The Suboptimal Power Allocation Scheme for V-BLAST System with Channel Feedback Delay

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***Abstract*-In this paper, an adaptive transmit power allocation (TPA) scheme is proposed for Vertical Bell Labs Layered Space-Time (V-BLAST) system with delayed channel state information (CSI) feedbacks. At the transmitter, the suboptimal adaptive power allocation scheme is presented to minimize bit error rate (BER) with total power constraint. At the receiver, the zero-forcing (ZF) rule is employed for signal detection. In order to get the power allocation parameters, the Lagrange multiplier method is used. Simulation results show that the proposed TPA scheme can effectively improve the system performance with lower computation complexity.**

Index Terms—V-BLAST; Power allocation; Feedback delay

I. Introduction

In this paper, a sub-optimal power allocation scheme is developed for V-BLAST system with channel feedback delay to minimize instantaneous BER by using the Lagrange multiplier method. The modified V-BLAST system model is presented and the instantaneous BER of the estimated value is obtained using mathematical transformation [1]. Finally, the closed expression of the TPA matrix is derived. Simulation results indicate that the BER performance can be well improved by the proposed TPA algorithm with delayed CSI.

II. System Model

The modified V-BLAST system model is equipped with M transmit and N receive antennas $(N \ge M)$ which is shown in Fig.1. At the transmitter, the data stream is first encoded, modulated and then demultiplexed into M independent substreams where each antenna is allocated with proper power on the basis of delayed feedback CSI. Finally the preprocessed symbols are transmitted by M transmit antennas.

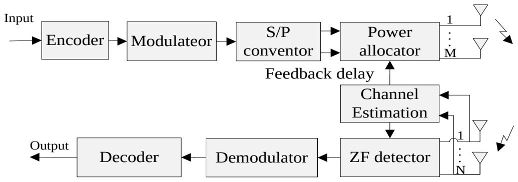

Fig. 1 The modified V-BLAST system with channel feedback delay

The model can be mathematically expressed as

$$\boldsymbol{y} = \boldsymbol{HPx} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{x}$ is the transmit symbol vector, $\boldsymbol{y}$ is the received vector, $\boldsymbol{P} = diag(\sqrt{P_1}, \sqrt{P_2}, \cdots, \sqrt{P_M})$ indicates a diagonal power allocation matrix. $\boldsymbol{H}$ stands for the channel matrix, $\boldsymbol{n}$ is the noise vector.

This work is supported by"Aviation Science Foundation of China (NO.20120152001)" and "China Postdoctoral Science Foundation (NO.2013M541661)"

At the transmitter, only the delayed feedback channel matrix $\hat{\boldsymbol{H}}$ is available. The relationship between real channel matrix $\boldsymbol{H}$ and $\hat{\boldsymbol{H}}$ can be modeled as[2]

$$\boldsymbol{H} = \sqrt{\rho}\hat{\boldsymbol{H}} + \boldsymbol{\Xi} \tag{2}$$

where $\rho = J_0^2(2\pi f_d \tau)$, $J_0(\bullet)$ indicates the zero-order Bessel function of the first kind, f_d denotes the maximum Doppler frequency and τ is the time delay. $\boldsymbol{\Xi}$ indicates the channel estimation error matrix, whose element satisfies $e_{ji} \sim CN(0,1\text{-}\rho)$.

III. The Distribution of SINR

The soft interference cancellation detector based on ZF principle is considered, the estimated value of transmit symbol vector is[1]

$$\hat{\boldsymbol{x}} = \langle \boldsymbol{G}_i \rangle_{k_i} \boldsymbol{y} = \boldsymbol{x} + ([\boldsymbol{HP}]_{\bar{k}_i})^{+} \boldsymbol{n} \tag{3}$$

where $\boldsymbol{G} = ([\boldsymbol{HP}]_{\bar{k}_i})^{+}$ is the ZF equalization matrices, $(\bullet)^{+}$ denotes the Moore-Penrose pseudo inverse, $[\bullet]_{\bar{k}_i}$ indicates nulling out the $k_1, k_2, \cdots k_i th$ columns of a matrix. Based on $\hat{\boldsymbol{H}}$, the estimated signal noise ratio (SNR) for the k_ith symbol is [3]

$$\hat{\gamma}_{k_i} = \frac{\bar{\gamma}}{[(([\hat{\boldsymbol{H}}\boldsymbol{P}]_{\bar{k}_{i-1}})^{\mathrm{H}}[\hat{\boldsymbol{H}}\boldsymbol{P}]_{\bar{k}_{i-1}})^{-1}]_{k_i k_i}} = \frac{\bar{\gamma} P_{k_i}}{[(([\hat{\boldsymbol{H}}]_{\bar{k}_{i-1}})^{H}[\hat{\boldsymbol{H}}]_{\bar{k}_{i-1}})^{-1}]_{k_i k_i}} \tag{4}$$

