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DOA Estimation by MUSIC Algorithm using Forward-Backward Spatial Smoothing with Overlapped and Augmented Arrays

Toshiki Iwai, Naoki Hirose, Nobuyoshi Kikuma, Kunio Sakakibara, and Hiroshi Hirayama
Department of Computer Science and Engineering, Nagoya Institute of Technology
Gokiso-cho, showa-ku, Nagoya 466-8555, Japan

***Abstract* – This paper proposes the forward-backward spatial smoothing with overlapped and augmented arrays (FBSSOA) to improve the conventional spatial smoothing for DOA (Direction-Of-Arrival) estimation. The computer simulation using FBSSOA with MUSIC demonstrates the effectiveness of the FBSSOA in improving the estimation accuracy and the angular resolution of MUSIC.**

***Index Terms* — DOA Estimation, MUSIC, Forward-Backward Spatial Smoothing, Augmented Array Aperture.**

I. Introduction

In estimating the directions of arrival (DOA) of multiple signals incident on an array antenna, the spatial smoothing (SS) is required as a preprocessing scheme when the number of snapshots is small or when the signals are correlated with each other [1]. However, SS has a disadvantage that it reduces the effective array aperture to the size of subarrays [1]. Therefore, we cannot avoid the deterioration in angular resolution and the decrease in the number of incident waves to be estimated. In this paper, we propose the forward-backward spatial smoothing with overlapped and augmented arrays (FBSSOA) which is the improved method of SSOA [2],[3]. Through computer simulation, we show that the proposed method with MUSIC provides higher accuracy of DOA estimation.

II. Analysis Model

We consider a uniform liner array (ULA) of M isotropic antennas in the far field of L signal sources. Let θ_l denote DOA of the lth signal, and then the snapshot vector $\mathbf{x}(t)$ at sample time of t and data matrix $\overline{\mathbf{X}}$ composed of Ns snapshots of $\mathbf{x}(t)$ are given by

$$\mathbf{x}(t) = \mathbf{As}(t) + \mathbf{n}(t) \quad \in \mathbf{C}^{M\times 1} \tag{1}$$

$$\overline{\mathbf{X}} = [\mathbf{x}(1) \quad \mathbf{x}(2) \quad \cdots \quad \mathbf{x}(Ns)] \quad \in \mathbf{C}^{M\times Ns} \tag{2}$$

$$\mathbf{A} = [\mathbf{a}(\theta_1) \quad \mathbf{a}(\theta_2) \quad \cdots \quad \mathbf{a}(\theta_L)] \quad \in \mathbf{C}^{M\times L} \tag{3}$$

where $\mathbf{A}$ is the mode matrix, $\mathbf{a}(\theta_l)$ is the mode vector, $\mathbf{s}(t)$ is the signal vector, and $\mathbf{n}(t)$ is the noise vector.

III. MUSIC Algorithm

The correlation matrix $\mathbf{R}_{xx}$ of data matrix $\overline{\mathbf{X}}$ is obtained from

$$\mathbf{R}_{xx} = \overline{\mathbf{X}}\overline{\mathbf{X}}^H / Ns \tag{4}$$

The mode vectors $\mathbf{a}(\theta_l)$ of incident signals are orthogonal to the eigenvectors in the noise subspace of $\mathbf{R}_{xx}$. Therefore, letting $\boldsymbol{E}_N$ denote the noise-subspace eigenvector matrix of $\mathbf{R}_{xx}$, MUSIC spectrum is given by[1]

$$P_{MU}(\theta) = \frac{\mathbf{a}(\theta)^H \mathbf{a}(\theta)}{\mathbf{a}(\theta)^H \mathbf{E}_N \mathbf{E}_N^H \mathbf{a}(\theta)} \tag{5}$$

We can obtain the DOA estimates from the peaks of the spectrum.

IV. Forward-Backward SSOA

When the number of subarray elements is K and the number of subarrays is N (=M–K+1), data matrices in which SS and FBSS are applied to $\mathbf{x}(t)$ are given by

$$\mathbf{X}_{SS}(t) = \left[\mathbf{J}_1^{(M)}\mathbf{x}(t) \quad \cdots \quad \mathbf{J}_n^{(M)}\mathbf{x}(t) \quad \cdots \quad \mathbf{J}_N^{(M)}\mathbf{x}(t)\right] \in \mathbf{C}^{K\times N} \tag{6}$$

$$\mathbf{X}_{SS}^{FB}(t) = \left[\mathbf{X}_{SS}(t) \quad \mathbf{\Pi}_K \mathbf{X}_{SS}^*(t)\right] \quad \in \mathbf{C}^{K\times 2N} \tag{7}$$

$$\mathbf{J}_n^{(M)} = \left[\mathbf{0}_{K\times(n-1)} \quad \mathbf{I}_K \quad \mathbf{0}_{K\times(N-n)}\right] \quad (n = 1,\cdots,N) \tag{8}$$

Here, $\mathbf{\Pi}_K$ is a $K\times K$ exchange matrix with ones on its antidiagonal and zeros elsewhere.

