Properties of Multiconstrained Multicast Trees in Ad-Hoc Networks with Topology Control

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***Abstract*—The article explores the quality of multicast trees constructed by heuristic routing algorithms in ad-hoc networks where topology control protocols operate. Network topology planning and performance analysis are crucial challenges for wire and wireless network designers. They are also involved in the research on routing algorithms and protocols for ad-hoc networks. In addition, it is worth to emphasize that the generation of realistic network topologies makes it possible to construct and study routing algorithms, protocols and traffic characteristics for ad-hoc networks.**

I. INTRODUCTION

The multicast transmission is one of the more popular techniques in modern networks. It enables simultaneous communication of a group of users which, when properly implemented, may offer great resource savings as compared to the basic point-to-point communication based approach. The real time multicast transmission of multimedia content is a widely-used traffic type, which is a challenging research subject as there is a great demand for it in the rapidly developing area of multimedia telecommunications.

Constrained Minimal Steiner Tree Problem (CMSTP) [1], [2] involves connecting a single source with multiple destinations in such way that one of the multiple metrics of the structure is minimal, under the restriction that the others do not violate required constraints. Therefore, when comparing different algorithms, one has to examine the costs of the multicast tree found in a given graph for given input parameters. The evaluation of the result is a non-trivial task. The metric which is to be minimized should obviously be the lowest, but the constrained metrics may be of greater or lesser importance depending on assumed goals. The CMSTP problem can be considered both in wired and wireless networks (ad-hoc, mesh, WSN, etc).

Ad-hoc networks are sets of nodes that form networks without any additional infrastructure and no centralized control. These nodes generate traffic to be forwarded to some other nodes (unicast) or a group of nodes (multicast) [4], [5]. Due to a dynamic nature of ad-hoc networks, traditional network routing protocols are not viable. Thus, nodes act both as the end system (transmitting and receiving data) and the router (allowing traffic to pass through), which results in multihop routing. Networks are *in motion*, i.e. nodes are mobile and may go out of range of other nodes in the network [3].

The efficient use of energy resources available to ad-hoc and sensor network nodes is one of the fundamental tasks for network designers [6]. Reduction of the energy consumed by radio communication is an important issue. Topology control mechanisms allow to maintain the lowest energy requirements of nodes and the maximum throughput of the network.

The article focuses on the quality of trees constructed by multicast routing algorithms in ad-hoc networks that use topology control mechanisms. It starts with an overview of the available algorithms and evaluation techniques in Section II. Section III defines topology control mechanisms and basic parameters describing network topology. In Section IV, the results of the simulation of the implemented topology control protocols along with their interpretation are described. Finally, Section V concludes the article.

II. ALGORITHMS DESCRIPTION

A. Aggr_MLARAC Algorithm

The Aggregated MLARAC [21] is an adaptation of a unicast algorithm to a multicast problem by performing an aggregation of the unicast results (paths from the source node to each of the destination nodes) into a multicast result (a tree that spans all of the multicast participants. The unicast technique selected as a base for this algorithm is MLARAC. The MLARAC algorithm is on the other hand a multidimensional generalization of the LARAC algorithm [12].

The LARAC algorithm is a technique that utilizes Lagrangian relaxation for optimizing a path optimization problem with a single constraints. The foundation of the Lagrangian relaxation is the maximization of the Lagrangian dual function. The Lagrangian dual is a concave function in the entire domain therefore only one maximum exists. The difficulty of finding the maximum is that the function is also piecewise linear, and thus the extreme cannot be found in the analytical way. In the LARAC algorithm two distant segments of the function are found and based on the intersections of the lines to which they belong an approximation of the optimum is found. Based on the approximation, another segment, closer to the optimum is determined and used to find another intersection. This procedure is repeated, and after each step a better approximation is obtained. The algorithm is guaranteed to find the optimum after finite number of steps.

The MLARAC algorithm is a generalization of the problem to multiple dimensions. Increasing the number of the optimization criteria increases the number of the dimensions of the Lagrangian dual function. In the MLARAC algorithm the intersection of lines has been replaced with the intersection of the hyperplanes. Also two problems that appear in the multidimensional space have been heuristically solved: the definition of the initial hyper-segments to intersect, and handling of the determined approximation. In the first case the one dimensional optimization is easier, because there are two sides of the hill of which the peak is to be found. There exists a robust way of selecting segments from the two sides of the hill. In the multidimensional case there is no straightforward equivalent method to determine the initial conditions. When the intersection of the hyperplanes is found presenting the new approximation of the result, there exists a condition that defines precisely, how it should be used in the consecutive intersections, but the exact equivalent for the multiple dimensions have not been found.

