}^t = \theta_k^t q_i^t - p_i^t - \gamma_i \tag{3}$$

where $\theta_k^t \in [0, \varphi^t]$ is the QoS valuation of user k at time slot t. Please note that θ_k^t is time-dependent, reflecting users' different preference for different time slot [6]. The value of θ_k^t is private information of users, but the distribution of θ_k^t is public information of NSPs. γ_i is the switching cost that charged by NSP S_i if user k changes from NSP S_i to NSP S_j. Please note that γ_i is not time-dependent. And different users may have different valuations on the same level of QoS. q_i^t is the QoS of NSP S_i's network, $\theta_k^t q_i^t$ is the benefit that the user

TABLE I
NOTATIONS SUMMARY.

Notation	Description
i	$i \in \{1,2\}$,which is NSP set
k	subscript of a user
S_i	the two duopoly NSP in the market, for $i = 1,2$
T	the total time slots
t	$t \in [1,T]$, the *t-th* time slot
N	the population of users
N_i^t	the number of users choose NSP S_i for $i = 1,2$ at time t.
x_i^t	$x_i^t = N_i^t/N$: the proportion of users who choose NSP S_i at time t
$u_{k,i}^t$	the payoff of user k in network S_i at time t
$h_i(x_i^t)$	the QoS function of NSP S_i
q_i^t	$q_i^t := h_i(x_i^t)$
p_i^t	the price set by NSP S_i at time slot t
θ_k^t	user k's valuation of QoS at time slot t
$f^t(\cdot)$	probability density function (PDF) of users' valuation of QoS at time t
$F^t(\cdot)$	cumulative density function (CDF) of users' valuation of QoS at time t
φ^t	the upper bound of the domain of the function $f^t(\cdot)$
$\tau_{S_i}^0$	the marginal point where users switch from getting negative payoff to deriving positive payoff from choosing NSP S_i
τ_k^1	the marginal point where users switch from using NSP S_2 to using NSP S_1
R_i^t	the revenue get by NSP S_i at time slot t
γ_i	the switching cost that charged by NSP S_i

can get from NSP S_i, and p_i^t is the price set by NSP S_i at time slot t, for $i = 1$ or 2. We have the following assumption for users' valuations of QoS.

Assumption 2: The users' valuations of QoS have the probability density function (PDF) $f^t(\cdot)$, which is strictly positive and continuous on $[0,\varphi^t]$ for $\varphi^t > 0$. The cumulative density function (CDF) is defined by $F^t(a) = \int_{-\infty}^{a} f^t(y)\mathrm{d}y$ for all $a \in R$.

Remark 2: It is assumed that the NSP has incomplete information with the users' valuations of QoS [12], which is much more reasonable compared with the assumption of having complete information with users' valuation of QoS [13]. The lower bound of the domain of the probability density function is set as zero to simplify the analysis.

At each decision-making time t, each user only chooses one NSP's network. Each user is a rational decision maker, which means that (1) individual-rationality constraint and (2) incentive-compatibility constraint should be satisfied.

Individual-rationality constraint means that each user chooses the NSP S_i only if he/she gets positive payoff by using NSP S_i. Incentive-compatibility constraint means that each user chooses the NSP who can provide a relative higher payoff to him/her. In other words, a user chooses an NSP under the conditions enumerated as follows.

$$\begin{cases} \text{NSP } S_1 & \text{if } u_{k,1}^t > 0 \text{ and } u_{k,1}^t > u_{k,2}^t \\ \text{NSP } S_2 & \text{if } u_{k,2}^t > 0 \text{ and } u_{k,2}^t > u_{k,1}^t \\ \text{Neither NSP } S_1 \text{ nor } S_2 & \text{if } u_{k,i}^t < 0 \text{ for } i \in \{1,2\} \end{cases}$$

Assumption 3: At each decision time t, each user makes decision independently.

Please refer to Fig. 1 for the network model. The notations used throughout this paper are summarized in Table I.

