# Concatenation of convolutional codes over ring and STBC codes for multiuser CPM transmission

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Abstract— In this paper, the concatenation of convolutional codes over ring and Space-Time Block Codes (STBC) is proposed for multiuser systems based on the Frequency Division Multiple Access (FDMA) method and the transmission signals with Continuous-Phase Modulation (CPM). In the presented FDMA system, the spacing between adjacent carriers are decreased to increase efficiency of accessible frequency band utilization. It makes the significant reduction of transmission quality due to the occurrence of Inter-Channel Interference (ICI). To improve the system performance and reduce the bit error rate (BER) a concatenation of CPM modulator, convolutional non-binary encoder and STBC encoder has been used on the transmitter side. In the receiver we used a lowcomplexity iterative algorithm for ICI cancellation a Minimum Mean Square Error (MMSE) detector. Finally, the computer simulations BER results for proposed solution are described and presented.

### Keywords — Inter-Channel Interference; Multiuser Detection, Receiver of Continuous-Phase Modulation

## I. INTRODUCTION

The very important feature of systems using wireless transmission is effective band utilization. The next attribute of wireless systems is enabling cheap consumer-grade transceivers to work with high power efficiency and low power consumption. This is the reason why radio transmitters operate close the saturation region of power amplifier and an appropriate design of the modulation scheme is required in order to properly receive the strongly distorted signal. Continuous Phase Modulations (CPM) are resistant to nonlinearity and are the modulation classes that enable obtaining of high spectral efficiency a wireless systems [1]. Multiuser (MU) CPM systems wherein all users employ a portion of the spectrum have been studied in, e.g., [2], [3], [4], [5].

In order to increase the reliability and bit rate of telecommunication systems, the multiantenna MIMO technique (Multiple Input Multiple Output) is used more and more often [6]. The technique involves multiple antennas on the transmitter and receiver sides. Employing the MIMO technique in wireless systems allows the use of Space Time Codes (STC), improving transmission quality [6]. The one of the most popular STC codes and widely used in MIMO systems are space time block codes (STBC). The space time block codes were designed to achieve the maximum diversity

order for the given number of transmit and receive antennas subject to the constraint of having a simple linear decoding algorithm. Space time block codes (STBC) [7] are a generalized version of Alamouti scheme [8], but have the same key features. These codes are orthogonal and can achieve full transmit diversity specified by the number of transmit antennas. In [9] and [10], the authors studied CPM signals encoded using the so-called orthogonal spacetime codes based on Alamouti's solution for linear modulations [8]. The CPM signals concatenated with STBC codes are characterized by a low bit error rate attainable at a simultaneously low receiver complexity. Neither of the above solutions for MIMO transmission concerned multiuser CPM systems.

In order to obtain a further improvement in energy efficiency, CPM was combined in classical systems with an external binary convolutional encoder (CE) and a mapper. It has been shown that a CPM scheme can be decomposed into a continuous phase encoder (CPE) followed by a memoryless modulator (MM) [11], where the CPE is a CE over a ring of integers [12]. Therefore, a natural way to combine CPM with an outer CE is to use a CE over the same ring of integers. Since the CE and CPE are over the same algebra, no mapper is needed, and an extra coding gain was reported [13].

The literature has discussed the aspects of increasing the spectral efficiency of systems employing CPM and FDMA (Frequency Division Multiple Access), e.g., in [14], but STBC coding concatenated with convolutional codes over ring in multiuser scenario has not been taken into consideration.

This paper considers a wireless MIMO transmission system in a multiple user environment. It involves a combination of non-binary convolutional codes with STBC and CPM signals for transmission in an uplink channel using the FDMA method. For a more efficient available band utilization, the distances between individual system user carriers are reduced. Such asolution causes the deterioration of transmission quality due to the occurrence of ICI. In order to reduce the bit error rate (BER) in such a system, a combination of CPM with non-binary convolutional codes over a ring has been used and a low-complexity iterative algorithm for ICI cancellation was employed on the receiver side.

The paper is organized as follows. Section 2 presents a brief description of CPM signals. Section 3 presents the

considered system. Section 4 provides the obtained simulation results. The paper is summarized and concluded in the final, 5-th Section.

### II. CONTINUOUS-PHASE MODULATION

The general equation describing CPM signals may be presented as follows [1]:

$$x(t,\boldsymbol{\alpha}) = \sqrt{\frac{2E_s}{T}} \cos(2\pi f_0 t + \phi(t,\boldsymbol{\alpha}) + \phi_0)(1)$$

where Es is the energy per symbol, T is the modulation interval, f0 is the carrier frequency,  $\phi 0$  is the initial phase, and  $\alpha = (..., \alpha-1, \alpha 0, \alpha 1, ...)$  refers to a sequence of data symbols adopting one of the values from the set:

$$\alpha_i \in \{\pm 1, \pm 2, \dots, \pm (M-1)\}$$
(2)

Phase  $\phi(t,\alpha)$ , which carries information, may be described by the following equation:

$$\phi(t, \boldsymbol{\alpha}) = 2\pi h \int_{-\infty}^{t} \sum_{t=-\infty}^{\infty} a_{i} g(\tau - iT) d\tau \quad (3)$$

where h is the modulation index defining the value by which the phase changes in each modulation interval.

