# Performance Improvement of Localization of Coherent Near-field Sources by Combined Use of DOA-Matrix Method and SAGE Algorithm

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***Abstract* - This paper deals with the near-field DOA-Matrix method for source localization using an array antenna. To apply this method for the estimation of the coherent sources, the spatial smoothing is incorporated into this method. The computer simulation along with SAGE algorithm demonstrates that the proposed method provides higher estimation accuracy of coherent source locations.**

***Index Terms* - source localization, near field, DOA-Matrix method, SAGE, coherent sources, spatial smoothing.**

## I. Introduction

The near-field DOA-Matrix method is used for source localization using an array antenna [1],[2]. This is an ESPRIT-like method and has the advantage of not requiring peak search of spectrum in obtaining the estimates. Also, the near-field DOA-Matrix method can be used jointly with the SAGE algorithm, and the combined algorithm (called the near-field SAGE-DOA-Matrix) can perform successful estimation of multiple sources impinging on the array [3].

However, when the coherent waves are incident on the array or the number of snapshots is small, the cross-correlation among the incident waves degrades the estimation accuracy of the near-field DOA-Matrix method. In this paper, therefore, we propose an improved algorithm which incorporates the spatial smoothing (SS) into the near-field DOA-Matrix method. Via computer simulation, we show that the proposed method attains higher accuracy of estimation for coherent sources.

## II. Signal Model

Consider a ULA (Uniform Linear Array) having $K = 2p + 1$ ($p$: positive integer) isotropic elements with element spacing of $d$, which is depicted in Fig. 1. As shown in Fig. 1, the array is placed in the coordinate system and the array center (origin) is designated as the phase reference point. We assume that there are $L$ source signals, the locations of which are given by both the ranges $r_{0,l}$ and DOAs $\theta_l$ ($l = 1, 2,\ldots, L$).

Then, the array response vector (mode vector) of the $l$th signal, $\mathbf{a}(\theta_l, r_{0,l})$, is defined as

$$\mathbf{a}\left(\theta_l, r_{0,l}\right) = \left[\frac{r_{0,l}}{r_{-p,l}}\exp\left(-j\tau_{-p,l}\right), \ldots, \frac{r_{0,l}}{r_{p,l}}\exp\left(-j\tau_{p,l}\right)\right]^T \quad (1)$$

$$\tau_{k,l} = \frac{2\pi}{\lambda}\left(r_{k,l} - r_{0,l}\right) \quad (k = -p, \ldots, p) \quad (2)$$

$$r_{k,l} = r_{0,l}\sqrt{1 + \left(\frac{kd}{r_{0,l}}\right)^2 - \frac{2kd\sin\theta_l}{r_{0,l}}} \quad (3)$$

where $r_{k,l}$ is the distance between the $l$th source and the $k$th element, $\tau_{k,l}$ is the phase lag of the $l$th source signal at the $k$th element with respect to the reference point, and $\lambda$ is the wavelength. When the $l$th signal at the reference point is denoted by $s_{0,l}(t)$, the array received signal vector $\mathbf{x}(t)$ is expressed as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (4)$$

$$\mathbf{A} = \left[\mathbf{a}\left(\theta_1, r_{0,1}\right), \mathbf{a}\left(\theta_2, r_{0,2}\right), \ldots, \mathbf{a}\left(\theta_L, r_{0,L}\right)\right] \quad (5)$$

$$\mathbf{s}(t) = \left[s_{0,1}(t), \ldots, s_{0,L}(t)\right]^T \quad (6)$$

where $\mathbf{A}$ is the mode matrix, and $\mathbf{n}(t)$ is the internal noise vector.

