Rearrangeability of 2×2 W-S-W Elastic Switching Fabrics with Two Connection Rates

Wojciech Kabaciński, Remigiusz Rajewski, Atyaf Al-Tameemi

***Abstract*—The rearreangeable conditions for 2×2 three-stage switching fabric of wavelength-space-wavelength architecture for elastic optical switches are considered in the paper. The required number of frequency slot units in interstage links is much lower than in the strict-sense nonblocking switching fabrics.**

***Index Terms*—Elastic optical networks, elastic optical switching nodes, interconnection networks, rearrangeable nonblocking conditions.**

I. INTRODUCTION

The Elastic Optical Network (EON) architecture has been proposed to utilize the bandwidth available in optical fiber more efficiently. By breaking the fixed-grid spectrum allocation limit of conventional WDM networks, EONs increase the flexibility in the connection provisioning [1], [2]. To do so, depending on the traffic volume, an appropriate-sized optical spectrum is allocated to connections in EON. This optical spectrum is called Frequency Slot Unit (FSU). Furthermore, unlike the rigid optical channels of conventional WDM networks [3], a light-path can expand or contract elastically to meet different bandwidth demands in EON. In this way, incoming connection requests can be served in a spectrum-efficient manner. This technological advance poses additional challenges on the networking level, specially on the efficient connection establishment. Similar to Wavelength Division Multiplexing (WDM) networks, an elastic optical connection must occupy the same spectrum portion between its end-nodes, that is, ensuring the so called spectrum continuity constraint. In addition, the entire bandwidth of the connections must be contiguously allocated. Bandwidth assigned to an optical channel depends on the required transmission data rate, distance to be cover, path quality, wave length spacing between channels, and/or the modulation scheme used [2], [4]–[6]. Several architectures of elastic optical switching nodes were proposed in literature [7]–[10], in this paper, we deal with one of these switching fabric architectures, it is the W-S-W (wavelength-space-wavelength) switching fabric, called WSW1 [11]. Strict-since nonblocking (SSNB) conditions for WSW1 architecture have been proved in [11]. We proposed rearrangeable nonblocking (RNB) conditions for this architecture in [12] for simultaneous connections routing with limited number of connection rates. Simultaneous connections means, at the same time all connections arrive at all inputs and they mast be served simultaneously. The number of simultaneous connection rates that can be served in our model is limited to z. The upper bound for RNB connections when $r > 2$ was derived in [12]. The aim for using RNB switching fabric is to reduce the required number of FSUs in the interstage links, which means to reduce the cost of this switching fabric. The necessary and sufficient RNB conditions have been derived in [12] for the special case when $r = z = 2$ and $\frac{n}{m_1}$,$\frac{n}{m_2}$, and $\frac{m_2}{m_1}$ are integers. In this paper, we generalize theses conditions to the general case, when $r = z = 2$ and for any value of n, m_1, and m_2. The obtained results contain those proved [12] for above mentioned special case.

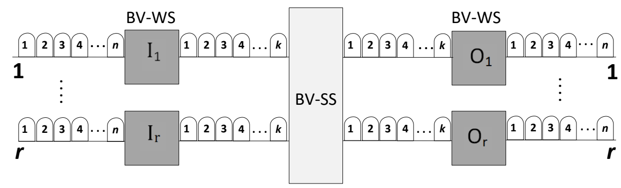

Fig. 1. The WSW1 switching fabric architecture

The rest of the paper is organized as follows. In the next section, the switching fabric is presented and the problem is described in a more detailed way. In Section III, the proposed model of RNB is included. The RNB results for the proposed model are discussed in Section IV. The paper ends with conclusions.

II. SWITCHING FABRICS AND THE MODEL

The WSW1 switching fabric considered in this paper was described in more details in [11], here, we will only provide a short description which will make the paper easier to follow. This architecture is presented in Fig. 1. In the first and third stages, there are r bandwidth-variable spectrum converting switches (BV-WSs), and one bandwidth-variable wavelength selective space switch (BV-SS) of capacity $r \times r$ in the second stage. Each BV-WS in the first stage has one input fiber with n FSUs and one output fiber with k FSUs, while each BV-WS in the third stage has one input fiber with k FSUs and one output fiber with n FSUs.

