# An Eight-Channel FDM System Using Mach-Zehnder Filters with Cosine Roll-Off Band-Limiting Characteristics

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***Abstract*—An eight-channel FDM system using Mach-Zehnder filters is equalized in order to be band-limited in cosine roll-off characteristics for no inter-symbol interference as an example of a multi-channel FDM system. The linearly equalization method is applied and the channel compensation is split equally between the transmitter and receiver filters. The equalizers consist of adaptively designed finite impulse response filters. The tap coefficients are obtained so as to satisfy the design objective transfer functions originated from both of the cosine roll-off characteristics and the Mach-Zehnder filter characteristics. As a result, the eye is fully open even when the neighboring channel spacing becomes narrow. The inter-symbol interference results in almost zero and the inter-channel interference powers decrease. Thus, the signal to inter-channel interference ratio increases more than 24 dB for the roll-off factor of 0.2.**

***Keywords*—Mach-Zehnder filter; finite impulse response filter; cosine roll-off; equalization; inter-symbol interference; inter-channel interference**

## I. INTRODUCTION

With the advances of the optical communication technology, the multiplexing schemes have been developed including time division multiplexing and wavelength division multiplexing schemes. Currently, frequency division multiplexing (FDM) scheme is expected as the key technology to increase the capacity and flexibility of those networks [1,2,3]. Applying the digital signal processing techniques to the techniques in the optical communications, the receiving sensitivity and the frequency utilization efficiency are increased by the coherent detection of the signals [4,5]. The current goal of multiplexing is the high speed transmission of 100 signals at 100 Gbit/s with the transfer capacity of 10 Tbit/s or more [3,4,5]. A four-channel FDM system using waveguide type Mach-Zehnder filters was proposed showing specifically a neighboring channel spacing of 5 GHz [6]. Since the Mach-Zehnder filter (MZF) has periodic spectral characteristics, it is feasible to use it for the FDM system [1,2].

The authors proposed the configuration of a $2^N$-channel FDM system using MZF ($2^N$-channel MZ-FDM) [7]. Transmission scheme was ASK. Thus far, by the analysis of the MZ-FDM characteristics, the inter-symbol interference (ISI) and the inter-channel interference (ICI) were observed and the eye aperture decreased when the channel spacing became narrower [7,8]. Thus, it is required to equalize the MZ-FDM in order to reduce the ISI. At present, we choose the linearly equalization method [9] and split the channel compensation equally between the transmitter and receiver filters.

The present paper deals with the effect of the equalization for the eight-channel MZ-FDM as an example of the $2^N$-channel MZ-FDM. The finite impulse response (FIR) filters are connected in series to each input and output port of the eight-channel MZ-FDM in order that a band-limiting scheme of the total system becomes cosine roll-off characteristics for no ISI. The eye pattern of the receive signal, and the signal to ICI ratio have been calculated numerically and evaluated.

## II. SYSTEM CONFIGURATION

Fig.1 shows the equivalent lowpass system configuration for the MZ-FDM with the FIR filters. The transmit and receive signals are designated as $g_T(t)$ and $g_R(t)$, respectively. The frequency spectra of those signals are designated as $G_T(\omega)$ and $G_R(\omega)$, respectively. The transfer functions of the FIR filter and the MZ-FDM at the transmitter are designated as $H_T^{(FIR)}(\omega)$ and $H_T^{(MZ)}(\omega)$, respectively. The transfer functions of the MZ-FDM and the FIR filter at the receiver are designated as $H_R^{(MZ)}(\omega)$ and $H_R^{(FIR)}(\omega)$, respectively. Thus, the receive signal spectrum can be expressed as follows:

$$G_R(\omega) = G_T(\omega) H_T(\omega) H_R(\omega)\,, \tag{1}$$

where

$$H_T(\omega) = H_T^{(FIR)}(\omega) H_T^{(MZ)}(\omega)\,, \tag{2}$$

$$H_R(\omega) = H_R^{(MZ)}(\omega) H_R^{(FIR)}(\omega)\,. \tag{3}$$

## III. AN EIGHT-CHANNEL MZ-FDM

Fig.2 shows an eight-channel FDM system using Mach-Zehnder filters (an eight-channel MZ-FDM) configuration. A square box indicates a Mach-Zehnder filter (MZF) model. A notched mark indicates a matched termination. The general expressions of the transfer functions for the MZF and MZ-FDM were presented in [7]. The left hand side of Fig.2 is assumed to be transmitter including 7 MZFs. The right hand side is assumed to be receiver including 7 MZFs. Channels between identical port numbers are assumed to be desired ones. Channels between different port numbers are interference ones.