IV. Revenue Maximization

The NSPs try to maximize their overall revenue by maximizing their revenue in each time slot. We model the NSP duopoly competition as a Betrand competition game for each time slot. Unique Nash equilibrium of the Betrand competition game is established under the assumption that the users' valuation of QoS is uniformly distributed. The revenue of the NSPs depends on the number of users in the network and the prices set by the NSPs, which is denoted as $R_i^t = (p_i^t + \gamma_i)x_i^t(p_1^t, p_2^t)$. Please note that the number of user in network S_i, $x_i^t(p_1^t, p_2^t)$, is a function of the price set by network S_1 and S_2, which can be calculated by the Proposition 1 established in this section. The overall revenue of each NSP can be expressed as $\sum_{t=1}^{T} R_i^t$.

At time slot t, user k will choose NSP S_1 if and only if the conditions shown in inequality (4) are satisfied.

$$\begin{aligned} &\theta_k^t h_1(x_1^{t-1}) - p_1^t - \gamma_1 \geq \theta_k^t h_2(x_2^{t-1}) - p_2^t - \gamma_2 \text{ and} \\ &\theta_k^t h_1(x_1^{t-1}) - p_1^t - \gamma_1 \geq 0 \end{aligned} \tag{4}$$

Similarly, user k will choose NSP S_2 if and only if the conditions shown in inequality (5) are satisfied.

$$\begin{aligned} &\theta_k^t h_2(x_2^{t-1}) - p_2^t - \gamma_2 \geq \theta_k^t h_1(x_1^{t-1}) - p_1^t - \gamma_1 \text{ and} \\ &\theta_k^t h_2(x_2^{t-1}) - p_2^t - \gamma_2 \geq 0 \end{aligned} \tag{5}$$

Or, user k will choose to neither of NSP S_1 and S_2 if and only if the conditions shown in inequality (6) are satisfied.

$$\theta_k^t h_1(x_1^{t-1}) - p_1^t - \gamma_1 < 0 \text{ and } \theta_k^t h_2(x_2^{t-1}) - p_2^t - \gamma_2 < 0 \tag{6}$$

Now we characterize the marginal points that identifying user's valuation of QoS associated with changes in their decision to choose either NSP. $\tau_{S_i}^0$ denotes the marginal point where users switch from getting negative payoff to deriving positive payoff from choosing NSP S_i, i.e., $\tau_{S_i}^0$ is a point such that $u_{k,i}^t = 0$. Similarly, τ_k^1 corresponds to the marginal point where users switch from using NSP S_2 to using NSP S_1, i.e. τ_k^1 is the point such that $u_{k,1}^t = u_{k,2}^t$. With the definition of $\tau_{S_i}^0$ and τ_k^1, we can have the following,

$$u_{k,i}^t = \theta_k^t h_i(x_i^{t-1}) - p_i^t - \gamma_i > 0 \text{ if } \theta_k^t > \tau_{S_i}^0 \tag{7}$$

$$u_{k,1}^t > u_{k,2}^t \text{ if } \theta_k^t > \tau_k^1 \tag{8}$$

Eq.(7) indicates that if $\theta_k^t > \tau_{S_i}^0$, then user with QoS valuation greater than $\tau_{S_i}^0$ can get positive payoff from choosing NSP S_i. Eq.(8) indicates that user with QoS valuation larger than τ_k^1 will choose NSP S_1 since he/she can get greater payoff from NSP S_1than that from NSP S_2. Therefore, it is very important to compute these marginal points, which determine the users' choice of NSP. Please note that although $\tau_{S_i}^0$ and τ_k^1 are also time-dependent, t is not written in the expression of $\tau_{S_i}^0$ and τ_k^1 for clear concern.

By setting $u^t_{k,i} = 0$, we can derive $\tau^0_{S_1}$ and $\tau^0_{S_2}$ as shown in Eq.(9) and Eq.(10), respectively.

$$\tau^0_{S_1} = \frac{p^t_1 + \gamma_1}{h_1(x^{t-1}_1)} \tag{9}$$

$$\tau^0_{S_2} = \frac{p^t_2 + \gamma_2}{h_2(x^{t-1}_2)} \tag{10}$$

By setting $u^t_{k,1} = u^t_{k,2}$, we can derive τ^1_k as shown in Eq.(11).

$$\tau^1_k = \frac{p^t_1 - p^t_2 + \gamma_1 - \gamma_2}{h_1(x^{t-1}_1) - h_2(x^{t-1}_2)} \tag{11}$$

Lemma 1: If $\frac{p^t_1+\gamma_1}{h_1(x^{t-1}_1)} < \frac{p^t_2+\gamma_2}{h_2(x^{t-1}_2)}$, then $\tau^1_k < \tau^0_{S_1} < \tau^0_{S_2}$. If $\frac{p^t_1+\gamma_1}{h_1(x^{t-1}_1)} \geq \frac{p^t_2+\gamma_2}{h_2(x^{t-1}_2)}$, then $\tau^1_k \geq \tau^0_{S_1} \geq \tau^0_{S_2}$.