The internal architecture of BV-WSs and BV-SS can be found in [11]. The switching fabric serves m-slot connections, FSUs in input/output fibers are numbered from 1 to n, BV-WSs in input/output–stages numbered from 1 to r, and FSUs in interstage fibers from 1 to k.

In our considerations we assume that BV-WSs have full range conversion capability, i.e., an m-slot connection which uses a set of m adjacent FSUs in the input fiber can be switched to a set of any other m adjacent FSUs in the output fiber. A new m-slot connection from input switch I_i to output switch O_j will be denoted by (I_i, O_j, m). When FSUs

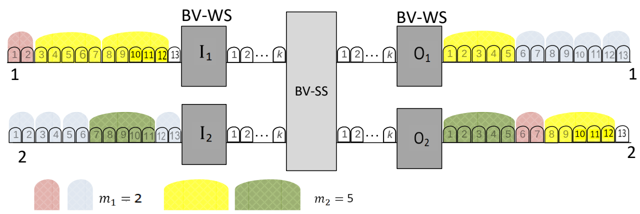

Fig. 2. The 2×2 WSW1 switching fabric with $\mathbb{C} = \{(I_1[1], O_2[6], 2); (I_1[3], O_1[1], 5); (I_1[8], O_2[8], 5); (I_2[1], O_1[6], 2); (I_2[3], O_1[8], 2); (I_2[5], O_1[10], 2); (I_2[7], O_1[1], 5); (I_2[12], O_1[12], 2)\}$

TABLE I
ASSIGNMENT OF FSUS FOR CONNECTIONS IN EXAMPLE 1.

matrices representing m_1 connections	FSUs in interstage links	matrices representing m_2 connections	FSUs in interstage links
$\mathrm{H}^{m_1} = \begin{bmatrix} 0 & 4 \\ 1 & 0 \end{bmatrix}$	—	$\mathrm{H}^{m_2} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$	—
$\mathrm{P}_1^{m_1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	1–2	$\mathrm{P}_1^{m_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	9–13
$\mathrm{P}_2^{m_1} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	3–4	$\mathrm{P}_2^{m_2} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	14–18
$\mathrm{P}_3^{m_1} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	5–6	—	—
$\mathrm{P}_4^{m_1} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$	7–8	—	—

numbers occupied by these connections are important, the number of the first FSU will be also provided. $(I_i[x], O_j[y], m)$ denotes the m-slot connection which in the input fiber of start I_i occupies FSUs from x to $x + m - 1$, and FSUs from y to $y + m - 1$ of output fiber of switch O_j. In the switching fabric, when a new connection (I_i, O_j, m) arrives, a control algorithm must find a set of m adjacent FSUs in interstage links which can be used for this connection, and these must be FSUs with the same numbers in the interstage links from I_i and to O_j, since BV-SS has no spectrum conversion capability. In the case of simultaneous connection model, we have a set of compatible connection requests which occupy most of FSUs in the input and output fibers, (i.e. the number of free FSUs in each input/output fiber is less than m_1). This set of connections is denoted by $\mathbb{C}$, and an example of such set in the 2×2 switching fabric with $n = 13$ is shown in Fig. 2. Eight connections of two types are to be set up: three 5-slot connections, five 2-slot connections, and one FSU remains free in input fiber 1 and output fiber 2. The problem now is, which FSUs in interstage links should be used by these connections, and how many FSUs are needed to set up all these connections, i.e., when the switching fabric is RNB. In [12] we proposed the control algorithm to assign FSUs to particular connection requests using the matrix decomposition algorithm, and showed the RNB conditions in case m_2/m_1, n/m_1, n/m_2 are integers. The case with any number of m_1, m_2, and n is considered in the next section.

