

# Call-level Evaluation of a Two-Link Single Rate Loss Model for Poisson Traffic

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**Abstract**—We study a teletraffic loss model of a two-link system that accommodates calls from a single service-class which follow a Poisson arrival process. Each link has two thresholds which refer to the number of in-service calls in the link. The lowest threshold, named support threshold, defines up to which point the link can support calls offloaded from the other link. The highest threshold, named offloading threshold, defines the point where the link starts offloading calls to the other link. The model does not have a product form solution for the steady state probabilities. However, an approximate formula exists in the literature for the calculation of call blocking probabilities. The accuracy of the formula is verified through simulation and found to be quite satisfactory.

## I. INTRODUCTION

Contemporary communication networks require Quality of Service (QoS) mechanisms in order to provide the necessary bandwidth needed by calls. In the case of call-level traffic in a single link, modeled as a loss system, such a QoS mechanism is a bandwidth sharing policy [1]. The simplest bandwidth sharing policy is the Complete Sharing (CS) policy, where a new call is accepted in the system if the call's bandwidth is available. Otherwise, the call is blocked and lost without further affecting the system. The simplest teletraffic loss model that adopts the CS policy is the classical Erlang model [1]. In this model, the call arrival process is Poisson while calls require one bandwidth unit (b.u.) to be accepted in the system and have generally distributed service times. The fact that Call Blocking Probabilities (CBP) are calculated via the classical Erlang B formula has led to numerous extensions of Erlang's model for the call-level analysis of wired (e.g., [2]-[15]), wireless (e.g., [16]-[27]), satellite (e.g., [28]-[30]) and optical networks (e.g., [31]-[36]).

In the recent work of [24], the Erlang B formula has been adopted in order to provide approximate CBP (compared to simulation) in a two-link system that accommodates Poisson arriving calls of a single service-class. Each link is modeled as a loss system (i.e., no queueing is permitted) and has two thresholds which refer to the number of in-service calls in the link. The lowest threshold, named support threshold, defines up to which point the link can support calls offloaded from the other link. The highest threshold, named offloading threshold, defines the point where the link starts offloading calls to the other link. By the term "offloaded call", we refer to a call that initially arrived in a link, but is served by the other link, if there exist available b.u.

The model of [24] does not have a Product Form Solution (PFS) for the steady state probabilities. This is due to the fact that the offloading mechanism destroys Local Balance (LB) between adjacent states (states that differ only by one call) of the system. In order to calculate the various performance measures of the system, e.g., CBP or link utilization, either a linear system of Global Balance (GB) equations should be solved or an approximate method that relies on the independence between the links and the classical Erlang B formula can be adopted. The system of GB equations leads to an accurate calculation of the performance measures but it requires the knowledge of the state space of the two-link system. Such a state space may consist of millions of states if the capacity of the links is high. Thus, the method of solving the GB equations can only be applied in small (tutorial) systems. On the other hand, the link independence assumption and the Erlang B formula facilitates the necessary calculations.

In this paper, we evaluate the model of [24] by studying the impact of the thresholds in CBP and by comparing this model with the single link Erlang loss model. In addition, we provide an approximate but recursive formula for the calculation of the link occupancy distribution for each link, which further simplifies the determination of the various performance measures.

This paper is organized as follows: In Section II, we review the system of [24]. In Section III, we review and provide insight to the analytical model of [24]. In addition, we show a recursive formula for the calculation of the link occupancy distribution and CBP. In Section IV, we provide analytical and simulation CBP results for the model of [24] and compare them with the corresponding analytical results obtained, for a single link, by the Erlang B formula. We conclude in Section V. In the Appendix, we provide a tutorial example.