Proof. The proof of this lemma is omitted due to the space limitation. Now, we consider the following two subsets of users who have valuations of QoS defined in $[0, \varphi^t]$,

$$\Theta_1(x^t) = \left\{\theta^t_k \in [0, \varphi^t] \middle| u^t_{k,1} \geq u^t_{k,2}, u^t_{k,1} > 0\right\} \tag{12}$$

$$\Theta_2(x^t) = \left\{\theta^t_k \in [0, \varphi^t] \middle| u^t_{k,2} > u^t_{k,1}, u^t_{k,2} > 0\right\} \tag{13}$$

Eq. (12) defines QoS valuation of users who choose NSP S_1. Eq. (13) defines QoS valuation of users who choose NSP S_2. We denote the number of users in each set at time slot t as $\Omega_i(x^{t-1})$.

Proposition 1: For any non-negative price pair (p^t_1, p^t_2), the number of users in NSP S_1 and S_2's networks are presented in Eq.(14) and Eq.(15), respectively.

$$\Omega_1(x^{t-1}) = \begin{cases} 1 - F^t(\tau^0_{S_1}) & \text{if} \quad \frac{p^t_1+\gamma_1}{h_1(x^{t-1}_1)} < \frac{p^t_2+\gamma_2}{h_2(x^{t-1}_2)} \\ 1 - F^t(\tau^1_k) & \text{if} \quad \text{otherwise} \end{cases} \tag{14}$$

$$\Omega_2(x^{t-1}) = \begin{cases} 0 & \text{if} \quad \frac{p^t_1+\gamma_1}{h_1(x^{t-1}_1)} < \frac{p^t_2+\gamma_2}{h_2(x^{t-1}_2)} \\ F^t(\tau^1_k) - F^t(\tau^0_{S_2}) & \text{if} \quad \text{otherwise} \end{cases} \tag{15}$$

Proof. The proof of this proposition is omitted due to the space limitation.

$\frac{p^t_i+\gamma_i}{h_i(x^{t-1}_i)}$ is the price per QoS of NSP S_i at the beginning of time slot t. If $\frac{p^t_1+\gamma_1}{h_1(x^{t-1}_1)} < \frac{p^t_2+\gamma_2}{h_2(x^{t-1}_2)}$, it means that the price per QoS of NSP S_1 is lower than that of NSP S_2. In this case, the number of users who choose NSP S_1 is positive, but the number of users who choose NSP S_2 is zero. If $\frac{p^t_1+\gamma_1}{h_1(x^{t-1}_1)} \geq \frac{p^t_2+\gamma_2}{h_2(x^{t-1}_2)}$, it means that the price per QoS of NSP S_2 is lower than that of NSP S_1. Both NSP S_1 and S_2 have positive number of users who use their networks. This proposition shows that the price per QoS rather than the price determines the market share of a NSP. When an NSP set its price, the competitor's price per QoS should be considered to keep its network competitive.

Each NSP tries to maximize their overall revenue by considering the following subproblem shown in Eq.(16) in each time slot t.

$$\max_{p^t_i} R^t_i \tag{16}$$

subject to $x^t_i \in D^t$

The above problem can be solved by considering the game played by NSP S_1 and S_2. The Nash Equilibrium point is the solution of the problems. Now we consider that two NSPs play a Bertrand competition (or price competition) game in each time slot t. The Bertrand competition game Γ(**Player, Strategy, Payoff**), is described as follows:

- **Player**: The NSP S_1 and S_2 are the two players in the game.
- **Strategy**: The strategy is the price set by NSP S_i for i=1,2.
- **Payoff**: The payoff is the revenue gotten by NSP S_i for i=1,2.

In this game, NSP S_1 and S_2 set their price p^t_1 and p^t_2 respectively, to get market share to maximize their revenue, which is the multiplication of price and the market share (or the number of users). The number of users in each NSP's network can be derived by Proposition 1.