where $\bar{\gamma}$ denotes the average SNR. The probability density function (PDF) of $\hat{\gamma}_{k_i}$ can be directly rewritten as[4]

$$f(\hat{\gamma}_{k_i}) = \frac{\hat{\gamma}_{k_i}^{\,k\text{-}1}}{\Gamma(K)} \left(\frac{P_{k_i}^{-1}}{\bar{\gamma}} \right)^{K} exp\left(-\hat{\gamma}_{k_i} \frac{P_{k_i}^{-1}}{\bar{\gamma}} \right), \hat{\gamma}_{k_i} \ge 0 \tag{5}$$

where $\Gamma(a) = \int_0^{\infty} e^{-t} t^{a-1} dt$ is the Gamma function, $K = N - M + 1$.

The joint PDF of real SNR γ_{k_i} and the estimated SNR $\hat{\gamma}_{k_i}$ is introduced in [5], the conditional PDF $f(\gamma_{k_i} \mid \hat{\gamma}_{k_i})$ can is calculated as

$$\begin{aligned} f(\gamma_{k_i} \mid \hat{\gamma}_{k_i}) &= \frac{f(\gamma_{k_i}, \hat{\gamma}_{k_i})}{f(\hat{\gamma}_{k_i})} = \frac{1}{1\text{-}\rho} \frac{P_{k_i}^{-1}}{\bar{\gamma}} \left(\frac{\gamma_{k_i}}{\rho \hat{\gamma}_{k_i}} \right)^{(K\text{-}1)/2} \bullet \\ & \mathrm{I}_{K\text{-}1}\left(\frac{2P_{k_i}^{-1}\sqrt{\rho \gamma_{k_i} \hat{\gamma}_{k_i}}}{(1\text{-}\rho)\bar{\gamma}} \right) \exp\left(-\frac{P_{k_i}^{-1}(\gamma_{k_i} + \rho \hat{\gamma}_{k_i})}{(1\text{-}\rho)\bar{\gamma}} \right) \end{aligned} \tag{6}$$

where $\mathrm{I}_n(\bullet)$ is the *n-order* modified Bessel function.

IV. BER of The Modified V-BLAST System

Assuming 2^R-QAM is employed, the BER can be tightly approximated by an exponential function of γ_{k_i} [1]

$$\mathrm{BER}(\gamma_{k_i}) \approx 0.2\,exp(-\frac{1.6\gamma_{k_i}}{2^{\mathrm{R}}-1}) = c_1\,exp(-\mathrm{c}_2\gamma_{k_i}) \quad (7)$$

where R is the modulation order, then the estimated BER is

$$\mathrm{BER}(\hat{\gamma}_{k_i}) = \int_0^{\infty} \mathrm{BER}(\gamma_{k_i})\, f(\gamma_{k_i} \mid \hat{\gamma}_{k_i})\mathrm{d}\gamma_{k_i} \quad (8)$$

Plug(6) and (7)into (8) , the instantaneous estimated BER is processed as

$$\mathrm{BER}(\hat{\gamma}_{k_i}) = c_1\left(\frac{1}{c_2(1\text{-}\rho)\overline{\gamma}P_{k_i}+1}\right)^K \exp\left(-\frac{c_2\rho\overline{\gamma}P_{k_i}}{\left\|\hat{v}_{k_i}\right\|^2 (c_2(1\text{-}\rho)\overline{\gamma}P_{k_i}+1)}\right) \quad (9)$$

where $\left\|\hat{v}_{k_i}\right\|^2 = [(([\hat{\boldsymbol{H}}]_{\bar{k}_{i-1}})^H [\hat{\boldsymbol{H}}]_{\bar{k}_{i-1}})^{-1}]_{k_i k_i}$. Since the transmit symbols are independent of one another, the average BER can be calculated as an arithmetic mean of the BER for every antenna. The Lagrangian multiplier method is used with the total transmit power constraint, the cost function is

$$J\left(P_1, P_2, \ldots, P_M\right) = \frac{1}{M}\sum_{i=1}^{M} BER(\hat{\gamma}_{k_i}) + \lambda\left(\sum_{i=1}^{M} P_i\text{-}M\right) \quad (10)$$