The aggregation of the results in the Aggregated MLARAC is performed by performing a union operation of the paths obtained from multiple MLARAC passes (from the source node to each of the destination nodes) which produces a subgraph containing all the multicast participants. Such structure is then pruned using the Prim algorithm [17]. A similar technique has been used earlier in [16].

B. HMCMC Algorithm

The HMCMC algorithm (*Heuristic Multi-Constrained MultiCast*) [10] is a relatively simple heuristic that has combines two main ideas. One is to handle the multiple criteria by aggregating them utilizing a nonlinear function:

$$m_{aggr}(t) = max\left\{\frac{m_1(t)}{c_1}, \frac{m_2(t)}{c_2}, \dots\right\}. \tag{1}$$

The second concept behind the HMCMC algorithm is performing the Dijkstra's algorithm [8] (with the application of the metric aggregation) multiple times. This algorithm defines the multicast participants as the seource and the destination nodes separately. The Dijkstra's algorithm is performed from the source first, and if the shortest paths to all destinations that are obtained this way fulfill the constraints defined in the probem they are accepted as the result. Otherwise the Dijkstra's algorithm is performed from all the destinations towards which the constraints have not been met.

When relaxing the graph from the destination node towards the source node, the information from the initial algorithm pass is used to heuristically improve the quality of the selected path. Such an approach is computationaly cheap as the number of times that the Dijkstra's algorithm needs to be performed is the same as the number of the multicast participants. The experiments have shown that it also provides a feasible result in many cases.

C. RDP Algorithm

The RDP algorithm [23] is an algorithm based on a simulation semantics applied a modified version of the Dijkstra's algorithm. The first of the two variations from the original algorithm is the multi source approach. It is based on a slight change that the relaxation is initialized in multiple sources rather than one. As the result the labeling of the costs of reaching particular nodes is perfomred from different sources. The costs of reaching the nodes are stored separately so they don't override each other. This way if the relaxation is performed for the entire graph, the cost labels for each of the graph's nodes will store the information about reaching the given node from each of the initial nodes. If the initial nodes are the same as the multicast participants, then these cost labels may play role of a weighted routing tables for each of the graph nodes. It is worth noting that in order to deal with multiple metric the same metric aggregation is utilized as in the HMCMC algorithm.

The second variation consists in the renaming of the original Dijkstra's algorithm's operations. It is performed in such a way that instead of describing the graph relaxation a simulation of the signal propagation in the graph is described. Introducing the notion of time into the consideration presents us with a means to define simultaneously of the node analysis operations.

Combining these two variations creates a context in which it is possible to treat the relaxations performed from the different sources as concurrently performed signal propagation processes. Therefore it is possible to state that at a certain point of the simulation time the signals propagating from all of the sources have reached a given node. In such conditions the given node is said to be equally or similarly close (in the topological metric) to all of the source nodes. The thesis behind the RDP algorithm is that such nodes (further referred to as the *rendez vouz points* or the RDPs) may be considered as the middle points for the multicast trees with a considerable probability.

In [22] two variants of the above technique have been presented and analyzed with the regard to quality of the obtained results. The quality is defined as the costs of the obtained multicast trees. The research has shown that there was no significant difference between the variants therefore the more performant algorithm should be used as the representative implementation of the general RDP technique.

III. AD-HOC NETWORK TOPOLOGY

Topology control mechanisms are used to ensure that certain parameters in the whole network are secure. Decisions in nodes are made locally to achieve a global goal. Both centralized and distributed techniques of topology control can be classified as topology control mechanisms.

A. Network Model

The ad-hoc network is represented by an undirected, connected graph $G = (V, E)$, where V is a set of nodes and E is a set of links. The existence of the link $e = (u, v)$ between node u and v entails the existence of the link $e' = (v, u)$ for any $u, v \in V$ (corresponding to two-way links in communications networks). In the most common power-attenuation model, the power needed to support a link $e = (u, v)$ is $p(e) = ||u, v||^{\beta}$, where $||u, v||$ is the Euclidean distance between u and v, and β is a real constant between 2 and 5 dependent on the wireless transmission environment (*path loss model*) [6].

B. Protocols of Distributed Topology Control

A practical approach to topology control requires a creation of distributed protocols that operate locally, without the knowledge of the global state of the network, and generate topologies close to the optimal. Topology graphs should provide desirable properties of a network using symmetric edges and should be consistent (if these properties are satisfied in the graph of the maximum power that contains the edges resulting from the maximum transmit power of the nodes) [18]. It is desirable then to build a graph of the least degrees of nodes, which reduces the probability of interference in the network. It is also desirable to create optimal topology based on inaccurate information. Providing accurate information on the nodes is often too expensive, because it requires GPS receiver in each node of the network.