Lemma 2: The necessary condition for existence of the Nash Equilibrium of the game Γ(**Player, Strategy, Payoff**) is

$$\varphi^t > \frac{p^t_1 + \gamma_1}{h_1(x^{t-1}_1)} \geq \frac{p^t_2 + \gamma_2}{h_2(x^{t-1}_2)} > 0 \tag{17}$$

Proof. As illustrated in Proposition 1, the price per QoS determines the market share of a NSP. If $\frac{p^t_1+\gamma_1}{h_1(x^{t-1}_1)} < \frac{p^t_2+\gamma_2}{h_2(x^{t-1}_2)}$, by Proposition 1,the number of users in NSP S_2's network is 0, thus the payoff of NSP S_2 is 0, while the payoff the NSP S_1 is positive. Therefore, the price strategy p^t_2 in this case is a dominated strategy for NSP S_2. NSP S_2 would not play the dominated strategy. In order to get a positive market share, NSP S_2 would decrease its price per QoS, which lead to the condition $\frac{p^t_1+\gamma_1}{h_1(x^{t-1}_1)} \geq \frac{p^t_2+\gamma_2}{h_2(x^{t-1}_2)}$. By Proposition 1, both NSP S_1 and S_2 have positive number of users in their network. Both NSP S_1 and S_2 can get positive payoff.

Q.E.D.

Proposition 2: If the following conditions are satisfied,

(1) Users' QoS valuation is distributed uniformly.

(2) The condition in Lemma 2 is satisfied.

(3) The QoS function has the following property, shown in inequality (19),

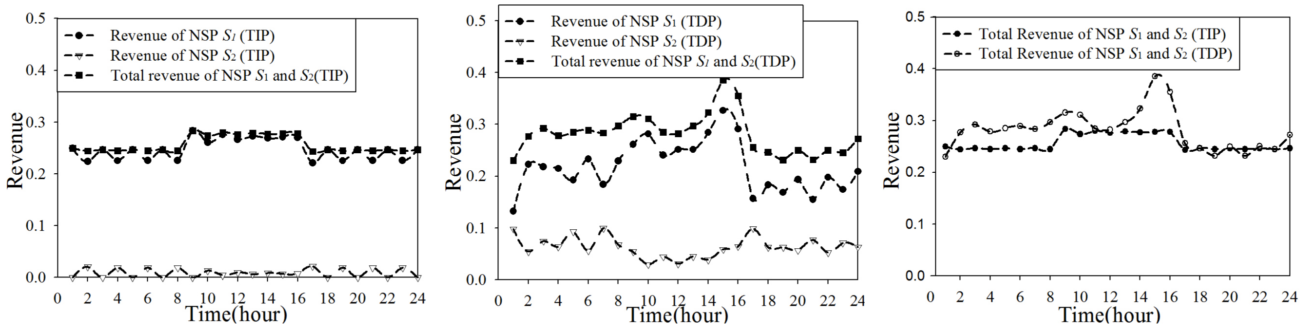
$$\frac{h_2(x^{t-1}_2)}{h_1(x^{t-1}_1)} < 4 \tag{18}$$

then,the Nash Equilibrium of the game Γ(**Player, Strategy, Payoff**) is unique, and the NE is determined by Eq. (19) and Eq. (20).

$$p^t_1 = BR_1(p^t_2) = \frac{1}{2}[\varphi^t(h_1(x^{t-1}_1) - h_2(x^{t-1}_2)) + (\gamma_2 - 2\gamma_1) + p^t_2] \tag{19}$$

$$p^t_2 = BR_2(p^t_1) = \frac{h_2(x^{t-1}_2))}{2h_1(x^{t-1}_1)} p^t_1 + \frac{h_2(x^{t-1}_2))}{2h_1(x^{t-1}_1)} \gamma_1 - \gamma_2 \tag{20}$$

where $BR_i(\cdot)$ is the best response function of player NSP S_i [14].



(a) Revenue of NSP S_1 and NSP S_2 under TIP. (b) Revenue of NSP S_1 and NSP S_2 under TDP. (c) Comparison of total revenue under TIP and TDP.

Fig. 3. Revenue vs. time.

Proof. The proof of this proposition is omitted due to the space limitation.

