Statistical Analysis of Adjacent Channel Interference in LTE Downlink

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Abstract

This paper analytically deals with the probability density function (pdf) of the adjacent channel interference (ACI) so that LTE downlink could be investigated. This analysis could remove the need for Monte Carlo simulations when calculating ACI numerically. The numerical verification shows that the analysis in this paper makes good agreement with Monte Carlo simulation. The expected values of ACI are also provided as a function of LTE guard band, which can be used for LTE network planning such as guard band selection as well as for their cell site selection.

Keywords: Long Term Evolution (LTE) probability density function (pdf) adjacent channel interference (ACI)

1. Introduction

This paper provides statistical analysis of adjacent channel interference (ACI) in LTE downlink without Monte Carlo simulations. An analytic derivation of the probability density function of ACI in the uplink of a cellular system is provided in [1] for a single-cell case. As the LTE has attracted significant attention, the isolation effect by guard band of LTE when it coexists with WiMAX system has been investigated in [2]. In this paper, we focus on the downlink ACI in LTE systems as a function of the guard band. The conditional pdf of downlink ACI given a level of receiving power is looked for analytically. We furthermore provide the pdf for multiple adjacent interfering cells. Numerical investigation is also made to verify the proposed analysis. The statistical property of ACI depends on the number of the adjacent cells and the receive filter characteristics of the victim MS. 10MHz guard band, compared with 4 MHz, is shown to reduce the expected ACI by more than 20 dB in LTE down link.

2. Statistical Analysis of ACI

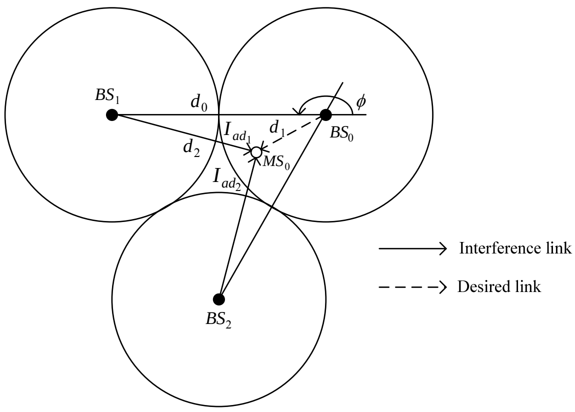


Figure 1: Cell Layout (the number of interfering BSs = 2).

We consider circular cells as shown in Figure 1 and assume that the links from the interfering BSs are mutually independent. BS_0 is the serving BS and MS_0 is the victim MS. The relationship between the parameters are depicted in Figure 1: transmitted power of serving cell BS_0

(P_{t_0}) and neighbour cell BS_1 (P_{t_1}), received power at victim MS_0 (P_r), interference power (I_{ad}), the distance between BS_0 and BS_1 (d_0), BS_0 and MS_0 (d_1), and BS_1 and MS_0 (d_2), respectively.

Let ξ_D and ξ_I be the random component due to shadowing on the desired link and the interference link, respectively, γ be the path-loss exponent and κ_I be the adjacent channel interference ratio (ACIR) in decibels. Then, the interference power I_{ad} is modelled by

$$I_{ad} = 10^{\frac{P_r}{10}} \left(\frac{d_1}{d_2} \right)^{\gamma} 10^{\frac{\xi_D - \xi_I - \kappa_I}{10}} . \tag{1}$$

The distance between interfering BS_1 and the victim MS d_2 can be expressed as a function of the polar coordinates of the MS, d_1, ϕ, and d_0,

$$d_2 = \sqrt{d_1^2 + d_0^2 + 2d_1 d_0 \cos(\phi)} . \tag{2}$$

Substituting (2) into (1) and using the substitution $\beta = \ln(10)/10$ yields

$$I_{ad} = 10^{\frac{P_r}{10}} \left[1 + \left(\frac{d_0}{d_1} \right)^2 + 2\left(\frac{d_0}{d_1} \right) \cos(\phi) \right]^{-0.5\gamma} \times \exp\left[\beta\left(\xi_D - \xi_I - \kappa_I \right) \right], \tag{3}$$

where the distance d_0 is a constant and d_1 is a random variable. Introducing a new random variable, $\upsilon = d_0/d_1$, we change I_{ad} in (3) in a logarithmic unit,

$$\hat{I}_{ad} = P_r + \underbrace{\xi_D - \xi_I}_{\xi} - \kappa_I - \frac{\gamma}{2\beta} \ln\left(\upsilon^2 + 2\upsilon \cos(\phi) + 1 \right). \tag{4}$$