III. REARRENGEABILITY CONDITIONS

We consider 2×2 WSW1 switching fabric, the number of connection rates is limited to 2, i.e., there are only m_x-slot connections, $x = 1, 2$. A set of compatible connections in $\mathbb{C}$ is represented by matrix H^{m_x}.

$$\mathrm{H}^{m_x} = \begin{bmatrix} h_{11}^{m_x} & h_{12}^{m_x} \\ h_{21}^{m_x} & h_{22}^{m_x} \end{bmatrix} \quad (1)$$

where $h_{ij}^{m_x}$ is equal to the number of m_x-slot connection requests from switch I_i to switch O_j. According to algorithm 1 in [12], this matrix can be decomposed to $c_{\max}^{m_x}$ permutation matrices $P_i^{m_x}$, where $c_{\max}^{m_x}$ represents the maximum number of m_x-slot connections in one input or output, while $c_{\min}^{m_x}$ represents the minimum number of such connections. We can use this algorithm to set up the connection requests presented in Fig. 2. The set of connection requests $\mathbb{C}$ consists of eight connections of two types: three 5-slot connections and five 2-slot connections. We used Tab. II to explain the decomposition steps, in the first row H^{m_1} and H^{m_2} that represent these connections. In the next rows matrices that result from decomposition steps are shown together with the numbers of FSUs which are assigned to these connections. The final arrangement of these connections is shown in Fig. 3

In [12] we proved that the WSW1 switching fabric presented in Fig. 1 with $r = 2$ is rearangeably nonblocking in case $m \in \{m_1; m_2\}$, $m_1 < m_2$, $\frac{n}{m_1}$, $\frac{n}{m_2}$, and $\frac{m_2}{m_1}$ are integers, if and only if:

$$k \geqslant n. \quad (2)$$

Now we will present a new theorem to find the value that makes this switching fabric RNB in more general case.

Theorem 1: The WSW1 switching fabric presented in Fig. 1 with $r = 2$ is rearangeably nonblocking for m-slot connections, where $m \in \{m_1; m_2\}$, if:

$$k \geqslant \left\lfloor \frac{n}{m_2} \right\rfloor \cdot m_2 + \left(\left\lfloor \frac{n}{m_1} \right\rfloor - \left\lfloor \frac{n}{m_2} \right\rfloor \cdot \left\lfloor \frac{m_2}{m_1} \right\rfloor \right) \cdot m_1 \quad (3)$$

Proof: Let $\mathbb{C}$ denote a set of compatible connections. We have two connection rates, m_1 and m_2, all connections can be represented by $\mathrm{H}^{m_1}$and H^{m_2}. According to the decomposing algorithm in [12], H^{m_1} and H^{m_2} can be decomposed into $c_{\max}^{m_1}$ and $c_{\max}^{m_2}$ permutation matrices P^{m_x}. Each $\mathrm{P}_i^{m_x}$ represents a set of m_x-slot connections which can be set up using the same m_x FSUs in interstage links. From these $\mathrm{P}_i^{m_x}$, only $c_{\min}^{m_1}$ and $c_{\min}^{m_2}$ matrices contains 1's in each row and each column, other matrices contains some rows and/or columns with only 0's. The number of matrices that do not contain 1 in each row and column are $(c_{\max}^{m_1} - c_{\min}^{m_1})$ and $(c_{\max}^{m_2} - c_{\min}^{m_2})$ matrices, which may be merged to permutation matrices with 0's in the same row and same column. In our case the number of matrices that can be merged is not generally more than $(c_{\max}^{m_1} - c_{\min}^{m_1}) - (c_{\max}^{m_1} - c_{\min}^{m_1})$. The required number of FSUs in interstage links to set up all connections simultaneously is given by the flowing formula:

$$\begin{aligned} k \geqslant \quad & c_{\min}^{m_1} \cdot m_1 + c_{\min}^{m_2} \cdot m_2 + (c_{\max}^{m_2} - c_{\min}^{m_2}) \cdot m_2 \\ & + (c_{\max}^{m_1} - c_{\min}^{m_1}) - (c_{\max}^{m_2} - c_{\min}^{m_2}) \cdot \left\lfloor \frac{m_2}{m_1} \right\rfloor \cdot m_1. \end{aligned} \quad (4)$$

