A new ADI-PE scheme with Fourth-order Accuracy for Radio Wave Propagation Prediction in Tunnels

Xianshi He, Dawei Li, Junhong Wang, Shouliang Li
Institute of Lightwave Technology, Beijing Jiaotong University, Beijing 100044, China
12120079@bjtu.edu.cn, wangjunh@bjtu.edu.cn

Abstract **– A new method for implementing the impedance boundary condition (IBC) of tunnel environment is presented, which is used in the Alternate Direction Implicit (ADI) method for solving the parabolic equation. The proposed method has fourth-order accuracy, and is used in the prediction of radio wave propagation in tunnels. Simulation results show that the proposed method not only has higher accuracy than available methods, but also retains the computational efficiency of the available methods with lower order accuracy.**

Index Terms **— Alternative Direction Implicit (ADI), impedance boundary condition, parabolic equation, radio wave propagation, tunnel environment.**

I. Introduction

The development of mobile communication leads to increasing interests in the research of radio wave propagation in confined spaces, such as tunnels. Among various methods for predicting wave propagation in tunnels, the Parabolic Equation (PE) method is the one with efficiency in computation time, and the Alternate Direction Implicit (ADI) technique, which is an unconditionally stable finite difference method, is usually used to solve the PE in tunnel scenarios. ADI technique transforms the three-dimensional problems to a series of two-dimensional problems, and the Peaceman-Rachford (PR)-ADI [1] and Mitchell-Fairweather (MF)-ADI [2] are the two typical ADI approaches. The (PR)-ADI method is a second-order accuracy method, while the (MF)-ADI method is a fourth-order accuracy method. It is difficult to implement the boundary condition of the ADI methods, because the field strengths on top and bottom boundaries of the intermediate marching planes cannot be obtained from the available ADI-PE algorithms, and the fields on the left- and right-side boundaries were supposed directly the same as the fields on the boundaries of the physical planes. Thus, the overall accuracy of the available work is only in the first order [1, 3]. This paper is dedicated to improve the overall accuracy of ADI-PE technique by modifying the discretized formulas of the impedance boundary condition.

II. Impedance Boundary Condition Scheme for Alternate Direction Implicit

Tunnels are usually made of concrete and soil, Leontovich boundary condition which is used to characterize the electrical parameters of tunnel walls is defined by [4]

$$\hat{n}\times\vec{E} = -\bar{Z}\hat{n}\times\left(\hat{n}\times\vec{H}\right) \tag{1}$$

In this paper, we introduce virtual points $u_{-1,l}$, $u_{m+1,l}$, $u_{m,l+1}$, $u_{m,-1}$ outside the left and right sidewalls by a distance of Δx, and upper and down the ceiling and floor by a distance of Δy, as shown in Fig.1. Using fourth-order accuracy finite-difference to approximate the first-order derivative, we get

$$\partial u/\partial\xi \simeq \left(-3u_{\xi_i-1} - 10u_{\xi_i} + 18u_{\xi_i+1} - 6u_{\xi_i+2} + u_{\xi_i+3}\right)/12\Delta\xi \tag{2}$$

Equation (2) is used to discretize the IBC in (1), e.g., at $x = 0$ of the intermediate plane, the values at virtual points can be expressed as

$$\tilde{u}_{-1,l}^{n+1/2} = 6\tilde{u}_{1,l}^{n+1/2} - 2\tilde{u}_{2,l}^{n+1/2} + \tilde{u}_{3,l}^{n+1/2}/3 - (10/3 + 4jk_0\Delta x/Z)\cdot\tilde{u}_{0,l}^{n+1/2} \tag{3}$$

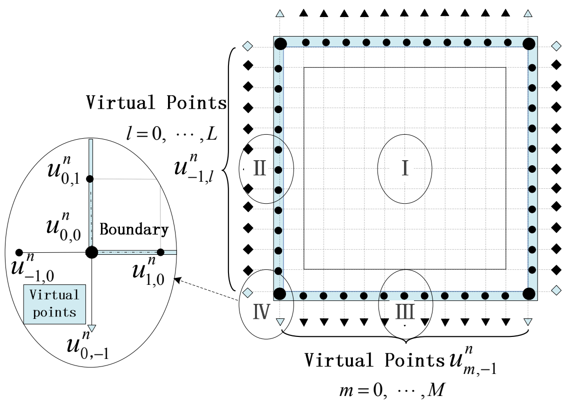

Fig. 1. Schematic diagram of the discretization of the impedance boundary condition for ADI method.