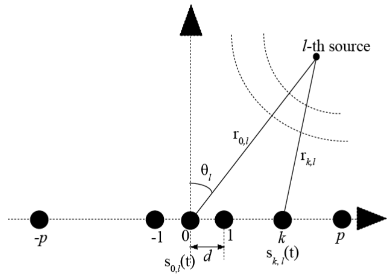


Fig. 1. $K$-element ULA and received signals

## III. Near-Field DOA-Matrix Method

If we make approximation of $\tau_{k,l}$, which exploits its second Taylor expansion, and also we assume $r_{k,l} = r_{0,l}$, then the received signal $x_k(t)$ at the $k$th element can be expressed as follows:

$$x_k(t) = \sum_{l=1}^{L} s_{0,l}(t)\exp\left\{j\left(\omega_l k + \phi_l k^2\right)\right\} + n_k(t) \quad (7)$$

$$\omega_l = \frac{2\pi d\sin\theta_l}{\lambda}, \quad \phi_l = -\frac{\pi d^2\cos^2\theta_l}{\lambda r_{0,l}} \quad (8)$$

With correlations $z_{-k-1,-k}$ and $z_{k+1,k}$ defined as

$$z_{-k-1,-k} = E\left[x_{-k-1}(t)x_{-k}^*(t)\right] \quad (9)$$

$$z_{k+1,k} = E\left[x_{k+1}(t)x_k^*(t)\right] \quad (10)$$

we construct two vectors $\mathbf{z}_1$ and $\mathbf{z}_2$ called the correlation vectors with length of $2p$, which are given by

$$\mathbf{z}_1 = \left[ z_{p-1,p}\ ,\ \dots\ ,\ z_{-p,-p+1} \right]^T = \mathbf{F}\mathbf{c}_s \in C^{2p\times 1} \quad (11)$$

$$\mathbf{z}_2 = \left[ z_{-p+1,-p}\ ,\ \dots\ ,\ z_{p,p-1} \right]^T = \mathbf{F}\mathbf{\Psi}\mathbf{c}_s \in C^{2p\times 1} \quad (12)$$

$$\mathbf{F} = \mathbf{B}\mathbf{\Phi}\mathbf{\Omega}^*,\ \mathbf{\Psi} = (\mathbf{\Omega})^2 \quad (13)$$

$$\mathbf{B} = \left[ \mathbf{b}(\phi_1), \dots, \mathbf{b}(\phi_L) \right] \quad (14)$$

$$\mathbf{b}(\phi_l) = \left[ e^{-2jp\phi_l},\ e^{-2j(p-1)\phi_l},\ \dots,\ e^{2j(p-1)\phi_l} \right]^T \quad (15)$$

$$\mathbf{\Omega} = \mathrm{diag}\left\{ e^{j\omega_1}, e^{j\omega_2}, \dots, e^{j\omega_L} \right\} \quad (16)$$

$$\mathbf{\Phi} = \mathrm{diag}\left\{ e^{j\phi_1}, e^{j\phi_2}, \dots, e^{j\phi_L} \right\} \quad (17)$$

$$\mathbf{c}_s = \left[ c_{s1}, c_{s2}, \dots, c_{sL} \right]^T \quad (18)$$

$$c_{sl} = E\left[ s_{0,l}(t) s^*_{0,l}(t) \right] \quad (19)$$

From the correlation vectors $\mathbf{z}_1$ and $\mathbf{z}_2$, we have the following equations [2]

$$\mathbf{RF} = \mathbf{F\Psi},\ \mathbf{R} = \mathbf{z}_2 [\mathbf{z}_1]^- \ \left( [\bullet]^- : \text{pseudo-inverse} \right) \quad (20)$$

As shown in (20), $\mathbf{F}$ and $\mathbf{\Psi}$ can be obtained from the eigenvalue decomposition (EVD) of $\mathbf{R}$, and therefore the estimates of $\theta_l$ and $r_{0,l}$ are directly calculated from $\mathbf{F}$ and $\mathbf{\Psi}$ through the above (13)–(17).

## IV. Proposed Method with Spatial Smoothing

Spatial smoothing (SS) is normally applied to the DOA estimation of coherent plane waves using ULA. In the case of the near-field sources, on the other hand, phase difference of the received signal between adjacent elements of the array is not constant because the range and DOA of near-field sources change depending on antenna elements. Consequently, it is impossible to employ SS directly in the near-field problem.