CPM signals are characterized by the following parameters: modulation index h, pulse length L and phase response function q(t), or its derivative g(t), the frequency response function. One of the parameters having an influence on the spectral characteristics of CPM signals is the shape of a frequency pulse g(t). In this paper, CPM signals with a raised cosine (RC) pulse are analysed. A raised cosine-shaped pulse guarantees better spectral features of a CPM signal than a rectangular pulse. This is caused by gentle phase changes in the CPM signal in consecutive modulation intervals.

The CPM modulator may be performed as a cascade concatenation of a continuous phase encoder (CPE) and a memoryless modulator (MM). Such a system is known as the Rimoldi decomposition [11]. The CPE is a convolutional encoder that performs the function of a CPM modulator memory. The memoryless modulator assigns relevant signal shapes to symbols received from the CPE.

To increase CPM signals' resistance to different types of interference arising during transmission, a connection between the CPM signal modulator system with an external encoder is introduced. Most frequently, it is a binary convolutional coder.

The binary convolutional encoder is connected to the modulator by an interleaver and a mapper. The mapper is not needed if the external convolutional encoder is connected to CPE of a CPM modulator over he same ring of integers modulo-M.

The external convolutional encoder CE with rate RCE may be equipped with a symbol puncturer with rate  $R_{pct}$ . The overall encoding rate R, with the puncturing operation allowed, is:

$$R = R_{CC}R_{pct} \tag{4}$$

The discussed method of serial concatenation of the external convolutional encoder and CPM guarantees a low bit error rate (BER) [1, 11]. It is obtained with an iterative

receiver that makes many iterations between the CPE decoder and the decoder of the external convolutional code CE to make a decision on the transmitted information [4].

### III. SYSTEM DESCRIPTION

In this paper, an FDMA system for uplink wireless transmission using ring encoded CPM signals is analyzed. To obtain the best spectral efficiency of the examined system with multiple users, the spaces between carrier frequencies are minor. The tight inter carrier frequency spacing between adjacent channels causes strong ICI. To eliminate the ICI in the receiver, an ICI cancelation algorithm is employed. Therefore, it is possible to obtain a high spectral efficiency (ASE) can be determined, assuming that the signal to noise ratio approaches infinity. *ASE* is directly proportional to the encoding rate of CE ( $R_{CE}$ ) and the number of bits falling on one symbol is inversely proportional to the value of normalized spacing between carrier frequencies. *ASE* is given by means of the following formula:

$$ASE = \lim_{\substack{E_b \\ N_0} \to \infty} SE = \frac{R \log_2 M}{\Delta_f T}$$
(5)

where  $E_b/N_0$  is the signal to noise ratio, *R* is the encoding rate, and  $\Delta_f T$  is the normalized spacing between inter carrier frequencies, *M*-ary modulation.

The studies were aimed at the evaluation of transmission quality in the system under analysis. Figure 1 shows a block diagram of the proposed system with STTC coding.

At the input, each *k*th user non-binary sequence  $a_0, ..., a_{k-1}$ , is Convolutional Encoded over ring (CE), interleaved (Int.) and converted into two ( $M_T$ , T=2) parallel streams in an STBC encoder. Each data stream is conveyed to one of  $M_T$  CPM modulators.

An STBC-CPM system analyzed in this paper is based on Alamouti's scheme [8].

The transmitted CPM signal vector can be expressed as it was shown in (6).

$$x_{t} = \left[x_{t}^{1}, x_{t}^{2}, \dots, x_{t}^{n_{T}}\right]^{T}$$
(6)

where  $n_T$  is the number of transmit antennas, and *T* denotes the transpose of the vector [].

The modified Alamouti scheme [8] with transmission matrix

$$S = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix}$$
(7)

is used in this paper for two transmit antennas FDMA CPM system. The signals transmitted by the first and second antennas into two time intervals are described by equations  $x\mathbf{1} = [s_1 - s_2^*]$  and  $x\mathbf{2} = [s_2 s_1^*]$  respectively.

In the analyzed system, a MIMO channel is considered. The system is composed of k users each of whom transmits signals via  $M_T$  transmit antennas. The receiver uses  $M_R$  antennas. The channel model used in the simulation takes into account multipath propagation. The channel model has been implemented as a Taped Delay Line (TDL) and it models a channel with flat fading and Additive White Gaussian Noise (AWGN) [6].



