

An Improved Sum-of-Sinusoids Channel Simulator Based on Brownian Motion

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Abstract – An improved channel simulator based on the sum-of-sinusoids method with exact Doppler spread (MEDS) is proposed. The parameters of our simulator updates at every discrete time according Brownian motion, which is more suitable with the time-variant channel under real environment. The new method yields good approximation to the desired statistical properties with limited simulation time constraints, and more important, every output channel fadings are independent, which is often demanded for simulating wideband channel and multiple input multiple output (MIMO) channels.

Index Terms — Channel simulator, Sum-of-sinusoids, Brownian motion, Power spectrum density

I. INTRODUCTION

The sum-of-sinusoids (SOS) method is widely used to simulate channel fading caused by multipath propagation. Many different parameter computation methods have been investigated, which can be classified as deterministic and stochastic. The classic deterministic method of exact Doppler spread (MEDS) [1] is simple and accurate, but inefficient for simulating large number of uncorrelated fadings. To overcome this drawback, Wang modified the method by adding an infinitesimal offset to the parameters of each fading waveform [2]. Based on this idea, other modified models such as generalized-MED (GMEDS) can be found in [3-4].

In this letter, an improved method originating from MEDS is proposed for stochastic SOS simulator. The parameters of new simulator updates every discrete time based on Brownian motion, which will achieve better approximation to the desired statistical properties under limited simulation time constraints.

II. IMPROVED MODEL

The reference model of Rayleigh fading is given by

$$\mu(t) = \mu_1(t) + j * \mu_2(t) \quad (1)$$

where $j = \sqrt{-1}$ and $\mu_i(t)$ is a Gaussian random process with zero mean. The inphase and quadrature components are uncorrelated. The SOS simulator for normalized Gaussian random process can be expressed as

$$\mu_i(t) = \frac{1}{\sqrt{N_i}} \sum_{n=1}^{N_i} \cos(2\pi f_{i,n} t + \theta_{i,n}) \quad i = 1, 2 \quad (2)$$

here N_i is the number of sinusoids, $f_{i,n}, \theta_{i,n}$ denote discrete frequency and phase. By invoking the central limit theory, $|\mu(t)|$ is a Rayleigh random process when $N_i \rightarrow \infty$. For arbitrary N_i , the time-averaged auto-correlation function is determined by discrete frequency of each sinusoid as follows

$$r_{\mu_i \mu_i}(\tau) = \sum_{n=1}^{N_i} \frac{1}{2N_i} \cos(2\pi f_{i,n} \tau) \quad (3)$$

While the corresponding power spectrum density(PSD) can be expressed as

$$S_{\mu_i \mu_i}(f) = \sum_{n=1}^{N_i} \frac{1}{4N_i} [\delta(f - f_{i,n}) + \delta(f + f_{i,n})] \quad (4)$$

A. MEDS

There are two typical Doppler power spectrum shapes, Jakes PSD and Gaussian PSD, which depend on the propagation environment. Jakes PSD is suitable for isotropic scattering environment, which is a U-shaped spectrum, as

$$S(f) = \begin{cases} \frac{\sigma_0^2}{\pi f_d \sqrt{1 - (f/f_d)^2}} & |f| \leq f_d \\ 0 & |f| > f_d \end{cases} \quad (5)$$

while Gaussian PSD is widely used in aeronautical communication field [5], which is

$$S(f) = \frac{\sigma_0^2}{f_c} \sqrt{\frac{\ln 2}{\pi}} e^{-\ln 2 (\frac{f}{f_c})^2} \quad (6)$$

where f_d is the maximum Doppler frequency, and $f_c = \sqrt{\ln 2} f_d$ is 3dB-cutoff frequency.

The solutions of discrete frequency for these two PSD shapes by MEDS can be written as

$$f_{i,n} = f_d \sin \left[\frac{\pi}{2N_i} \left(n - \frac{1}{2} \right) \right] \quad (7)$$

and

$$f_{i,n} = f_d \operatorname{erf}^{-1} \left(\frac{2n-1}{2N_i} \right) \quad (8)$$

where erf^{-1} is the inverse of Gauss error function.

B. Improved method

The parameters of MEDS are fixed during full simulation, but the real propagation environment is time-variant duo to the movement of transmitter and receiver. To overcome this disadvantage, we introduce a stochastic factor originating from the Brownian motion. Brownian movement is a continuous process B_t , with the properties that $B_0 = 0$ and

for $0 \leq s < t$, the increment $B_t - B_s$ is normally distributed with mean zero and variance $t - s$ [6].

So, We propose to redefine the discrete frequency as

$$f'_{i,n}(kT_s) = f'_{i,n}((k-1)T_s) + B_{i,n}(kT_s), k = 1, \dots, \infty \quad (9)$$

where T_s, k are channel sampling interval and discrete time number, the initial value of new discrete frequency equal to $f_{i,n}$, and $B_{i,n}(kT_s)$ is generated by Brownian motion, as

$$B_{i,n}(kT_s) = B_{i,n}((k-1)T_s) + \sqrt{\delta T_s} N_k(0,1) \quad (10)$$

where $B_{i,n}(0) = 0$, $N_k(0,1)$ is a normalized Gaussian random variable and δ is a scale factor. One sample of Brownian motion is showed in Fig.1, where $\delta = 1, T_s = 1$. As we can see that each sample path is a random process, while the amplitude PDF for all sample paths at kT_s is Gaussian distribution when k is fixed.