Mathematically, the condition (3) in proposition 2 ensure that Eq. (19) and Eq. (20) can intersect at a unique point, which is the unique Nash Equlibrium of the game. Actually, this condition (3) means that the QoS difference should not too large.

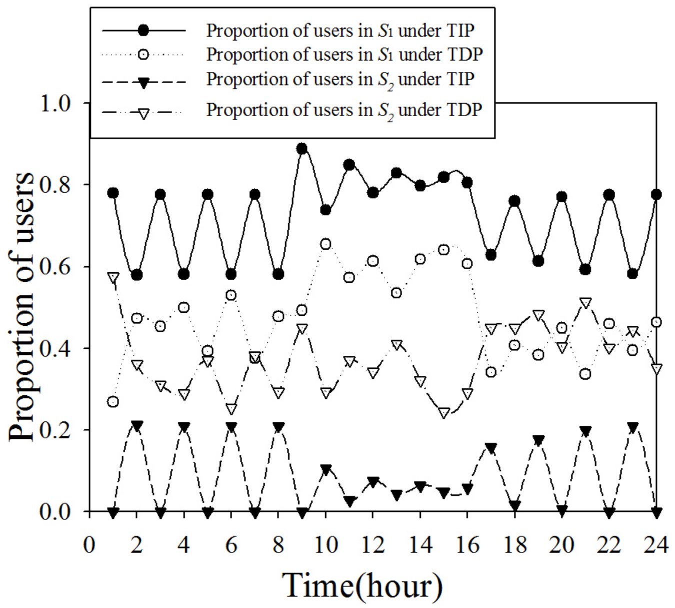


Fig. 4. Comparison of the proportion of users in network of NSP S_1 and S_2 under TIP and TDP.

V. Simulation

This section makes simulation analysis In this section, we present the analytic results through simulations to validate our analytical results. The simulation analysis includes the following aspects:

- Compare the revenue from Time-Dependent Pricing (TDP) scheme with the revenue from Time-Independent Pricing (TIP) scheme for the duopoly case.
- Compare the number of users for TDP scheme with the number of users for TIP scheme for the duopoly case.

TABLE II
Parameters in the simulation.

User $k's$ valuation of QoS at time slot t: θ_k^t	(1) when $t \in [1,8]$ or $t \in [17,24]$, θ_k^t is from uniform distribution defined on $[0,2]$, $f(\theta_k^t) = \frac{1}{2}, F(\theta_k^t) = \frac{\theta_k^t}{2}, \theta_k^t \in [0,2]$; (2) when $t \in [9,16]$, θ_k^t is from uniform distribution defined on $[0,4]$, $f(\theta_k^t) = \frac{1}{4}, F(\theta_k^t) = \frac{x}{4}, \theta_k^t \in [0,4]$
NSP S_1's QoS function $h_1(x_1^t)$	$h_1(x_1^t) := 0.8 - 0.15 * x_1^t$
NSP S_2's QoS function $h_2(x_2^t)$	$h_2(x_2^t) := 0.6 - 0.3 * x_2^t$
The NSP S_i's price at time slot t: p_i^t	Set as the NE point by solving Eq. (19) and Eq. (20) in Proposition 2.
Switching cost	$\gamma_1 = 0.03, \gamma_2 = 0.03$
Number of time slots	24 time slots per day

- Compare the total revenue gets by NSP S_1 and S_2 with that from monopoly NSP.

The analysis for the case of monopoly NSP is omitted due to the space limitation.

The parameters for simulations are summarized in Table. II. The QoS functions for simulation are shown in Fig. 2. We assume that the distribution of the users' valuation θ_k^t of QoS follows an uniform distribution $[0, \varphi^t]$. When $t \in [1,8]$ or $t \in [17,24]$, $\varphi^t = 2$, and when $t \in [9,16]$, $\varphi^t = 4$. Therefore, the users averagely have much higher valuation of QoS during time slots $[9,16]$ than that during other time slots. It can be expected that the peak traffic will occur during time slots $[9,16]$. The QoS function is defined as simple affine function satisfying Assumption 1 aforementioned. This kind of affine QoS function has been also adopted in [15], and also satisfies satisfy the conditions in Proposition 2. The two NSPs set the prices in each time slot according to the NE established in Proposition 2.