LMST protocol (*Local Minimum Spanning Tree*) calculates the local approximation of the minimum spanning tree [13]. It is performed in three, or optionally four, stages.

The first stage is the exchange of information. All nodes send messages to their visible neighbors containing their identities and locations (visible neighbor nodes that are within range when transmitting at the maximum power).

In the second stage of topology creation, each node performs locally Prim's algorithm [17] taking their Euclidean length of edge as cost – the minimum spanning tree $T_u = (VN_u, E_u)$ contains all visible neighbors of node u (VN_u) in the max-power graph $G_{\varepsilon} = (N, V_{\varepsilon})$. Then, each node defines a set of neighbors.

The node v is treated as a neighbor of node u ($u \to v$) if a node v is within range of node u and is available in one step in a minimum spanning tree computed in this node $T_u = (VN_u, E_u)$:

$$u \longrightarrow v \Longleftrightarrow (u, v) \in E_u. \quad (2)$$

A set of neighbors of node u is defined as:

$$N(u) = \{v \in VN_u | u \longrightarrow v\}. \quad (3)$$

Network topology defined in the LMST protocol is represented by a directed graph $G_{LMST} = (N, E_{LMST})$, where directed edge $(u, v) \in E_{LMST}$ exists only if $u \longrightarrow v$.

In the last (required) step of the protocol, power levels of signals required for the communication with neighboring nodes are calculated. This can be obtained by measuring the power of incoming messages sent to the nodes in the first stage of protocol with the maximum power received from the visible neighbors.

The fourth (optional) step creates a topology with symmetric links. This is achieved either by replacing the asymmetric edges of symmetric ones or by removing asymmetric edges.

DistRNG protocol (*Distributed Relative Neighborhood Graphs*) [7] constructs a RNG graph built on a set of nodes N that has an edge between a pair of nodes $u, v \in N$ if and only if there is a node $w \in N$ such that:

$$\max\{\delta(u, w), \delta(v, w)\} \leq \delta(u, v). \quad (4)$$

The DistRNG protocol uses the concept of *coverage area*. If node v is a neighbor of node u, the coverage area of node v: $Cov_u(v)$ is defined as the clipping plane with the center at node u and width $a\hat{u}b$, where a and b are the points of intersection of the circles with the radius $\delta(u, v)$ and midpoints in the nodes of u and v. The total coverage area of node u is the sum of the areas of all of its neighbors.

IV. EXPERIMENTAL RESULTS

The comparison of the multicriterial algorithms is a hard task not only because of the complexity of the algorithms themselves, but also because of the multitude of detail involved in the performance of the simulation, let alone its initiation. In a simulation study we compared the cost of the multicast trees obtained in different network topologies for routing algorithms without constraint (m_0), with one constraint (m_1) and two constraints (m_2).

Simulations were performed for the sets of graphs of 200 nodes generated with LMST and DistRNG protocol, and compared with Waxman model (in two scenarios: with $k = 100$ edges and $k = 200$ edges). In order to achieve the high statistical quality of the results 1000 graphs were generated for each of the topology model. Three metrics (constraints) were randomly generated from the range $\langle 1, 1000 \rangle$ for each edge in the graph. Each of the generated topologies was tested for connecting $4, 8, ..., 28$ multicast nodes. The technique presented in [9] was used to pick the constraints for the MCMST problem.

The results presented on Figure 1 show that the average cost of multicast trees increases with the increase of the number of multicast nodes in the network within each constraint. The influence of different network topologies is observable. Analysis of the results presented in Figure 1 indicate strong similarities in the results obtained with the algorithms generated network topologies using a LMST protocol and Waxman model ($k = 100$), as well as the protocol DistRNG and Waxman model ($k = 200$). In the second case the costs of obtained trees are comparable and smallest for each examined algorithms. Aggr_MLARAC and HMCMC multicast algorithms have the best performance in DistRNG ad-hoc networks and networks generated with an application of Waxmax model ($k = 200$). This leads to the conclusion that in simulations studies on ad-hoc networks it is possible to use fast methods that generate random graphs.