In SSOA, R groups of subarrays are extracted from $\mathbf{X}_{SS}(t)$ while shifting each group by Δ elements in the column direction of $\mathbf{X}_{SS}(t)$, and R groups obtained are connected in the row direction of matrix. Thus, the data matrix of SSOA is given by

$$\mathbf{X}_{SSOA}(t) = \begin{bmatrix} \mathbf{X}_{SS}(t)\mathbf{J}_{SS}^{(1)} \\ \vdots \\ \mathbf{X}_{SS}(t)\mathbf{J}_{SS}^{(R)} \end{bmatrix} \in \mathbf{C}^{RK\times N'} \quad (9)$$

$$\mathbf{J}_{SS}^{(r)} = \left[\mathbf{0}_{N'\times(r-1)\Delta} \quad \mathbf{I}_{N'} \quad \mathbf{0}_{N'\times(R-r+1)\Delta}\right]^T \quad (r=1,\cdots,R) \quad (10)$$

where $N' = N-(R-1)\Delta$. Since different values of R and Δ yield different augmented data matrix of SSOA, we must choose the optimum values of R and Δ depending on the situation [3]. Furthermore, in this paper, we use FBSSOA which is an improved method of SSOA, and the augmented data matrix is configured by

$$\mathbf{X}_{SSOA}^{FB}(t) = \left[\mathbf{X}_{SSOA}(t) \quad \mathbf{\Pi}_{RK}\mathbf{X}_{SSOA}^{*}(t)\right] \in \mathbf{C}^{RK\times 2N'} \quad (11)$$

The correlation matrix $\mathbf{R}_{xx}$ is calculated by using the data matrix after the spatial smoothing processing, and DOA estimation is performed by MUSIC algorithm.

V. Computer Simulation

The simulation of DOA estimation is carried out under the conditions shown in Table I. We evaluate DOA estimation accuracy by means of root mean square errors (RMSE). The values of number of R and Δ are (R=2, Δ=1) for MUSIC with SSOA, and (R=2, Δ=4) for MUSIC with FBSSOA which are an optimum combination obtained from simulation. Fig. 2 shows the RMSE of DOA estimates as a function of SNR. It is found that MUSIC with FBSSOA gives the best performance. Particularly, the performance is remarkably improved at SNRs more than 10dB. Fig. 3 shows the RMSE of DOA estimates when DOA of the first wave is fixed and angle separation of 9 waves are changed equally. It is confirmed that MUSIC with FBSSOA maintains the highest estimation accuracy even when the angle separation of 9 waves are small, which means that it has higher angular resolution than the other methods.

VI. Conclusion

In this paper, we have proposed forward-backward spatial smoothing with overlapped and augmented arrays (FBSSOA) in order to improve aperture reduction in conventional spatial smoothing. As a result of DOA estimation simulation, it is shown that MUSIC with FBSSOA holds superiority in DOA estimation performance over MUSIC with the other SS schemes. Also, FBSSOA is found to be able to enhance the angular resolution capability, which means that FBSSOA provides the increased effective array aperture.

Future work is to examine the way of determining the number of subarray elements and (R, Δ) of SSOA and FBSSOA for obtaining their optimum performance.

References

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TABLE I
SIMULATION CONDITIONS

Array configuration	Uniform liner array
Antenna element	Isotropic
Number of array elements (M)	34
Number of subarray elements (K)	17
Number of waves (L)	9 (coherent)
DOAs $[\theta_1,\ldots,\theta_L]$	$[-30°,-24°,-18°,-12°, -6°,0°,6°,12°,18°]$
Signal waveform	Gaussian signal
Number of snapshots	3
Number of trials	2000

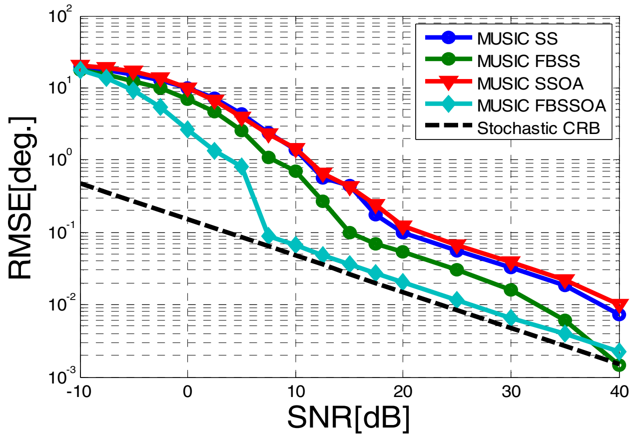

Fig. 2. RMSE of DOA estimates vs. SNR

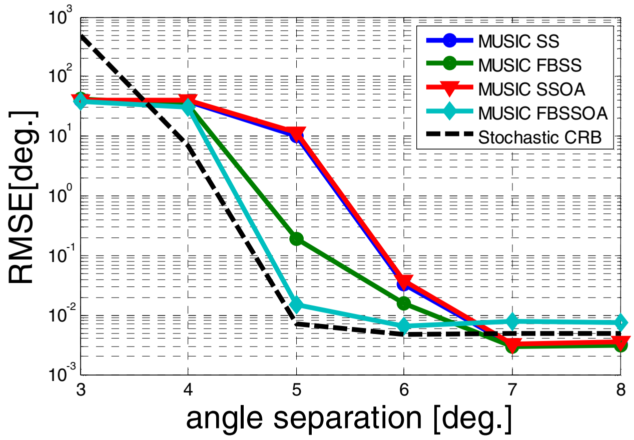

Fig. 3. RMSE of DOA estimates vs. angle separation (SNR=10dB)