New Analytical Design Methods for Two-Dimensional Filters

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The field of two-dimensional filters and their design methods has been developed by many researchers, for more than three decades. This tutorial reviews several 2D filter design methods that I have developed based on frequency transformations. Starting from 1D prototype filter with a desired characteristics (low-pass maximally-flat, selective low-pass or band-pass etc.), some specific spectral transformations are applied in order to obtain the 2D filter with a desired shape of the frequency response.

Various types of 2D filters are approached: maximally-flat low-pass or sharp-band-pass filters with circular or elliptically-symmetric frequency response, directional filters, oriented wedge-shaped and fan filters, diamond-shaped filters, multi-directional filters in polar coordinates etc. All these filters have already found specific applications in image processing. The design is focused on recursive (IIR) filters, since they are the most efficient, but also some FIR filters are discussed. Generally, the design starts from a 1D prototype which is a common digital filter, either maximally-flat or equiripple (Butterworth, Chebyshev etc.) given by a transfer function in variable z, decomposed into a product of functions of first or second order.

In this case the design consists in finding the specific complex frequency transformation from the variable z to the complex plane (z_1, z_2). Once found this mapping, the 2D filter function results directly through substitution. The case of zero-phase 2D filters is treated as well since they are very useful in various image filtering applications due to the absence of phase distortions. This method is at the same time simple, efficient and versatile, since once found the adequate frequency transformation, it can be applied to different prototype filters obtaining the same type of 2D filter. The latter inherits the selectivity properties of its 1D counterpart (bandwidth, flatness, transition band etc.). Changing the prototype filter parameters will change the properties of the obtained 2D filter. All the proposed design techniques are mainly analytical but also involve numerical optimization, in particular rational approximations (Padé or Chebyshev-Padé). As the design starts from a factorized transfer function, the 2D filter also results directly factorized, a major advantage in its implementation.

Some methods use the bilinear transform as an intermediate step. The designed filters may present non-linearity distortions towards the margins of the frequency plane, due to the frequency warping effect. In order to compensate for these errors, a pre-warping is applied, which however will increase the filter order. Other proposed methods avoid from the start the use of bilinear transform and the filter coefficients result through a change of frequency variable and a bi-variate Taylor or Chebyshev expansion of the filter frequency response. Finally, the filter transfer function in z_1 and z_2 results directly by identification of the 2D Z transform terms.

Some applications of the designed 2D filters are also shown. Simulation results are provided for some test images and also real-life greyscale images, especially for the case of directional and wedge-shaped filters. These filters may be used in detecting and separating lines and other image details with a specified orientation. The lines whose spectrum overlaps with the filter frequency response are preserved, while the others are more or less blurred through low-pass filtering, depending on their orientation. Some filtering examples on biomedical images are also given. For instance, directional filter banks, composed of several selective filters on different directions, may be used in analyzing angiography images, by detecting vessels with a given orientation, which could provide valuable information to clinicians.