V. CONCLUSION

Multicriterial constrained multicast routing problems presents a non-trivial level of complexity. Following this concept, a need for a broad analysis techniques spectrum

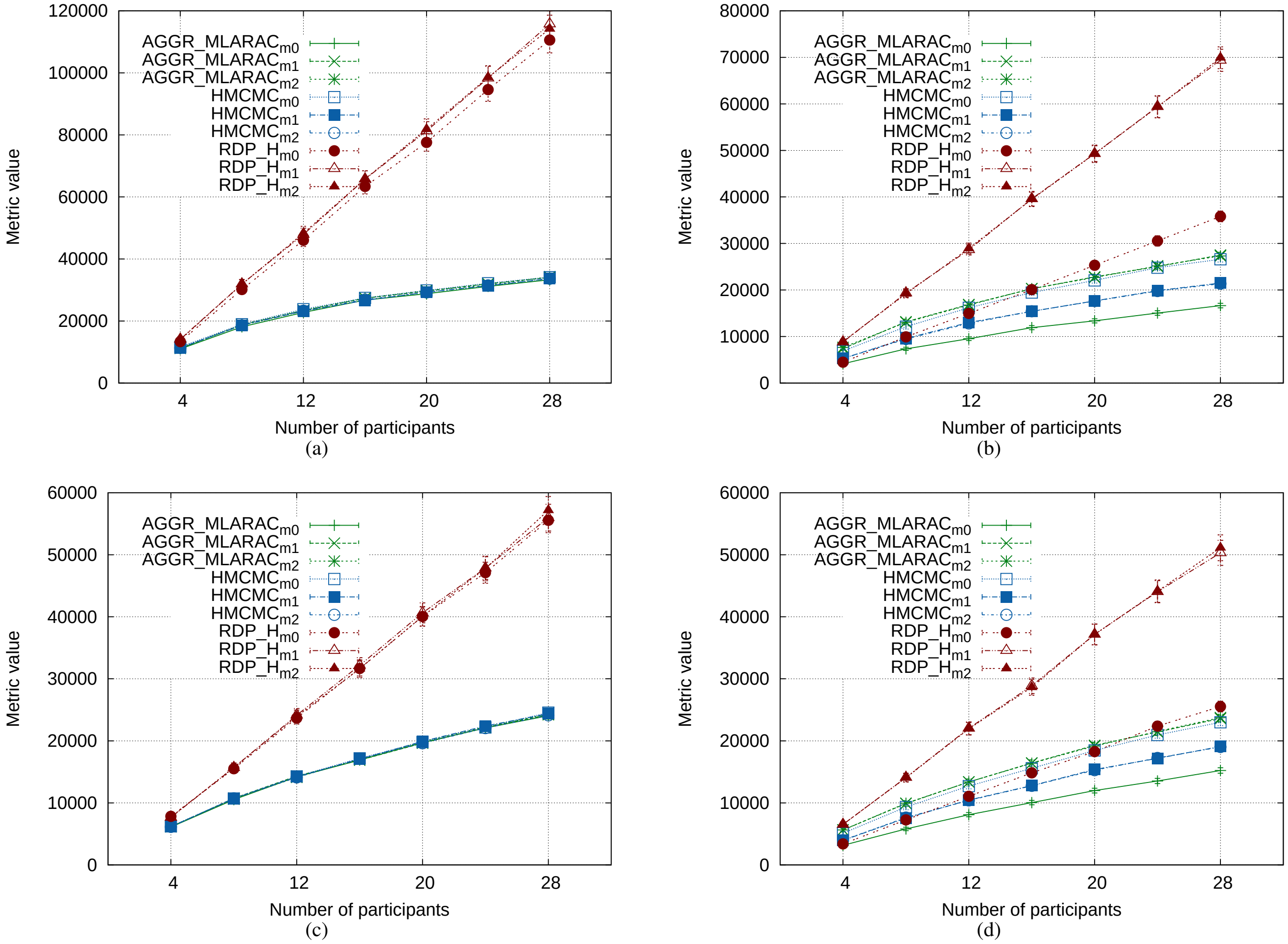

Fig. 1. Average cost of constrained multicast trees obtained in networks with 200 nodes generated according to LMST protocol (a), DistRNG protocol (b), Waxman model with $k = 100$ (c) and Waxman model with $k = 200$ (d).

arises. It has been shown that exploring not only the space of the algorithms, but also the space of their comparison is worth an increased amount of effort as the conclusions may render different algorithms useful in different situations. In addition, the stability of the algorithms against changes in different conditions can be shown with the use of the innovative and non-standard analysis.

REFERENCES

[1] S. Chen and K. Nahrstedt. An overview of quality of service routing for next-generation high-speed networks: problems and solutions, *IEEE Network*, vol. 12, pp. 64-79, 1998.

[2] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, *Network flows: theory, algorithms, and applications*. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 1993.

[3] P. Santi. Mobility Models for Next Generation Wireles Networks: Ad Hoc, Vehicular, and Mesh Networks. *John Wiley and Sons, Chichester, UK*, 2012.