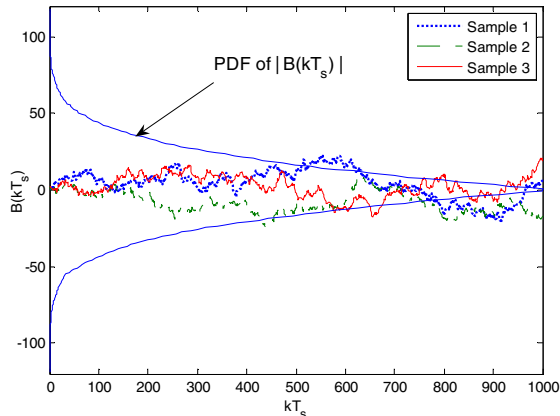


Fig. 1. Brownian motion include three sample paths

III. NUMERICAL SIMULATION

In this section we use new model to generate Rayleigh fading with Gaussian and Jakes PSD. A maximum Doppler frequency of $f_d = 100\text{Hz}$ is selected, which corresponding to mobile speed of 120km/h and a carrier of 900MHz . The output PDFs are given in Fig.2, where the Rayleigh theoretical distribution is also showed for comparison. It can be observed the simulation results agree well with the reference curves.

Fig.3 compares the output PSDs, where the theoretical ones come from (5) and (6) with $\sigma_0^2 = 1$. We only plot the curves between $-f_m$ and f_m , because the outside zone is meaningless according the Doppler frequency selection method of (9). As we can see, the simulation curves show good convergence to the desired ones.

IV. CONCLUSION

In this paper, an improved parameter computation method based on Brownian motion is proposed. The output channel of this method is independent and time-variant, which is suitable for mobile communication channel Simulation results for Rayleigh channels with Jakes PSD and Gaussian PSD show that the new simulator provides good approximation to the desired stochastic features of reference model.

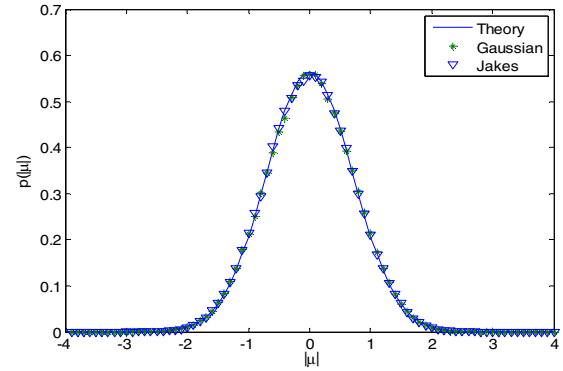


Fig. 2. The PDFs of output fading for Gaussian and Jakes scene

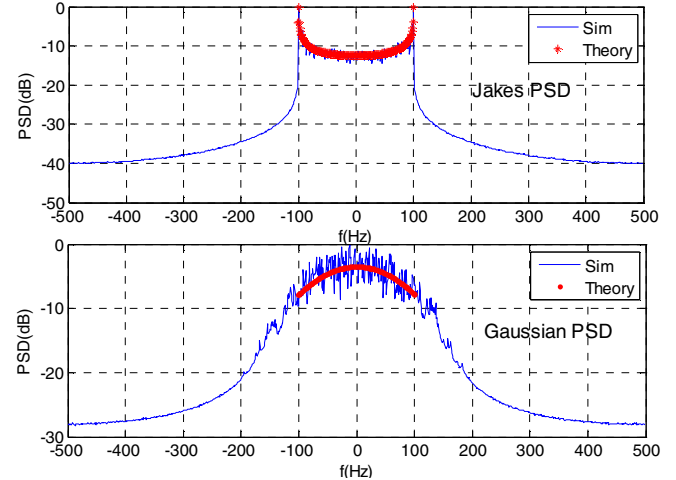


Fig. 3. The PSDs of output fading for Gaussian and Jakes scene

ACKNOWLEDGMENT

This work is supported by Aeronautical Science Foundation of China (20120152001) and China Postdoctoral Science Foundation (2013M541661).

REFERENCES

- [1] M. Pätzold, *Mobile Fading Channels*. Chichester, UK: John Wiley & Sons, 2002.
- [2] Wang C, Yuan D, Chen H H, et al. "An improved deterministic SoS channel simulator for multiple uncorrelated Rayleigh fading channels". *IEEE Transactions on Wireless Communications*, vol.7, No.9, pp3307-3311, 2008.
- [3] Patzold M, Wang C X, Hogstad B O. "Two new sum-of-sinusoids-based methods for the efficient generation of multiple uncorrelated Rayleigh fading waveforms". *IEEE Transactions on Wireless Communications*, vol.8, no.6, pp3122-3131, 2009.
- [4] Gan Y, Xu Q. "An improved SoS method for generating multiple uncorrelated Rayleigh fading waveforms". *IEEE Communications Letters*, vol.14, no.7, pp.641-643, 2010.
- [5] Su Z, Zhang T. An improved model of Rayleigh fading channel with Gaussian's PSD for aeronautical En-Route scenario[C]//*Microwave, Antenna, Propagation and EMC Technologies for Wireless Communications (MAPE)*, 2013 IEEE 5th International Symposium on. 2013: 16-20.
- [6] Karatzas I. *Brownian motion and stochastic calculus* [M]. Springer, 1991