Three independent random variables are involved in (4): υ, ξ, and ϕ. Let us assume that the pdf of these random variables are modelled in a similar way in [1]. The conditional pdf of $\hat{I}_{ad}$ given P_r can be obtained analogously as in [1]

$$p\left(\hat{I}_{ad} \middle| P_r \right) = \frac{\beta d_0^2}{\gamma R^2 \sigma \sqrt{2\pi^3}} \int_0^{2\pi} \int_{-\infty}^{\infty} \left(\sum_{i=1}^{2} \left| -\frac{\upsilon_i^2 + 2\upsilon_i \cos(\phi) + 1}{\upsilon_i^3 \left(\upsilon_i + \cos(\phi) \right)} \right| \right) \exp\left(-\frac{\xi^2}{2\sigma^2} \right) d\xi d\phi , \tag{5}$$

$$\upsilon_i\left(\hat{I}_{ad}, \xi, \phi \right) = \begin{cases} \underbrace{-\cos(\phi)}_{u} + (-1)^i \sqrt{y}, & \text{if } y \geq 0 \text{ and } \frac{d_0}{R} \leq \left(u + (-1)^i \sqrt{y} \right) \leq \infty, \\ 0, & \text{otherwise,} \end{cases} \tag{6}$$

where $y = \cos^2(\phi) + \exp\left[-\frac{2\beta}{\gamma} \left(\hat{I}_{ad} - \xi - P_r + \kappa_I \right) \right] - 1$ and $p(X\,|\,Y)$ denote the conditional pdf of random variable *X* given *Y*.

Let I_{ad_i} denote the ACI from interfering BS_i, $i = 1, 2, \cdots, N$, where N is the number of adjacent interfering cells. The total interference received by victim MS is then $I_{ad\_tot} = I_{ad_1} + I_{ad_2} + \cdots + I_{ad_N}$. By assuming I_{ad_i}'s are mutually independent when P_r is given, we can obtain the conditional pdf of I_{ad_tot} through the convolution of the conditional pdfs of I_{ad_i},

$$p(I_{ad\_tot} | P_r) = p(I_{ad_1} | P_r) * p(I_{ad_2} | P_r) * \cdots * p(I_{ad_N} | P_r), \tag{7}$$

where the operator ($*$) means convolution operation.

3. LTE Spectrum Mask and ACIR

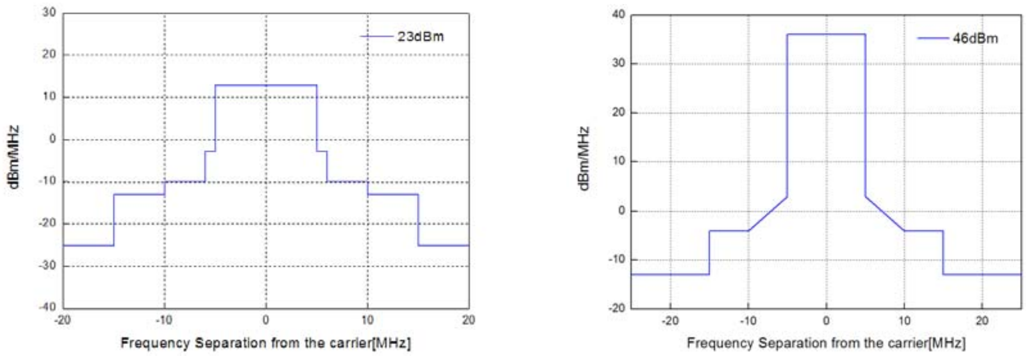
ACI is the interference caused by extraneous power from interfering signals in an adjacent channel. ACI may be caused by imperfect filtering in either the transmitted signals or the received signals. ACLR (Adjacent Channel Leakage Power Ratio) is associated with imperfect filters at the transmitter. On the other hand, ACS (Adjacent Channel Selectivity) accounts for the characteristics of receiver. The total interference caused by ACLR and ACS is given as an ACIR (Adjacent Channel Interference Ratio) and is defined by

$$ACIR = \left(\frac{1}{ACLR} + \frac{1}{ACS} \right)^{-1}, \tag{8}$$

In this paper, we assume that ACIR is equal to ACLR by assuming ACS=∞.