## II. THE SYSTEM

We consider a system of two links with capacities  $C_1$  and  $C_2$  b.u., respectively. Each link accommodates Poisson arriving calls of a single service-class which require one b.u. in order to be connected in a link. Let  $\lambda_1$  and  $\lambda_2$  be the arrival rates in the 1<sup>st</sup> and 2<sup>nd</sup> link, respectively. We also denote by  $j_1$  and  $j_2$  the occupied b.u. in the 1<sup>st</sup> and 2<sup>nd</sup> link, respectively. Then,  $0 \leq j_1 \leq C_1$  and  $0 \leq j_2 \leq C_2$ . Since calls require one b.u., the values of  $j_1, j_2$  also represent the number of in-service calls in the 1<sup>st</sup> and the 2<sup>nd</sup> link, respectively.

Each link  $l$  ( $l=1, 2$ ) has two different thresholds: the support threshold  $th_{1l}$  and the offloading threshold  $th_{2l}$ , with  $th_{1l} < th_{2l}$  and  $0 \leq th_{1l}, th_{2l} \leq 1$ . Assuming that  $\lfloor x \rfloor$  is the largest integer not exceeding  $x$ , then the role of these thresholds, in the  $l$ th link, is the following (see Fig. 1):

- a) If  $0 \leq j_l < \lfloor th_{1l} C_l \rfloor$  then the  $l$ th link is in a *support mode* of operation, i.e., it accepts and serves not only new calls that initially arrive in the  $l$ th link but also new calls offloaded from the  $m$ th link ( $m=1, 2, m \neq l$ ).
- b) If  $\lfloor th_{1l} C_l \rfloor \leq j_l < \lfloor th_{2l} C_l \rfloor$  then the  $l$ th link is in a *normal mode* of operation, i.e., it does not accept calls offloaded from the  $m$ th link. It only accepts calls that initially arrive in the  $l$ th link.
- c) If  $\lfloor th_{2l} C_l \rfloor \leq j_l$  then the  $l$ th link is in an *offloading mode* of operation, i.e., a new call that initially arrives in the  $l$ th link will be offloaded to the  $m$ th link. If the  $m$ th link is in *support mode* (i.e.,  $0 \leq j_m < \lfloor th_{1m} C_m \rfloor$ ) then the call will be accepted in the  $m$ th link. If the  $m$ th link is not in *support mode* and  $j_m \leq C_m - 1$  the call will be accepted in the  $l$ th link. Otherwise the call will be blocked and lost.

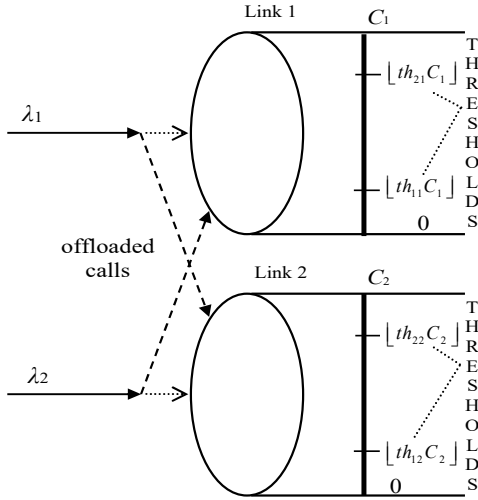


Figure 1. The system of the two links.

Based on the above, the call admission of a new call that initially arrives in the  $l$ th link ( $l=1, 2$ ) is summarized in the following steps:

- 1) If  $(0 \leq j_l < \lfloor th_{2l} C_l \rfloor)$  then the call is accepted by the  $l$ th link and remains for a generally distributed service-time with mean  $\mu^{-1}$ .
- 2) If  $\lfloor th_{2l} C_l \rfloor \leq j_l$  then: 2a) if  $0 \leq j_m < \lfloor th_{1m} C_m \rfloor$  the call is offloaded to the  $m$ th link and remains for a generally distributed service-time with mean  $\mu^{-1}$ , 2b) if  $\lfloor th_{1m} C_m \rfloor \leq j_m$ , the  $m$ th link is in a *normal mode* of operation and does not support offloaded calls from the  $l$ th link. In that case, the call will try to be accepted in the  $l$ th link. If  $j_l \leq C_l - 1$ , then the call is accepted in the  $l$ th link and remains for a generally distributed service-time. Otherwise, the call is blocked and lost without further affecting the system of the two links.

