Broadband Characterization of Planar Transmission Line Substrate Permittivity up to 67 GHz

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***Abstract*—In this paper, it is shown how different planar transmission lines (TL) such as microstrip (MS), coplanar waveguide (CPW) and grounded CPW (GCPW) can be used to characterize substrate permittivity using on-wafer probes and a thru-reflect-line (TRL) calibration. From measured S-Parameters, the effective permittivity $\varepsilon_{\mathrm{r,eff}}$ can be extracted and fast and precisely mapped to the physical value ε_{r} of the TL's substrate using simulations. The results up to 67 GHz for the aforementioned TL on a conventional RF substrate are presented and show very good agreement with each other as well as data supplied by the substrate manufacturer.**

***Index Terms*—Permittivity measurement, dielectric properties, planar transmission lines, TRL, dielectric constant.**

I. INTRODUCTION

For electronic applications, knowledge of the dielectric properties of the used materials is crucial for allowing a proper design procedure. Most material manufacturers and simulation tools only provide frequency-limited or static material data, which decreases simulation accuracy as the frequency increases and material dispersion becomes more significant.

Transmission/reflection methods have been widely investigated and used for dielectric material characterization using rectangular waveguide or coaxial transmission line cells [1], [2]. These methods can be adapted to planar transmission lines under certain conditions [2]. Using these techniques on TL, $\varepsilon_{\mathrm{r,eff}}$ can be calculated from S-Parameter measurements. A subsequent calculation can then evaluate the dispersion effects and map the $\varepsilon_{\mathrm{r,eff}}$ to the actual, physical ε_{r} of the substrate.

For the most relevant TL with respect to applications, dispersion models have been published, which often rely on curve fits (such as [3] for MS or [4] for CPW) or are mere quasi-static approximations. These models can only provide limited accuracy, especially if frequency and thus dispersion is increasing. To circumvent this issue and be able to evaluate the dispersion even for higher frequencies, we will use FEM simulations to correct for dispersion and do a fast and easily implemented mapping of the permittivity.

II. THEORY

Using signal flow graph theory, useful relations between S-Parameters and transmission (T) as well as reflection (Γ) coefficient can be found [1]:

$$S_{11} = S_{22} = \frac{(1-T^2)\Gamma}{1-\Gamma^2 T^2} \tag{1}$$

$$S_{21} = S_{12} = \frac{(1-\Gamma^2)T}{1-\Gamma^2 T^2} \tag{2}$$

Combining (1) and (2), the coefficients can be expressed by the S-Parameters:

$$T = \frac{(S_{11}+S_{21})-\Gamma}{1-(S_{11}+S_{21})\Gamma} \tag{3}$$

$$\Gamma = K \pm \sqrt{K^2-1} \quad \text{with} \quad K = \frac{S_{11}^2 - S_{21}^2 + 1}{2S_{11}}, \tag{4}$$

where the sign in (4) is chosen so that $|\Gamma| \leq 1$, which is required for causal, passive materials.

In general, for TL the transmission coefficient is given by

$$T = e^{-\gamma\, l}, \tag{5}$$

where $\gamma = \alpha + j\beta$ is the propagation constant and *l* the length of the transmission line. The propagation constant is

$$\gamma = -\frac{\ln(T)}{l} = -\frac{1}{l}\left[\ln(|T|) + j[\varphi(T) + 2\pi n]\right] \tag{6}$$

and the phase constant can be written as

$$\mathrm{Im}(\gamma) = \beta = \frac{\omega}{c_0}\sqrt{\varepsilon_{\mathrm{r}}}, \tag{7}$$

where $n \in \mathbb{Z}$, ω denotes the angular frequency, c_0 the velocity of light in vacuum and ε_{r} the relative permittivity of the transmission line.

Using (3) and (6), one obtains the following relation for the relative permittivity:

$$\varepsilon_{\mathrm{r}} = \left[\frac{c_0}{\omega}\beta\right]^2 = \left[\frac{c_0}{\omega}\frac{\varphi'(T)}{l}\right]^2, \tag{8}$$

where $\varphi'(T) = \varphi(T) + 2\pi n$ denotes the corrected phase.

Eq. (7) only holds true on condition of a single transversal-electromagnetic (TEM) mode of propagation, which cannot be satisfied in a straight manner by a planar transmission line. However, most lines of this kind can be assumed to work in a quasi-TEM mode with effective material parameters modeling a homogeneous cross-section, as long as single mode operation is maintained and higher order modes are sufficiently suppressed. In this case, ε_{r} in Eq. 7 and 8 is replaced by $\varepsilon_{\mathrm{r,eff}}$ of the TL's equivalent cross-section.