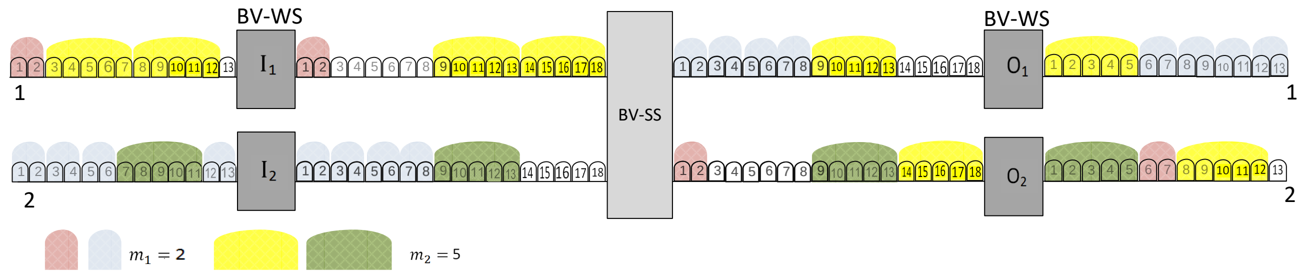

Fig. 3. The 2×2 WSW1 switching fabric of Fig. 2 with $\mathbb{C}$ set up through 18 FSUs according to algorithm mention in [12]

Equation (4) can be simplified to the following one:

$$k \geqslant \left(c_{\max}^{m_1} - c_{\max}^{m_2} \cdot \left\lfloor \frac{m_2}{m_1} \right\rfloor + c_{\min}^{m_2} \cdot \left\lfloor \frac{m_2}{m_1} \right\rfloor \right) \cdot m_1 + c_{\max}^{m_2} \cdot m_2 \tag{5}$$

Equation (5) mast be maximize through all possible sets $\mathbb{C}$. Since $c_{\max}^{m_x}$ represents the maximum number of m_x-slot connections in one of inputs or outputs, the number of such connections in one input/output will never be greater than $\lfloor \frac{n}{m_x} \rfloor$. When $c_{\max}^{m_x}$ values maximize $c_{\min}^{m_x}$ values minimize. when we put in (5) $c_{\max}^{m_x} = \lfloor \frac{n}{m_x} \rfloor$ and $c_{\min}^{m_x} = 0$ we get:

$$k \geqslant \left\lfloor \frac{n}{m_2} \right\rfloor \cdot m_2 + \left(\left\lfloor \frac{n}{m_1} \right\rfloor - \left\lfloor \frac{n}{m_2} \right\rfloor \cdot \left\lfloor \frac{m_2}{m_1} \right\rfloor \right) \cdot m_1 \tag{6}$$

■

In the next section, the idea of the proof will be more clear explained by using an example.

IV. Example

As an example, let us consider the 2×2 WSW1 switching fabric with $n = 16$, $z = 2$, $m_1 = 3$, and $m_2 = 5$, the set $\mathbb{C}$ of connection requests through this switching fabric is shown in Fig. 4. It can be represented by matrices $\mathrm{H}^{m_1} = \begin{bmatrix} 0 & 2 \\ 0 & 3 \end{bmatrix}$ and $\mathrm{H}^{m_2} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix}$. The number of permutation matrices for H^{m_1} is $c_{\max}^{m_1} = 5$, and for H^{m_2} is $c_{\max}^{m_2} = 3$. The matrices that are not contain 1's in each row and each column can be merged together because it represents connections independent to each other and are directed to different outputs. After decomposition the number of permutation matrices from H^{m_1} that cannot be merged with other matrices, is equal to $c_{\min}^{m_1} = 0$ and for H^{m_2} is $c_{\min}^{m_2} = 0$. But that not mean all of the permutation matrices can be merged together because of the value of n/m_1,n/m_2 and m_2/m_1 are not integer (or at lest the third value is not integer which is important). So we can not merge matrices only ratio $\lfloor \frac{m_2}{m_1} \rfloor$ and such is in 4 this example. For H^{m_1}, $\mathrm{P}_1^{m_1} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, we get $\mathrm{H}_1^{m_1} = \mathrm{H}^{m_1} - \mathrm{P}_1^{m_1} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}$. Then the next permutation matrix is $\mathrm{P}_2^{m_1} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, and $\mathrm{H}_2^{m_1} = \mathrm{H}^{m_1} - \mathrm{P}_2^{m_1} = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}$.

TABLE II
Assignment of FSUs for connections in Example 1.