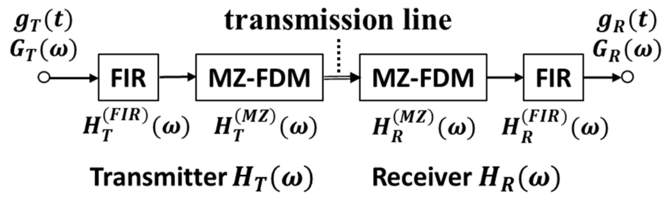


Fig.1 System configuration for MZ-FDMs with the FIR filters.

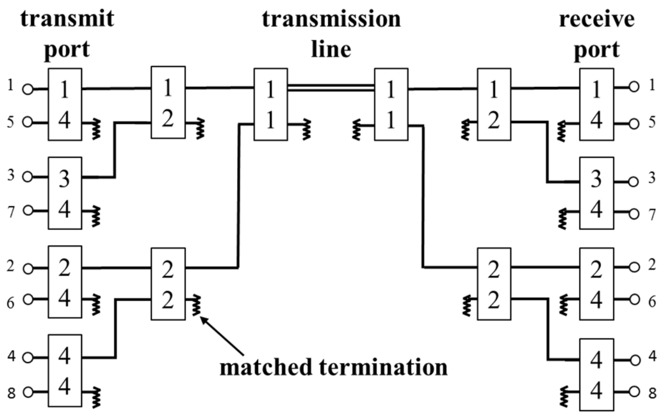


Fig.2 Configuration of an eight-channel FDM system using Mach-Zehnder filters (an eight-channel MZ-FDM).

## IV. Design of FIR Filters

The tap coefficient of FIR filter is assumed to be $c_n$ where n is 0, ±1, ±2, ⋯, and ±M. The unit delay time is assumed to be $T_0$. The total transfer function $G_R(\omega)$ is forced to agree with cosine roll-off characteristics $G_{cr}(\omega)$ which satisfy the first Nyquist condition [9]. Since we have split the channel compensation equally between the transmitter and receiver filters, the transfer function characteristics at the transmitter and the receiver are forced to agree with the root cosine roll-off characteristics as shown in (5) and (6).

$$G_R(\omega) = G_{cr}(\omega) \tag{4}$$

$$G_T(\omega)H_T(\omega) = \sqrt{G_{cr}(\omega)} \tag{5}$$

$$H_R(\omega) = \sqrt{G_{cr}(\omega)} \tag{6}$$

Thus, the tap coefficients of FIR filters are determined in order that the transfer functions satisfy the following equations:

$$H_T^{(FIR)}(\omega) = \sqrt{G_{cr}(\omega)}\Big/\left\{G_T(\omega)H_T^{(MZ)}(\omega)\right\}, \tag{7}$$

$$H_R^{(FIR)}(\omega) = \sqrt{G_{cr}(\omega)}\Big/H_R^{(MZ)}(\omega). \tag{8}$$

The equations (7) and (8) are the design objective transfer functions of FIR filters at the transmitter and the receiver.

A baseband signal $g_T(t)$ is assumed to be a rectangular pulse with unit amplitude and pulse width of $T$. The frequency spectrum $G_T(\omega)$ of $g_T(t)$ is given as follows:

$$G_T(\omega) = T \cdot \mathrm{sinc}(\omega/\omega_r), \tag{9}$$

where $\omega_r$ $(= 2\pi/T)$ is the clock radian frequency. $H_T^{(MZ)}(\omega)$ and $H_R^{(MZ)}(\omega)$ are the equivalent lowpass transfer functions of MZ-FDM at the transmitter and receiver, respectively. The general expressions of the transfer functions of the MZ-FDM and MZF are given in [7].