III. MEASUREMENT AND SIMULATIONS

The measurements were carried out on a wafer prober station using GGB Pico Probes Model 67A and a Network Analyzer up to 67 GHz. The multiline TRL calibration [5] using a custom set of calibration standards has been fabricated on the same substrate as the transmission lines (Rogers Corp. 4003C$^{\text{TM}}$ [6]). Using the TRL algorithm for calibration, the reference plane of the measurement phase can be shifted onto the TL, which effectively cancels out the probe-TL interface during measurement and allows us to use the theory presented in Section II.

Each TL type was fabricated in different lenghts. The results displayed in this paper are averaged over those measurements of the same TL type.

The MS and GCPW have been fabricated on the same substrate with a height of 203µm, whereas the CPW has been fabricated on a substrate with height 508µm. Due to fabrication issues concerning the probe-MS interface structure, the MS lines could only be measured properly up to 24 GHz. Results for $\varepsilon_{\text{r,eff}}$ can be seen in Fig. 1.

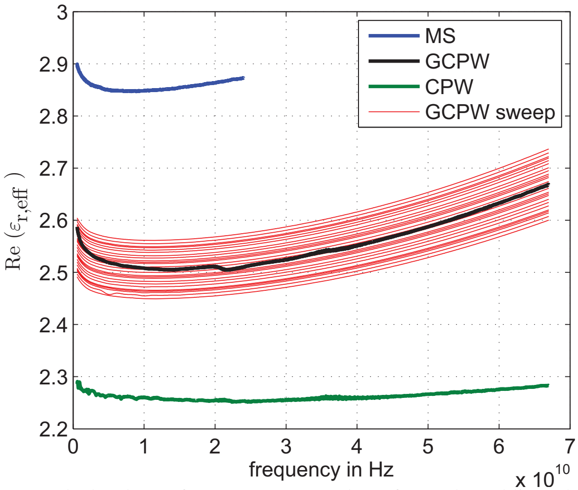

Fig. 1: Calculated $\varepsilon_{\text{r,eff}}$ from measurement data of TL. The red lines show the curves obtained by our parameter sweep exemplarily for GCPW.

To find ε_{r}, the TLs have been modeled and simulated using Ansys HFSS. A parameter sweep of ε_{r} with a step size of 0.01 has been carried out for every type of TL and the resulting S-Parameters have been used for calculation of $\varepsilon_{\text{r,eff}}$. A simple search algorithm compares the $\varepsilon_{\text{r,eff}}$ obtained during measurement with the ones calculated from simulation data and maps it to the closest result[1], see Fig. 2 for resulting ε_{r}.

Since we used a substrate of the same material but different height for fabrication of the CPW, there is a slight difference between ε_r for the substrate with MS and GCPW and the substrate with CPW. We assume this can be explained by RO4003C$^{\text{TM}}$ fabrication tolerances. In general, the results obtained by our method agree well with the data given by Rogers Corp. [6].

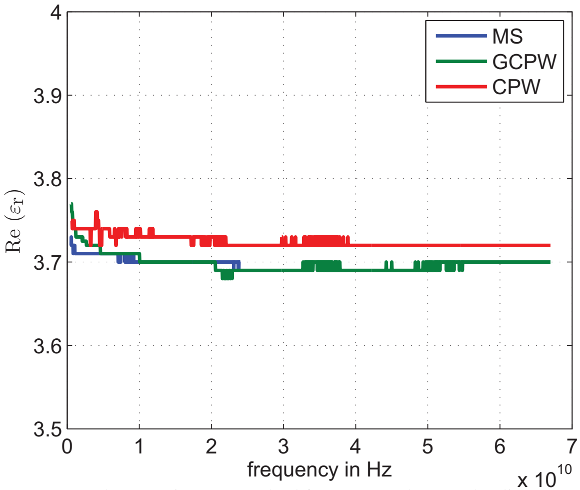

Fig. 2: Mapped $\varepsilon_{\text{r,eff}}$ using our method for measured TLs. The discrete peaks in the data are due to the parameter sweep with 0.01 step size and not of physical origin. This effect can be compensated by a smaller step size or a slight curve fitting to smoothen the curve, both of which seemed not reasonable to the authors, as 0.01 gives a precise enough value and is being assumed to lie near the accuracy bounds of the actual measurement.

IV. CONCLUSION

Different TLs have been measured up to 67 GHz and the related substrate's ε_{r} could be determined using our method. The results for MS and GCPW on RO4003C$^{\text{TM}}$ agree very well. The result for CPW on a substrate of the same material, but different height, shows a comparable value and differences are assumed to arise only from fabrication tolerances. Our results represent the physical conditions of the measured TLs properly, since full-wave analysis of the actual structures has been used for the dispersion mapping. Since we use the same method to map S-Parameters to $\varepsilon_{\text{r,eff}}$ for measurement as well as simulation data, possible errors may only be introcuded by measurement or simulation inaccuracies.

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[1] Since we use the same method to map S-Parameters to $\varepsilon_{\text{r,eff}}$ for measurement as well as simulation data, possible errors introduced by our method are neglectable up to this second mapping procedure.