where λ is the Lagrange multiplier, from $\partial J/\partial P_i = 0$, a set of equations can be obtained. However, note in (9) and(10), it is difficult to get a closed form solution, and then a sub-optimal expression is considered. The impact of P_{k_i} in $\mathrm{c}_2(1\text{-}\rho)\overline{\gamma}P_{k_i}$ is much smaller, we consider it as the equal power allocation element. Hence the approximate expression can be written as

$$\mathrm{BER}(\hat{\gamma}_{k_i}) = \mathrm{c}_1\left(\frac{1}{\mathrm{c}_2(1\text{-}\rho)\overline{\gamma}+1}\right)^K exp\left(-\frac{\mathrm{c}_2\rho\overline{\gamma}P_{k_i}}{\left\|\hat{v}_{k_i}\right\|^2 (\mathrm{c}_2(1\text{-}\rho)\overline{\gamma}+1)}\right) \quad (11)$$

By solving(10), the power allocation elements and the Lagrange multiplier can be calculated

$$P_{k_i} = -\frac{\left\|\hat{v}_{k_i}\right\|^2 (\mathrm{c}_2(1\text{-}\rho)\overline{\gamma}+1)}{\mathrm{c}_2\rho\overline{\gamma}}\, ln\left(\frac{M\lambda\left\|\hat{v}_{k_i}\right\|^2}{\mathrm{c}_1\mathrm{c}_2\rho\overline{\gamma}}(\mathrm{c}_2(1\text{-}\rho)\overline{\gamma}+1)^{K+1}\right) \quad (12)$$

$$\lambda = \exp\left(-\frac{\dfrac{M\mathrm{c}_2\rho\overline{\gamma}}{\mathrm{c}_2(1\text{-}\rho)\overline{\gamma}+1} + \sum_{i=1}^{M}\left\|\hat{v}_{k_i}\right\|^2 ln\left(\dfrac{M\left\|\hat{v}_{k_i}\right\|^2}{\mathrm{c}_1\mathrm{c}_2\rho\overline{\gamma}}(\mathrm{c}_2(1\text{-}\rho)\overline{\gamma}+1)^{K+1}\right)}{\sum_{i=1}^{M}\left\|\hat{v}_{k_i}\right\|^2}\right) \quad (13)$$

When $f_d\tau = 0$, equation (12) is consistent with the solution in perfect CSI.

V. Simulation Results

In this section, the BER performance of the modified V-BLAST system with the proposed sub-optimal TPA scheme base on delayed CSI is evaluated. Fig.2 shows the effect of the channel feedback delay on the TPA scheme. The transmit and receive antennas are both 6, 4QAM is adopted. The channel coding is (2,1, 2) convolutional code. EPA denotes the equal power allocation scheme, TPA stands for the proposed adaptive transmit power allocation scheme and $f_d\tau$ is the normalized feedback delay. We can observe significant improvement in BER performance with a small feedback delay, besides the BER performance are getting worse gradually with the increase of channel feedback delay, which result from that the TPA becomes focus on the accuracy of CSI. When $f_d\tau = 0.01$ the SNR gains about 2dB over $f_d\tau = 0.03$ for BER=10^{-3} , which verifies that the smaller $f_d\tau$, the better BER performance with TPA can be expected.

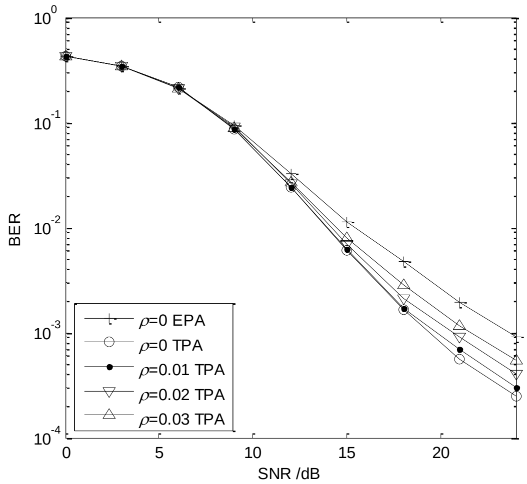

Fig. 2 The effect of channel feedback delay on the BER performance.

VI. Conclusions

In this paper, a sub-optimal TPA scheme of the modified V-BLAST system is presented to minimize the overall coded BER with relatively small CSI feedbacks overhead. The closed expression of the TPA matrix has been presented. It has been found that the proposed algorithm can evaluate the effect of the channel feedback delay effectively and reduce the overall BER significantly. Numerical results have shown that the V-BLAST system with the proposed transmit power allocation scheme achieves more SNR gain over the EPA scheme.

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