Otherwise, the call is blocked and lost without further affecting the system of the two links.

A tutorial example in the Appendix, presents in detail the call admission mechanism and the required calculations for the CBP determination.

### III. THE ANALYTICAL MODEL

Due to the *support* and *offloading modes* of operation of the two links, the 2-D Markov chain of the system is not reversible and therefore LB between adjacent states (states that differ only by one call) is destroyed. Thus, the steady state distribution,  $P(\mathbf{j}) = P(j_1, j_2)$ , of this system cannot be described by a PFS. To determine the values of  $P(j_1, j_2)$  (and consequently CBP) there exist two different methods.

The first method provides accurate results (compared to simulation) but requires the knowledge of the state space of the system and the solution of the set of linear GB equations for each state  $\mathbf{j} = (j_1, j_2)$  expressed as *rate into state  $\mathbf{j}$  = rate out of state  $\mathbf{j}$* :

$$\begin{aligned} & \lambda_1(j_1-1, j_2)P(j_1-1, j_2) + \lambda_2(j_1, j_2-1)P(j_1, j_2-1) + \\ & (j_1+1)\mu P(j_1+1, j_2) + (j_2+1)\mu P(j_1, j_2+1) = \\ & \lambda_1(j_1, j_2)P(j_1, j_2) + \lambda_2(j_1, j_2)P(j_1, j_2) + (j_1\mu + j_2\mu)P(j_1, j_2) \end{aligned} \quad (1)$$

where:

$$\lambda_l(j_1, j_2) = \begin{cases} \lambda_l + \lambda_m, & \text{if } (j_l < \lfloor th_{1l} C_l \rfloor) \cap (j_m \geq \lfloor th_{2m} C_m \rfloor) \\ 0, & \text{if } (j_l \geq \lfloor th_{2l} C_l \rfloor) \cap (j_m < \lfloor th_{1m} C_m \rfloor) \\ 0, & \text{if } (j_1, j_2) \text{ is a boundary state} \\ \lambda_l, & \text{otherwise} \end{cases} \quad (2)$$

Having obtained the values of  $P(j_1, j_2)$ , we can determine the CBP in the 1<sup>st</sup> and the 2<sup>nd</sup> link,  $P'_{b_1}$  and  $P'_{b_2}$  via (3) and (4), respectively [24]:

$$P'_{b_1} = \sum_{j_2=\lfloor th_{21} C_1 \rfloor}^{C_2} P(C_1, j_2) \quad (3)$$

$$P'_{b_2} = \sum_{j_1=\lfloor th_{12} C_2 \rfloor}^{C_1} P(j_1, C_2) \quad (4)$$

In addition, we can calculate the total blocking probability in the system via the following weighted summation [24]:

$$P'_b = \frac{\lambda_1}{\lambda_1 + \lambda_2} P'_{b_1} + \frac{\lambda_2}{\lambda_1 + \lambda_2} P'_{b_2} \quad (5)$$

Finally, we can also determine the average number of occupied b.u. in the  $l$ th link,  $E'_l$ , and the total number of occupied b.u. in the system,  $E'_{tot}$ , via the formulas:

$$E'_l = \frac{\lambda_l}{\mu} (1 - P'_{b_l}), \quad l=1, 2 \quad (6)$$

$$E'_{tot} = \frac{\lambda_1 + \lambda_2}{\mu} (1 - P'_b) \quad (7)$$

Before we proceed with the second method, we emphasize that the state space determination and the solution of the set of GB equations can be quite complex even for systems of moderate size and therefore is only practically used for small tutorial examples (see Appendix).

The second method provides approximate CBP results (compared to simulation) by assuming that the two links operate independently from one another. Such an assumption simplifies the necessary calculations for the determination of CBP.