Here, we focus on the correlation vectors $\mathbf{z}_1$, $\mathbf{z}_2$ in the near-field DOA-Matrix method. Phase difference between the adjacent components of each correlation vector is constant. Therefore, we try to apply SS to the correlation vectors. Subarray data are extracted from both correlation vectors $\mathbf{z}_1$, $\mathbf{z}_2$ to create the data matrix as $\mathbf{Z}_1 = [\mathbf{J}_1 \mathbf{z}_1, \dots \mathbf{J}_N \mathbf{z}_1]$, $\mathbf{Z}_2 = [\mathbf{J}_1 \mathbf{z}_2, \dots \mathbf{J}_N \mathbf{z}_2]$ ($\mathbf{J}_n$: matrix for $n$th-subarray extraction, $N$: the number of subarrays), and we calculate $\mathbf{R} = \mathbf{Z}_2[\mathbf{Z}_1]^-$ for (20).

## V. Computer Simulation

The computer simulation of location estimation of near-field sources is carried out under the conditions shown in Table I. Estimation performances of the near-field SAGE-DOA-Matrix methods (SAGE-NFD) with and without SS are compared. We evaluate the location estimation accuracy by means of root mean square errors (RMSE). Figs. 2 to 5 show the RMSE of location estimates and stochastic CRB as a function of SAGE iterations and SNR. The initial value of SAGE algorithm is set equal to the true value plus CRB for basic investigation.

It is found from Figs. 2 to 5 that the method without SS reveals degraded estimation accuracy because of cross-correlation of coherent waves. On the other hand, the method with SS can provide successful location estimation.

## VI. Conclusion

In this paper, the near-field DOA-Matrix method for source localization using spatial smoothing has been proposed. The computer simulation results have shown that the proposed method can provide successful location estimation for coherent sources.

## References

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TABLE I. Simulation conditions

| | | | |
|---|---|---|---|
| Number of elements | 7 | Number of incident signals | 2 |
| Element spacing | $0.5\lambda$ | DOA[ $\theta_1$, $\theta_2$ ] | [ 0°, 30° ] |
| Number of subarrays | 3 | range[ $r_1$ , $r_2$ ] | [ $3\lambda$, $5\lambda$ ] |
| Number of snapshots | 100 | SNR[dB] | 20 |
| Number of SAGE iterations | 30 | Number of trials | 100 |

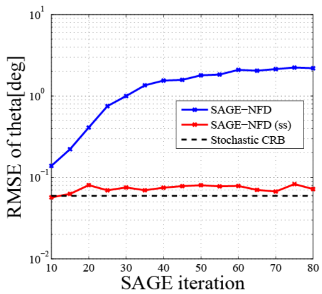


Fig. 2. RMSE of DOA estimates vs. the number of SAGE iterations

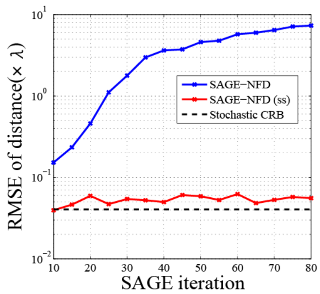


Fig. 3. RMSE of range estimates vs. the number of SAGE iterations

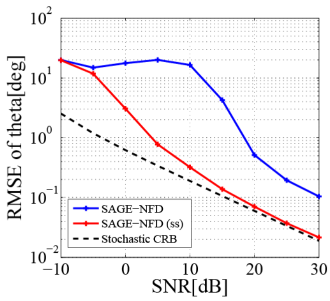


Fig. 4. RMSE of DOA estimates vs. SNR

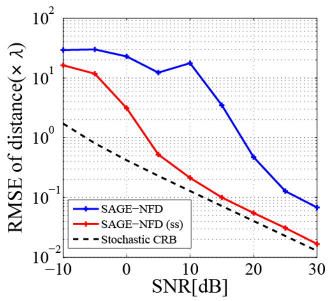


Fig. 5. RMSE of range estimates vs. SNR