Perm. matrix	Merged perm. matr.	Connection	Merged Connection	Assigned FSUs
$\mathrm{P}_1^{m_1}$	— —	$(I_1[7],\ O_2[4],\ 1)$ $(I_2[1],\ O_1[7],\ 1)$	—	1
$\mathrm{P}_1^{m_2}$	— —	$(I_1[4],\ O_2[1],\ 3)$ $(I_2[8],\ O_1[4],\ 3)$	—	2—4
$\mathrm{P}_2^{m_2}$	— —	$(I_1[8],\ O_2[5],\ 3)$ $(I_2[11],\ O_1[14],\ 3)$	—	5—7
$\mathrm{P}_3^{m_2}$	$\mathrm{P}_5^{m_1}$ $\mathrm{P}_6^{m_1}$ $\mathrm{P}_7^{m_1}$	$(I_1[11],\ O_2[8],\ 3)$	$(I_2[2],\ O_1[8],\ 1)$ $(I_2[3],\ O_1[9],\ 1)$ $(I_2[4],\ O_1[10],\ 1)$	8—10 8 9 10
$\mathrm{P}_4^{m_2}$	$\mathrm{P}_8^{m_1}$ $\mathrm{P}_9^{m_1}$ $\mathrm{P}_{10}^{m_1}$	$(I_1[14],\ O_2[11],\ 3)$	$(I_2[5],\ O_1[11],\ 1)$ $(I_2[6],\ O_1[12],\ 1)$ $(I_2[7],\ O_1[13],\ 1)$	11—13 11 12 13
$\mathrm{P}_5^{m_2}$	$\mathrm{P}_2^{m_1}$ $\mathrm{P}_3^{m_1}$ $\mathrm{P}_4^{m_1}$	$(I_2[14],\ O_2[14],\ 3)$	$(I_1[1],\ O_1[1],\ 1)$ $(I_1[2],\ O_1[2],\ 1)$ $(I_1[3],\ O_1[3],\ 1)$	14—16 14 15 16

From the last $Hmi21$ we can get three equal permutation matrices $\mathrm{P}_3^{m_1} = \mathrm{P}_4^{m_1} = \mathrm{P}_5^{m_1} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$. For $Hm2$ the first and second permutation matrices are equal $\mathrm{P}_1^{m_2} = \mathrm{P}_2^{m_2} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. And the next permutation matrix we get from $\mathrm{H}_2^{m_2} = \mathrm{H}^{m_2} - \mathrm{P}_1^{m_2} - \mathrm{P}_2^{m_2} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \mathrm{P}_3^{m_2}$. For each permutation matrices that we got, the assigned FSUs is

V. Conclusions

We considered WSW1 switching fabrics for elastic optical switching nodes. For switching fabrics of capacities $r = 2$ and with two connection rates $z = 2$. We derived the formula for rearrangeablility in any values of n, m_1, and m_2. A step by step example to explain the idea of merging operation is stated with details.

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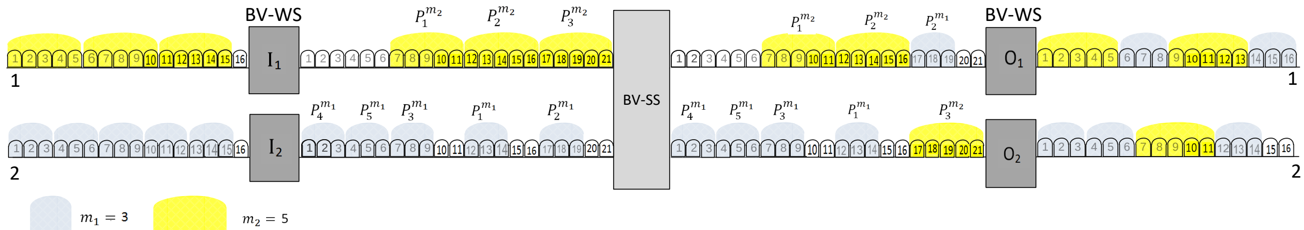

Fig. 4. The 2×2 WSW1 switching fabric with $\mathbb{C} = \{(I_1[1], O_1[1], 5); (I_1[6], O_1[9], 5); (I_1[11], O_2[7], 5); (I_2[1], O_1[6], 3); (I_2[4], O_1[14], 3); (I_2[7], O_2[1], 3); (I_2[10], O_2[4], 3); (I_2[13], O_2[12], 3)\}$ set up through 30 FSUs

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