Since each independent link behaves as an Erlang loss system, the CBP in the 1<sup>st</sup> and the 2<sup>nd</sup> link can be approximated by (8) and (9), respectively [24]:

$$P_{b_1} = P_1(C_1)P_2(j_2 \geq \lfloor th_{12}C_2 \rfloor) \quad (8)$$

$$P_{b_2} = P_2(C_2)P_1(j_1 \geq \lfloor th_{11}C_1 \rfloor) \quad (9)$$

where  $P_l(C_l)$  refers to the CBP in the  $l$ th link ( $l=1, 2$ ) which can be determined by the classical Erlang B formula:

$$P_l(C_l) = \frac{a_l^{C_l}}{C_l!} \frac{1}{\sum_{i=0}^{C_l} \frac{a_l^i}{i!}}, \quad a_l = \lambda_l / \mu \quad (10)$$

As far as the values of  $P_l(j_l \geq \lfloor th_{l1}C_l \rfloor)$  are concerned they are given by:

$$P_l(j_l \geq \lfloor th_{l1}C_l \rfloor) = \sum_{j_l = \lfloor th_{l1}C_l \rfloor}^{C_l} P_l(j_l) \quad (11)$$

where  $P_l(j_l)$  is determined by the truncated Poisson distribution:

$$P_l(j_l) = \frac{a_l^{j_l}}{j_l!} \frac{1}{\sum_{i=0}^{C_l} \frac{a_l^i}{i!}}, \quad a_l = \lambda_l / \mu \quad (12)$$

The rationale behind (8), (9) is that a call that initially arrives in the  $l$ th link will be blocked if there are no available b.u. in that link and the  $m$ th link is not in support mode of operation.

Finally, the total blocking probability can be determined via (5), where  $P'_{b_1}$  and  $P'_{b_2}$  are replaced by  $P_{b_1}$  and  $P_{b_2}$ , respectively.

In what follows, we propose an alternative way for the determination of  $P_l(j_l)$ ,  $j_l = 1, \dots, C_l$ , which is recursive and is based on the assumption that each link  $l$  operates independently from one another. In the Erlang loss model, used to describe each link  $l$ , there exist LB between the adjacent states  $j_l - 1$  and  $j_l$  and has the form:

$$\lambda_l P'_l(j_l - 1) = j_l \mu P'_l(j_l) \quad \text{or} \quad j_l P'_l(j_l) = a_l P'_l(j_l - 1) \quad (13)$$

Based on (13), we can recursively determine the unnormalized values of  $P'_l(j_l)$ 's assuming as an initial value that  $P'_l(0) = 1$ . The normalized values of  $P'_l(j_l)$ 's will be given by:

$$P_l(j_l) = \frac{P'_l(j_l)}{\sum_{x=0}^{C_l} P'_l(x)} \quad (14)$$

Based on (14), we can calculate  $P_{b_1}$ ,  $P_{b_2}$  and the total blocking probability, via (8), (9) and (5), respectively. By replacing  $P'_{b_l}$  with  $P_{b_l}$  in (6), we can determine the values of  $E_l$  and consequently the value of  $E_{tot}$ .

#### IV. NUMERICAL EXAMPLES - EVALUATION

In this section, we present an application example and provide analytical and simulation CBP results of the model of [24] and analytical CBP results of the Erlang B model assuming that: 1) each link works separately from the other and 2) we have a link of capacity  $C = C_1 + C_2$  that serves the total offered traffic-load,  $\alpha = \alpha_1 + \alpha_2$ . Simulation results are derived via the Simscript III simulation language [37] and are mean values of 7 runs. As far as the reliability ranges are concerned they are less than two order of magnitude, and therefore are not presented in the following figures. All simulation runs are based on the generation of twenty million calls per run. To account for a warm-up period, the first 5% of these generated calls are not considered in the CBP results.

As an application example, consider a system of two links of capacities  $C_1 = C_2 = 30$  b.u., that accommodates Poisson arriving calls with  $\lambda_1 = 14$  calls/min,  $\lambda_2 = 12$  calls/min and let  $\mu^{-1} = 1.0$  min. We consider three different support thresholds: 1)  $th_{11} = th_{12} = 0.1$ , 2)  $th_{11} = th_{12} = 0.3$  and 3)  $th_{11} = th_{12} = 0.5$ . In all three cases, we assume that the offloading thresholds do not alter and are equal to:  $th_{21} = th_{22} = 0.7$ .