Observation 1: The revenue is more stable in the TDP scheme than in TIP scheme.

We can see from Fig.3(a) and Fig.3(b) that, the revenue of NSP is oscillated in the TIP scheme. In the TIP scheme, the price is fixed initially, then the number of users in a network (for example, network of S_1) with smaller price per QoS keeps increasing, which makes the network of S_1

congested. The users who get negative in network of S_1 due to congestion will switch to network of S_2, which makes the number of users in network of S_1 decreases. The above process happens iteratively, which makes the revenue oscillated in TIP scheme. However, in the TDP scheme, NSPs can use price as a congestion management tool to control the number of users choosing their network at each time slot.

Observation 2: The revenue from TDP scheme is higher than that from the TIP scheme in the duopoly competition environment.

In TDP scheme, NSPs can set new prices for each time slot, which is the Nash Equilibrium of the game Γ, thus the NSPs revenue get maximized at each time slot. However, in TIP scheme, the price can only set initially without considering the competition from the rival NSP and the congestion effect. Please see the Fig.3(c).

Observation 3: TDP Scheme has congestion control effect in the duopoly case.

We can see from Fig.4, in the NSP S_1's network, the number of users in "peak hours" under TIP Scheme is much more than the number of users in "peak hours" under TDP Scheme. The reason is that when the price is time-dependent, the NSPs can increase its price to make less users to use its network. It is interesting to see that, the number of users in "peak hours" under TIP scheme is much less than the number of users in "peak hours" under TDP scheme for NSP S_2's network. The reason is that the QoS provided by network of NSP S_2 is lower than that of NSP S_1 (see Assumption 1). Under TIP scheme, high QoS valuation users tend to choose network of NSP S_1. However, under TDP scheme, the competition effect push the users from NSP S_1's network to NSP S_2's network.

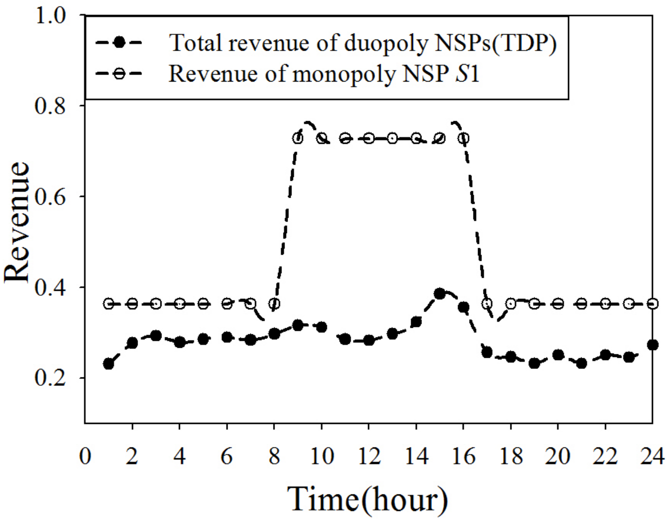


Fig. 5. Comparison of total revenue from monopoly NSP and duopoly NSPs under the TDP Scheme.

Observation 4: The revenue from TDP scheme in the duopoly NSP case is smaller than that in a monopoly NSP case.

In the monopoly case, the NSP has market power, all the surplus can be extracted by the NSP. However, due to the competition effect, all the surplus cannot be extracted by the duopoly NSPs, a part of the surplus goes to users. Thus, The revenue from TDP scheme in the duopoly NSP case is smaller than that in a monopoly NSP case. Please refer to Fig.5.

VI. Conclusion

This paper analyzes the time-dependent pricing scheme in a duopoly competition environment. We model the NSP duopoly competition as a Betrand competition (price competition) game, in which each NSP sets price to compete for market share (number of users) to maximize its revenue. The sufficient condition for existence of the Nash equilibrium of the Bertrand is established. Unique Nash equilibrium is also established under the assumption that the users' valuation of QoS is uniformly distributed. The simulation results reflect that the revenue from a time-dependent pricing scheme is higher that from the time-independent pricing scheme in the duopoly case. However, due to competition effect, the NSPs could not extract all the surplus from users. In this sense, we can conclude that the time-dependent pricing scheme in a competitive environment can also benefit NSPs, but the revenue improvement is limited due to competition effect.

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