Figure 2 shows a spectrum emission mask of MS and BS for LTE system, respectively [3], [4]. With the information of this mask, ACLR is calculated by $P_{TX_{inband}} / P_{TX_{outband}}$, where $P_{TX_{inband}}$ is the transmission power of interfering BS and $P_{TX_{outband}}$ is the received power of victim MS, respectively. Let α be a guard-band length, we can obtain $P_{TX_{outband}}$ as follows:

$$P_{TX_{outband}} = \begin{cases} \int_{\alpha}^{5} 10^{\frac{3-7/5a}{10}} da + 10^{-0.4} \times 5 + 10^{-1.3} \times \alpha, & 0 < \alpha \le 5, \\ 10^{-0.4} \times (1-\alpha) + 10^{-1.3} \times \alpha, & 5 < \alpha \le 10. \end{cases} \tag{9}$$



(a) MS (b) BS

Figure 2: Spectrum Emission Mask of LTE System [3], [4]

4. Simulation Results

The comparison between the proposed pdf and Monte Carlo simulation is carried out for three different guard band parameters when $N = 1$ and $N = 2$ in order to obtain sufficient evidence as to whether both approaches lead to similar results. The parameters used for the verification are selected as a cell radius of $R = 50$ m, $d_0 = 50$ m, $\gamma = 3$, $\sigma = 8$ dB, and we fix the received signal $P_r = -111$ dBm.

Figure 3 presents the pdf of $\hat{I}_{ad\_tot}$ obtained by the analytical approach and by Monte Carlo simulation. It is seen that the pdfs from the methods are well matched. Used ACIR values are 12.24, 22.27, and 49.00 dB for guard band 0, 4, and 10 MHz, respectively. When the guard band increases, the pdfs move to the left, which means that the adjacent channel interference decreases by setting great guard band. Comparing 10 MHz with 4 MHz, 10MHz reduces the expected ACI by more than 20 dB as shown in Table 1. In Table 2, variance for N = 2 is less than that for N = 1. This is due to the effect of numerical conversion between nominal and logarithmic values.

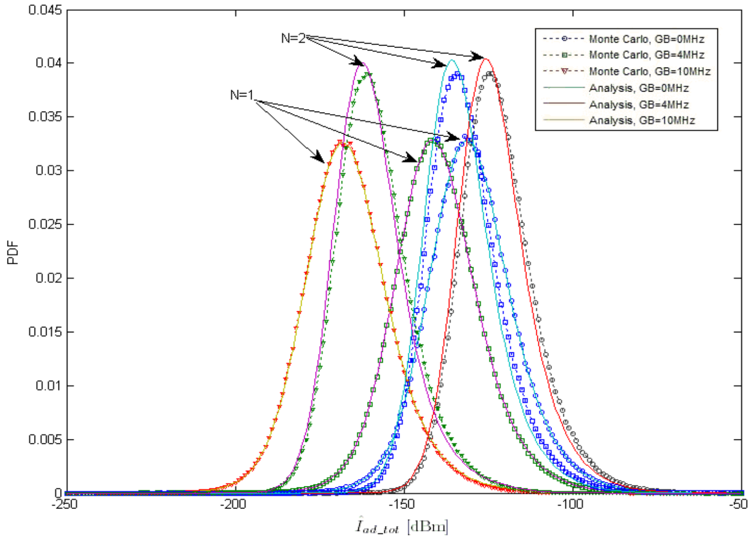


Figure 3: A Comparison of the PDF by the Analytical Approach and the Monte Carlo Simulations

Table 1: Comparison of Mean and Variance of $\hat{I}_{ad\_tot}$

	Monte Carlo				PDF Analysis			
Guard Band [MHz]	**N=1**		**N=2**		**N=1**		**N=2**	
	$E(\hat{I}_{ad\_tot})$	$Var(\hat{I}_{ad\_tot})$	$E(\hat{I}_{ad\_tot})$	$Var(\hat{I}_{ad\_tot})$	$E(\hat{I}_{ad\_tot})$	$Var(\hat{I}_{ad\_tot})$	$E(\hat{I}_{ad\_tot})$	$Var(\hat{I}_{ad\_tot})$
0	-129.7	178.9	-127.7	130.7	-129.8	178.5	-122.9	123.2
4	-139.8	178.5	-131.7	130.7	-139.8	179.2	-132.9	123.9
10	-168.5	178.8	-158.4	131.2	-166.2	184.5	-159.3	129.7

5. Conclusion

This paper has presented the pdf of ACI so that LTE downlink ACI could be statistically modelled. This derivation also could remove the need for extensive Monte Carlo simulations and provides insight into the impact of certain system parameters on the shape of the ACI. The numerical verification of the given analysis has shown good agreement with the Monte Carlo simulations. We also provide the expected values of ACI as a function of LTE guard bands.

References

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