Assuming that the transfer function of the FIR filter is $H^{(FIR)}(\omega)$ with the tap coefficients $c_{m-M}$, the following expressions (10) and (11) hold in general. They are a Fourier transform pair in a discrete time signal system [10].

$$H^{(FIR)}(\omega) = \sum_{m=0}^{2M} c_{m-M} e^{-jmT_0\omega}, \tag{10}$$

$$c_{m-M} = \frac{1}{2\Omega_0}\int_{-\Omega_0}^{\Omega_0} H^{(FIR)}(\omega) e^{j(m-M)T_0\omega} d\omega, \tag{11}$$

where

$$T_0 = \pi/\Omega_0, \tag{12}$$

and $\Omega_0$ is assumed to be the radian frequency specifying the approximate spectral range for the design. Let $H^{(FIR)}(\omega)$ in (11) be replaced by (7) in the case of the filter design at the transmitter. And in the case of the filter design at the receiver, let $H^{(FIR)}(\omega)$ in (11) be replaced by (8). And then, when those functions of (7) and (8) are squarely approximated, equation (11) can be changed into (13) which shows the form of an inverse discrete Fourier transform.

$$c_{m-M} = \frac{1}{N}\sum_{k=-N/2}^{N/2-1} H^{(FIR)}(k\omega_s) W_N^{(m-M)k}, \tag{13}$$

where

$$\omega_s = 2\Omega_0 / N, \tag{14}$$

$$W_N = e^{j2\pi/N}, \tag{15}$$

$\omega_s$ is the sampling radian frequency, and N is the sampling number. Equation (13) is used to calculate tap coefficients.

## V. Inter-Channel Interference

### A. Interference Signal PSD

The transmission scheme is assumed to be ASK. A channel 1 is assumed to be a desired channel here. The transmit signal $s_{T1}(t)$ at the channel 1 can be represented as follows:

$$s_{T1}(t) = \sum_{k=-\infty}^{\infty} a_k g_T(t - kT)\cos\omega_1 t, \tag{16}$$

where $a_k$ is an independent and identically distributed (i.i.d.) random variable that takes 1 or 0, and $\omega_1$ is the center radian frequency of the channel 1.

The transmit signal $s_{Ti}(t)$ at the other channel $i$ is wide sense stationary random process due to the incoherent transmission to the desired channel 1 and is assumed to be given as follows:

$$s_{Ti}(t) = \sum_{k=-\infty}^{\infty} b_{ik} g_T(t - kT + \xi_i)\cos(\omega_i t + \phi_i), \tag{17}$$

where $\omega_i$ is the center radian frequency at the channel $i$, $b_{ik}$ are i.i.d. random variables that take 1 or 0, $\xi_i$ is uniformly distributed over the range of $(0, T)$, $\phi_i$ is uniformly distributed over the range of $(0, 2\pi)$, and $i$ takes from 2 to 8. The power spectral densities (PSDs) of the signals at the channels from 2 to 8 are obtained from the Fourier transform of the autocorrelation functions of those signals [9,10]. As final results of calculation, their PSDs are given as follows:

$$W_{Ti}(\omega) = \frac{1}{16T}\left\{\left|G_T(\omega-\omega_i)\right|^2 + \left|G_T(\omega+\omega_i)\right|^2\right\} + \frac{\pi}{8}\left\{\delta(\omega-\omega_i) + \delta(\omega+\omega_i)\right\}, \tag{18}$$

where $G_T(\omega)$ is the frequency spectrum of $g_T(t)$ as in (9), and $\delta(\omega)$ is the Dirac delta function.