In the x-axis of Figs 2-3 the offered traffic load in the first and in the second link increases in steps 0.5 erl. So, point 1 is:  $(\alpha_1, \alpha_2) = (14.0, 12.0)$  while point 7 is:  $(\alpha_1, \alpha_2) = (17.0, 15.0)$ .

In Figs. 2-3, we present analytical and simulation CBP results of the model of [24]. As a reference, we provide the analytical CBP results for a single link with: 1)  $C_1 = 30$  b.u. and offered traffic-load  $\alpha_1$  erl (Fig. 2), 2)  $C_2 = 30$  b.u. and offered traffic-load  $\alpha_2$  erl (Fig. 3) and 3)  $C = 60$  b.u. and offered traffic-load  $\alpha_1 + \alpha_2$  erl (Figs. 2, 3). Both figures show that the analytical CBP results of [24]: a) are close to the corresponding simulation results, b) cannot be approximated by the CS system of  $C=60$  b.u., c) decrease as the support thresholds increase, an intuitively expected fact since both links cooperate with each other and d) are almost identical to the corresponding Erlang B single link CBP when the support thresholds are equal to 0.1. This is because when  $th_{11} = th_{12} = 0.1$ , both links work almost in isolation.

In Fig. 4, we consider that  $(\alpha_1, \alpha_2) = (17.0, 15.0)$ ,  $th_{11} = 0.1$  and present the analytical CBP results in the 1<sup>st</sup> link for various support thresholds of the 2<sup>nd</sup> link and various values of  $C_2$ . We see that by increasing  $th_{12}$ , the values of

$P_{b_1}$  decrease since the 2<sup>nd</sup> link starts supporting calls of the 1<sup>st</sup> link. However, the decrease of  $P_{b_1}$  is higher if the capacity of the 2<sup>nd</sup> link is high too. This is also expected, since a high value of  $C_2$  and a high value of  $th_{12}$  results in more b.u. provided to support calls from the 1<sup>st</sup> link. Contrary to the decrease in  $P_{b_1}$ , the CBP in the 2<sup>nd</sup> link increases from (0.00022 when  $C_2=30$ , to 0.005 when  $C_2=25$  and 0.046 when  $C_2=20$  b.u.). This is anticipated since the support threshold in the 1<sup>st</sup> link remains the same ( $th_{11} = 0.1$ ) and therefore arriving calls to the 2<sup>nd</sup> link do not actually have access to the b.u. of the 1<sup>st</sup> link.

## V. CONCLUSION

In this paper we evaluate a single-rate two-link loss system that accommodates Poisson arriving calls. A link can share a part of its capacity in order to support calls from the other link and vice versa. The model does not have a PFS for the steady state distribution. However, an approximate method based on the classical Erlang B formula exists that provides quite satisfactory results in terms of CBP and average number of occupied b.u. As a future work, we intend to study this two-link system under the assumption that it serves many service-classes with different bandwidth-per-call requirements and various call arrival processes.