[4] Frost & Sullivan. Wireless sensors and integrated wireless sensor networks. *Frost & Sullivan Technical Insights*, 2004.

[5] M. Głabowski, B. Musznicki, P. Nowak, P. Zwierzykowski. An algorithm for founding Shortest Path Tree using Ant Colony Optimization metaheuristic. *Image Processing and Communications Challenges 5, Advances in Intelligent Systems and Computing*, vol. 233, pp. 317-326, 2014.

[6] P. Santi. Topology Control in Wireless Ad Hoc and Sensor Networks. *John Wiley and Sons, Chichester, UK*, 2005.

[7] S. Borbash, E. Jennings. Distributed Topology Control Algorithm for Multihop Wireless Networks. *Proceedings of 2002 World Congress on Computational Intelligence (WCCI 2002)*, pp. 355-360, 2002.

[8] E. Dijkstra. A note on two problems in connexion with graphs. *Numerische Mathematik*, 1:269-271, 1959.

[9] F. Gang. The revisit of QoS routing based on non-linear Lagrange relaxation. *Int J Commun Syst*, 2007;20:922.

[10] F. Gang. A multi-constrained multicast QoS routing algorithm. *Computer Communications*, vol. 29, no. 10, pp. 1811-1822, 2006.

[11] K. Gomez, R. Riggio, T. Rasheed, D. Miorandi, I. Chlamtac, F. Granelli. Analysing the Energy Consumption Behaviour of WiFi Networks. *Proceedings of IEEE Greencom 2011*, pp. 98-104, 2011.

[12] A. Jüttner, B. Szviatovszki, I. Mecs, and Z. Rajko. Lagrange Relaxation Based Method for the QoS Routing Problem. *IEEE INFOCOM' 2001*, 2001.

[13] N. Li, J. Hou, L. Sha. Design and Analysis of an MST-based Topology Control Algorithm. *INFOCOM 2003. Twenty-Second Annual Joint Conference of the IEEE Computer and Communications*, pp. 1702-1712, 2003.

[14] A. Medina, A. Lakhina, I. Matta, J. Byers. BRITE: An Approach to Universal Topology Generation. *IEEE/ACM MASCOTS*, pp. 346-356, 2001.

[15] M. Piechowiak, M. Stasiak, P. Zwierzykowski. Analysis of the Influence of Group Members Arrangement on the Multicast Tree Cost. *5-th*

Advanced International Conference on Telecommunications AICT 2009, Venice, Italy, pp. 429-434, 2009.

[16] M. Piechowiak and P. Zwierzykowski, “A New Delay-Constrained Multicast Routing Algorithm for Packet Networks,” in *IEEE AFRICON*, Nairobi, Kenya, sept. 2009, pp. 1-5.

[17] R. Prim. Shortest Connection Networks and Some Generalizations. *Bell Systems Tech J.*, 36:1389-1401, 1957.

[18] R. Rajaraman. Topology Control and Routing in Ad Hoc Networks: A Survey. *ACM SIGACT News*, 33:60–73, June 2002.

[19] R. Riggio, T. Rasheed, S. Testi, F. Granelli, I. Chlamtac. Interference and Traffic Aware Channel Assignment in WiFi-based Wireless Mesh Networks. *Elsevier Ad Hoc Networks*, pp. 864-875, 2010.

[20] P. Santi. Topology Control in Wireless Ad Hoc and Sensor Networks. *ACM Computing Surveys*, 37:164-194, June 2005.

[21] K. Stachowiak, J. Weissenberg, and P. Zwierzykowski. Lagrangian relaxation in the multicriterial routing. in *IEEE AFRICON*, Livingstone, Zambia, pp. 1-6, 2011.

[22] K. Stachowiak, P. Zwierzykowski. Innovative Method of the Evaluation of Multicriterial Multicast Routing Algorithms. *Journal of Telecommunications and Information Technology*, vol. 1, pp. 49-55, 2013.

[23] K. Stachowiak, P. Zwierzykowski. Rendezvous point based approach to the multi-constrained multicast routing problem. *AEU – International Journal of Electronics and Communications*, 01/2014.

[24] B. Waxmann. Routing of multipoint connections. *IEEE Journal on Selected Area in Communications*, 6:1617-1622, 1988.

[25] A. Zamożniewicz. Methods for generating topologies of ad-hoc networks. *MSc thesis*, Poznan University of Technology, 2009 (in Polish).

[26] E. W. Zegura, K. L. Calvert, S. Bhattacharjee. How to Model an Internetwork. *IEEE INFOCOM 96*, pp. 592-602, 1996.