The inter-channel interference signal PSDs at the channel 1 output of receiver from transmitting channels 2 to 8 are designated as $W_{Ri1}(\omega)$. The PSDs are given as follows:

$$W_{Ri1}(\omega) = W_{Ti}(\omega)\left|H_{Ti}(\omega)\right|^2 \left|H_{R1}(\omega)\right|^2 , \tag{19}$$

where

$$H_{Ti}(\omega) = H_{Ti}^{(FIR)}(\omega - \omega_i) H_{Ti}^{(MZ)}(\omega) , \tag{20}$$

$$H_{R1}(\omega) = H_{R1}^{(MZ)}(\omega) H_{R1}^{(FIR)}(\omega - \omega_1) . \tag{21}$$

*B. A Signal to Inter-Channel Interference Ratio*

The signal to ICI ratio $\rho_1$ at the desired channel 1 of receiver can be expressed as follows:

$$\rho_1 = (A_e^2 / 2) \Big/ \sum_{i=2}^{8} P_{Ri1} , \tag{22}$$

where $P_{Ri1}$ is the interference power at the channel 1 output of receiver from transmitting channels 2 to 8, and can be obtained as follows:

$$P_{Ri1} = \frac{1}{2\pi} \int_{-\infty}^{\infty} W_{Ri1}(\omega) \mathrm{d}\omega . \tag{23}$$

The other signal to ICI ratios can be obtained in the same manner.

Fig. 3 shows the signal to ICI ratio $\rho_1$ at channel 1 output of receiver as a function of $\omega_d/\omega_r$. Thick solid line and thick dotted line show $\rho_1$ characteristics in the case of roll-off factor of 0.2 and 0.5, respectively. Thin solid line shows $\rho_1$ characteristics with no equalizer. Totally, $\rho_1$ is increasing as $\omega_d/\omega_r$ increases. This is due to the increase in channel spacing.

$\rho_1$ with equalizer is increasing extremely due to the fact that the eye is fully open and the interference powers decrease considerably. $\rho_1$ in the case of roll-off factor 0.2 increases more rapid as compared to the case of 0.5. This is due to the lower interference in the case of 0.2 originated from the sharp roll-off characteristics.

As results of calculation in the cases of the other desired channels, it is found that $\rho_1$ equals $\rho_8$, and their values are higher than the other signal to ICI ratios by 3 dB due to the interference from one sided neighboring channel.

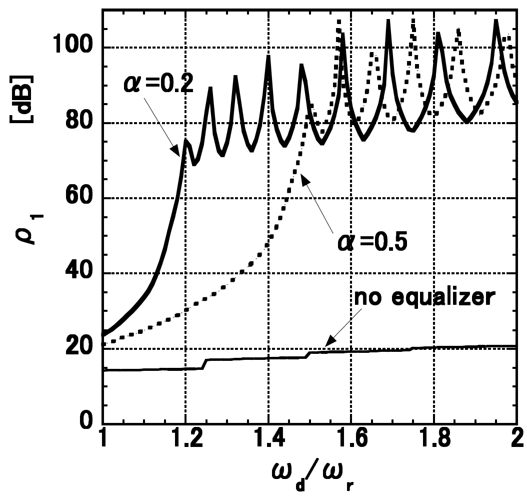


Fig. 3 The signal to inter-channel interference ratio $\rho_1$.

## VI. Conclusion

We equalize the eight-channel MZ-FDM as an example of the $2^N$-channel MZ-FDM by connecting adaptively designed FIR filters serially to the system to realize cosine roll-off band-limiting characteristics for no ISI. At present, we split the channel compensation equally between the transmitter and receiver filters. The tap coefficients are determined in order to satisfy the design objective transfer functions originated from the transfer functions of both MZ-FDM and cosine roll-off characteristics $G_{cr}(\omega)$.

The eye is fully open even when the channel spacing becomes narrow as compared to the system with no equalizer. The ISI results in almost zero and the interference powers decrease. Thus, the signal to ICI ratio increases extremely in both roll-off factors of 0.2 and 0.5. The ratio of channel 1 equals to that of channel 8. It is possible to equalize the MZ-FDM with the other channel number in the same manner.

In the future, we intend to consider the other transmission scheme such as QPSK, and to concentrate the equalizer in receiver side and to apply the other equalization method such as minimum mean square method in order to equalize the MZ-FDM with AWGN. We are also going to consider the phase changes at every filter.

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