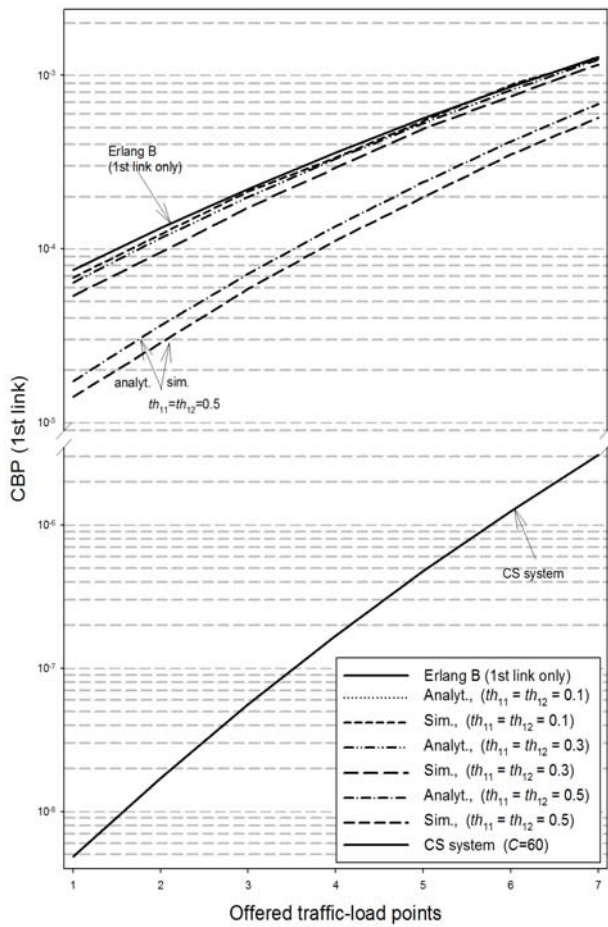


Figure 2. CBP – 1<sup>st</sup> link.

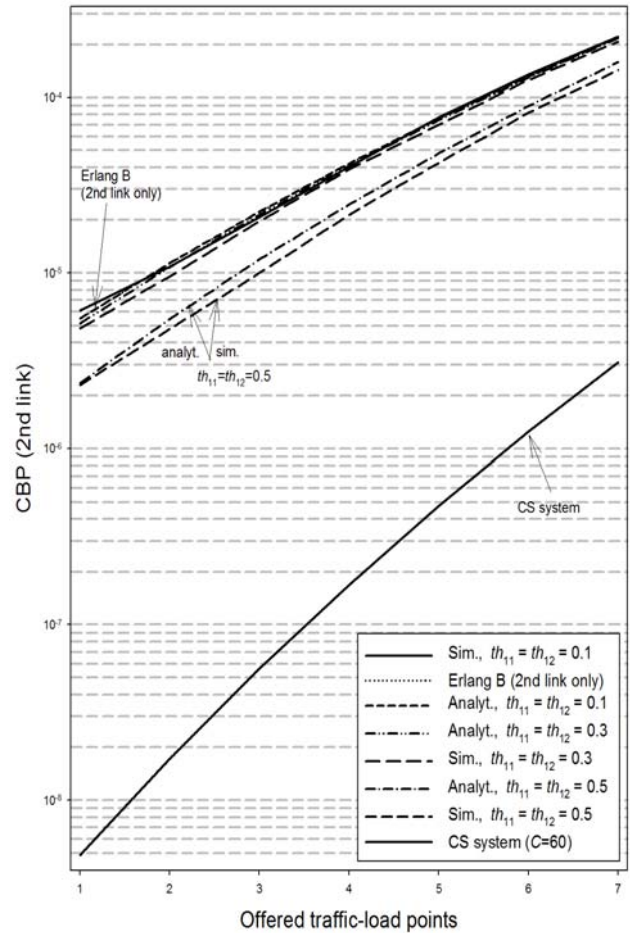


Figure 3. CBP – 2<sup>nd</sup> link.

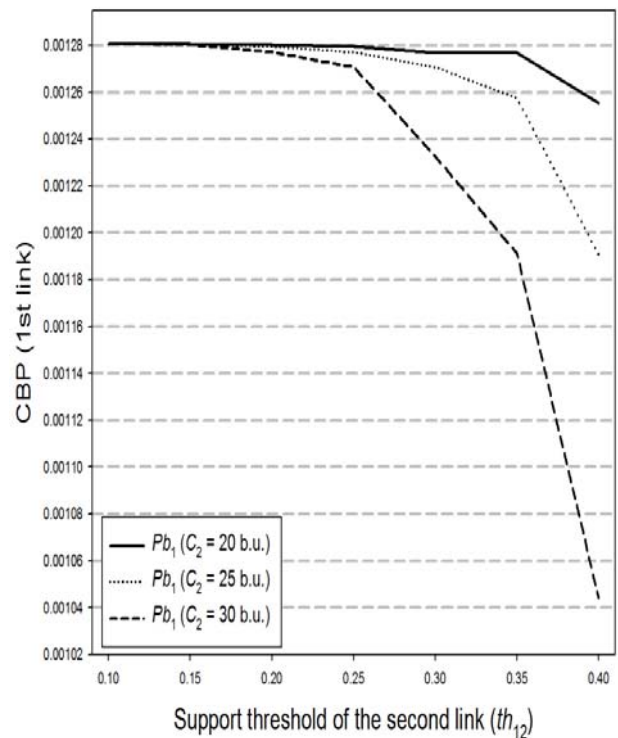


Figure 4. CBP– 1<sup>st</sup> link (for different values of  $th_{12}$ ).