The values got from (3) are then utilized to replace the values at virtual points occurring in the difference equation of ADI-PE equation [2]. Then, we obtain the discretizing formulas for the impedance boundary conditions of the intermediate plane

$$\left[1-\left(r_x/(4jk_0)-1/12\right)\beta_x\right]\cdot\tilde{u}_{m,l}^{n+\frac{1}{2}} = \left[1+\left(r_y/4jk_0+1/12\right)\delta_y\right]u_{m,l}^{n} \tag{4}$$

$$\left[1-\left(r_x/(4jk_0)-1/12\right)\delta_x\right]\cdot\tilde{u}_{m,l}^{n+\frac{1}{2}} = \left[1+\left(r_y/4jk_0+1/12\right)\beta_y\right]u_{m,l}^{n} \tag{5}$$

$$\left[1-\left(r_x/(4jk_0)-1/12\right)\beta_x\right]\cdot\tilde{u}_{m,l}^{n+\frac{1}{2}} = \left[1+\left(r_y/4jk_0+1/12\right)\beta_y\right]u_{m,l}^{n} \tag{6}$$

where

$$\beta_x \cdot \tilde{u}_{m,l}^{n+\frac{1}{2}} = 7\tilde{u}_{1,l}^{n+\frac{1}{2}} - 2\tilde{u}_{2,l}^{n+\frac{1}{2}} + \tilde{u}_{3,l}^{n+\frac{1}{2}} / 3 - \left(16/3 + 4jk_0\Delta x / Z\right)\cdot \tilde{u}_{0,l}^{n+\frac{1}{2}} \quad (7)$$

$$\beta_y \cdot u_{m,l}^{n} = 7u_{m,1}^{n} - 2u_{m,2}^{n} + u_{m,3}^{n} / 3 - \left(16/3 + 4jk_0\Delta y Z\right)\cdot u_{m,0}^{n} \quad (8)$$

In this paper, the tunnel boundaries are divided into three regions: sidewall regions II, ceiling/floor regions III and the corner regions IV, and the fields in these regions are calculated by impedance boundary condition (4), (5) and (6) respectively, and the overall accuracy is fourth order. The formulas of IBC are so similar to that of the (MF)-ADI scheme that the IBC can be implemented as part of the (MF)-ADI iteration process. Here, we introduce a matrix A, whose main diagonal entries are 1, and $a_{1,2}$ = [4/3－1/3/(r/4jk_0－1/12))], $a_{1,3}$ = －1/3. By premultiplying the coefficient matrix of fields on the intermediate plane with the matrix A, we can use Thomas algorithm to solve the tri-diagonal matrix. Thus, the new approach maintains the computational efficiency. The impedance boundary conditions for the second step of (MF)-ADI method can be derived similarly.

III. Results

In this section, we compare the simulation results of the new approach with the known analytical solutions of lossy tunnel and the measurement results of a realistic tunnel.

The real tunnel can be seen as an imperfect waveguide, in which hybrid modes involving TE and TM modes exist. For the vertical polarized fundamental mode $EH_{1,1}$ at frequency of 900 MHz [5], the transverse discretized grids of a tunnel with 6×6 m cross-section are Δx = Δy = 0.24λ, and the discretized grid along tunnel axis is Δz = 5.556λ, the electrical parameters for the tunnel walls are ε_r = 6.8 and σ_o = 0.034 S/m. In order to see the improvement of the proposed fourth-order accuracy impedance boundary condition, the RMS error [1] of the proposed method is compared to that of the (MF)-ADI method with first-order accuracy, as shown in table I.

TABLE I
The Normalized RMS Error for the Rectangular Waveguides with Impedance Boundary Conditions

RMS (%)	Axial Distance (m)			
	50	200	500	1000
First order	3.75	8.34	21.45	47.73
Fourth order	0.52	0.69	1.62	3.22

Evidently, from table I we can see that the RMS error varies with the propagation distance, the longer distance between transmitting and receiving antennas, the larger RMS error of the ADI scheme. However, the proposed method has a much smaller RMS error compared with the first-order accuracy scheme. The field strength of the hybrid modes $EH_{1\text{-}7,1\text{-}7}$ in the centre line of the tunnel as function of the axis distance is depicted in Fig.2 (a). The analytical result is also given for comparison. It is seen that the magnitudes of the analytical and numerical results are in good agreement.

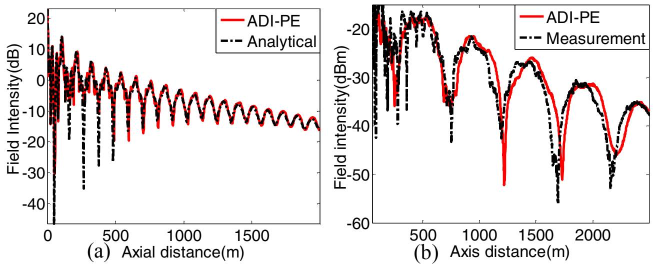

Fig. 2. Comparison of the field magnitudes of (a) hybrid modes along axial direction and (b) measurements and simulation.

The proposed method is also applied to model the signal propagation at 900MHz in Massif Central tunnel in south-central France studied previously by Liénard [6]. The transverse section of the tunnel is 7.8 × 5.3 m^2, and the electrical parameters are ε_r = 5 and σ_o = 0.01 S/m. The electric-field intensities are shown in Fig.2 (b), and we can find good agreement between the (MF)-ADI simulated result and measured results.

IV. Conclusion

An ADI method combined with the fourth-order accuracy impedance boundary conditions for solving parabolic equation in tunnels is presented. The new ADI-PE approach has much better performance than the available ADI-PE methods with lower order accuracy boundary condition. The computational efficiency of the proposed scheme maintains the same as that of the lower order accuracy methods.

Acknowledgment

The research work was supported by National Natural Science Foundation of China under Grant No. 61331002. The authors would like to appreciate Prof. Martin Liénard for providing us the experimental data as shown in Fig. 2 (b).

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