## APPENDIX. TUTORIAL EXAMPLE

Consider a system of two links with  $C_1=6$  and  $C_2=5$  b.u., that accommodates calls of a single service-class. Let  $\lambda_1 = 4$  calls/min,  $\lambda_2 = 2$  calls/min and  $\mu^{-1} = 1$  min. The thresholds for this system are the following:

1<sup>st</sup> link ( $l=1$ ):  $th_{11} = 0.2, th_{21} = 0.7$ .

2<sup>nd</sup> link ( $l=2$ ):  $th_{12} = 0.2, th_{22} = 0.7$

Based on the thresholds' values we have:

### 1<sup>st</sup> link

- If  $0 \leq j_1 < \lfloor th_{11}C_1 \rfloor \Rightarrow 0 \leq j_1 < 1$  then the 1<sup>st</sup> link is in a *support mode* of operation.
- If  $\lfloor th_{11}C_1 \rfloor \leq j_1 < \lfloor th_{21}C_1 \rfloor \Rightarrow 1 \leq j_1 < 4$  then the 1<sup>st</sup> link is in a *normal mode* of operation.
- If  $\lfloor th_{21}C_1 \rfloor \leq j_1 \Rightarrow 4 \leq j_1$  then the 1<sup>st</sup> link is in an *offloading mode* of operation.

### 2<sup>nd</sup> link

- If  $0 \leq j_2 < \lfloor th_{12}C_2 \rfloor \Rightarrow 0 \leq j_2 < 1$  then the 2<sup>nd</sup> link is in a *support mode* of operation.
- If  $\lfloor th_{12}C_2 \rfloor \leq j_2 < \lfloor th_{22}C_2 \rfloor \Rightarrow 1 \leq j_2 < 3$  then the 2<sup>nd</sup> link is in a *normal mode* of operation.
- If  $\lfloor th_{22}C_2 \rfloor \leq j_2 \Rightarrow 3 \leq j_2$  then the 2<sup>nd</sup> link is in an *offloading mode* of operation.

The state space of the system consists of 42 states of the form  $(j_1, j_2)$ , depicted in Fig. 5 together with the corresponding transition rates. To help a reader understand the state transition diagram of Fig. 5 and the offloading mechanism, assume that the system is in state  $(0, 2)$  when a new call arrives in the 2<sup>nd</sup> link. Then, the call will be accepted in the 2<sup>nd</sup> link and the new state will be  $(0, 3)$ . If another new call arrives in the 2<sup>nd</sup> link then the call will be offloaded to the 1<sup>st</sup> link (and served by that link) and the new state will be  $(1, 3)$ . If now, another call arrives in the 2<sup>nd</sup> link, then this call cannot be offloaded to the 1<sup>st</sup> link (since  $j_1 = 1$ ) but it can be served by the 2<sup>nd</sup> link due to bandwidth availability. In that case the new state will be  $(1, 4)$ . A similar rationale exists when we consider call arrivals in the 1<sup>st</sup> link and the states  $(3, 0)$ ,  $(4, 0)$ ,  $(4, 1)$  and  $(5, 1)$ .

Based on the solution of the 42 GB equations of Fig. 5, the CBP in the 1<sup>st</sup> and 2<sup>nd</sup> link is given by:

$$P'_b = \sum_{j_2=\lfloor th_{12}C_2 \rfloor}^{C_2} P(C_1, j_2) = \sum_{j_2=1}^5 P(6, j_2) = 0.10370$$

$$P'_{b_2} = \sum_{j_1=\lfloor th_{11}C_1 \rfloor}^{C_1} P(j_1, C_2) = \sum_{j_1=1}^6 P(j_1, 5) = 0.03758$$

On the same hand, the total blocking probability in the two-link system is determined by:

$$P_b = \frac{\lambda_1}{\lambda_1 + \lambda_2} P'_b + \frac{\lambda_2}{\lambda_1 + \lambda_2} P'_{b_2} \stackrel{\lambda_1=4}{\lambda_2=2} = 0.08166.$$

As far as the values of  $E'_1$ ,  $E'_2$  and  $E'_{tot}$  are concerned, we have:

$$E_1 = 3.5852 \text{ b.u.}, E_2 = 1.9248 \text{ b.u. and } E_{tot} = 5.51 \text{ b.u.}$$

Based on the approximate method of link independence and (8), (9), we have:

$$P_b = P_1(C_1)P_2(j_2 \geq \lfloor th_{12}C_2 \rfloor) = P_1(6)P_2(j_2 \geq 1) = 0.11716 * 0.862386 \Rightarrow P'_b = 0.10104.$$

$$P_{b_2} = P_2(C_2)P_1(j_1 \geq \lfloor th_{11}C_1 \rfloor) = P_2(5)P_1(j_1 \geq 1) = 0.03670 * 0.979405 \Rightarrow P'_{b_2} = 0.03594.$$

The total blocking probability in the two-link system is determined by:

$$P_b = \frac{\lambda_1}{\lambda_1 + \lambda_2} P'_b + \frac{\lambda_2}{\lambda_1 + \lambda_2} P'_{b_2} \stackrel{\lambda_1=4}{\lambda_2=2} = 0.07934.$$

As far as the (approximate) values of  $E_1$ ,  $E_2$  and  $E_{tot}$  are concerned, we have:

$$E_1 = 3.5958 \text{ b.u.}, E_2 = 1.9281 \text{ b.u. and } E_{tot} = 5.524 \text{ b.u.}$$

The previous results reveal that the approximate method provides quite satisfactory results compared to the exact values, even in small tutorial examples.

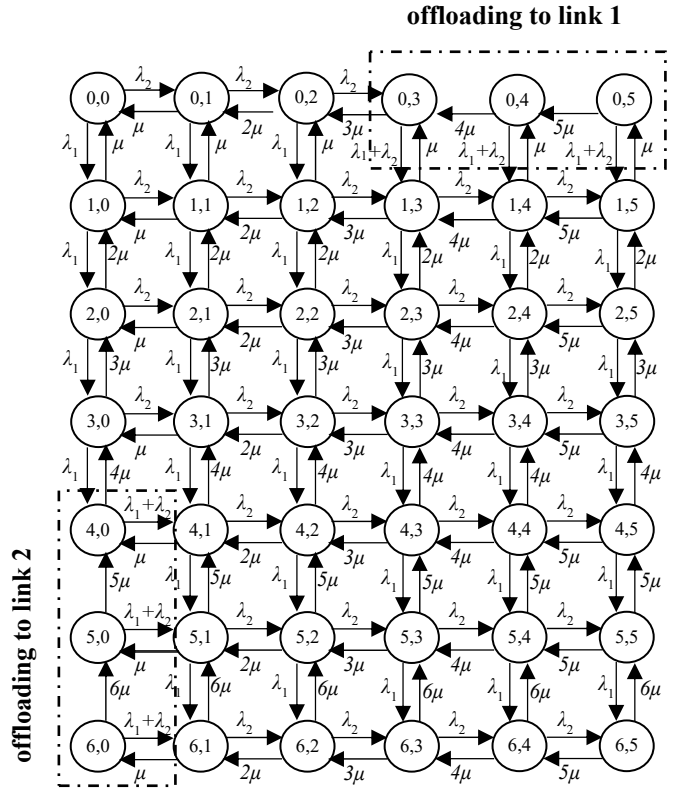


Figure 5. State transition diagram of the tutorial example.

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