Fig. 1. Block diagram of the MU STBC FDMA-CPM system

Each receive antenna receives a faded superposition of  $M_T$  simultaneously transmitted signals corrupted by additive white Gaussian noise. The fading is assumed to be flat and distributed according to a Rayleigh *pdf*. The random path gains between transmit antenna *m* and receive antenna *p*,  $h_{m,p}(t)$  are independent complex Gaussian random variables with zero mean and variance per dimension 1/2. The fading is slow, such that the  $M_T x M_R$  fading coefficients are constant during a frame, but vary from frame to frame. The AWGN noise components  $n_p(t)$  are independent zero-mean complex Gaussian random processes with power spectral density  $N_0$ .

At the receiving end, the system consists of a MIMO MMSE/ STBC detector/decoder and a low-complexity iterative algorithm to ICI cancellation [14].

The MMSE/STC block realizes the STC decoding and computes the cost function, i.e., minimizes:

$$G_{MMSE} = \sqrt{\frac{n_T}{E_s}} \left( \boldsymbol{H}^H \boldsymbol{H} + \frac{n_T N_0}{E_s} I_{n_T} \right)^{-1} \boldsymbol{H}^H (8)$$

where *H* is the matrix of channel impulse response estimates, *I* – identity matrix,  $N_0$  – spectral density of noise power, upper index of quantity  $H^H$  denotes the Hermitian transpose of matrix H and it is the sum of the operations of transpose and complex conjugate of the matrix. Signal y from MMSE/STC reaches the ICI cancellation block. The receiver carries out ICI cancellation through a set of single-user MAP detector/ remodulator blocks, as described by Perotti et al. [14]. The remodulators make use of the output of the MAP detector to compute the remodulated signal  $s_k^{\tilde{t}_i}(t)$  relative to the kth user and *i*th iteration. The channel decoder performs two iterations loops. The inner loop is formed by the ICI canceller, the MAP detector, the CPE SISO decoder and the remodulator, while the outer loop involves the CPE SISO decoder, the non-binary CE SISO decoder, the interleaver and the deinterleaver between the inner CPE decoder and the outer CE decoder. ICI cancellation can be performed while executing the decoding iterations to enhance the receiver performance. In such a case, after the inner CPE decoder is executed, remodulation is performed. Then, interference cancellation is performed and the CPM receiver, including the inner CPE decoder, is again executed. The decoder starts decoding a received code word executing  $N_{IC}$  inner iterations. Then, it executes  $N_D$  times an outer iteration followed by an inner iteration. This way, ICI cancellation is performed as part of the decoding iterations and it results inan improved ICI cancellation [14]. On the final *outer* iteration, a decision is made on the transmitted data symbols  $\hat{a}_0, ..., \hat{a}_{k-1}$ .

## IV. SIMULATION RESULTS

The Monte-Carlo simulation method was used to determine the BER for the described FDMA system with STBC CPM modulation concatenated with binary CE or CE over a ring.

The simulations have been performed with the aid of a simulation program written in C++ using IT++ libraries, ver. 4.2 [15]. The analysis assumed that each user transmits CPM signals with the same modulation and coding parameters, and of equal strength. It was assumed that the standardized value of the interval between the carrier frequencies of consecutive channels  $\Delta_f T$  in the system is equal to 3/4, and that the transmission is performed via 8 neighboring channels (FDMA users). The signals are transmitted through an AWGN channel with spectral power density  $N_0/2$ . The transmitted packages were 1000 bits long. The signalis of was stopped if at least 100 errors occurred.

The analysis, for an FDMA system with CPM signals and CC over ring  $Z_4$ , has been performed with reference tobinary encoded CPMmodulation with the parameters h=1/4, h L=1 and the REC shape frequency pulse. The number of iterations in the receiver was experimentally fixed as a good trade-off between receiver performance and complexity. Two iterations  $(N_I=2)$  in the interference cancellation loop have been made and six iterations  $(N_O=6)$  in the soft output non-binary MAP decoder. The CPM modulator was concatenated with CE over ring [1, D+3, 3D+3] or a binary [7, 5] encoder (from [11]), both with four states. The obtained BER results for the CPM-FDMA systems with CE over a ring were compared with BER for CPM-FDMA systems employing binary CE which has the same spectral efficiencies.

Fig. 2 presents BER for systems employing convolutional encoded CPM modulation concatenated with STBC encoding for different numbers of receive antennas. In this case, each of the eight users transmitted signals via two transmit antennas, and the receiver used one, two or four receive antennas. In this scenario, the error rate at the BER level of  $10^{-4}$  obtained for the 2x2 system was worse than the one obtained for the 2x4 system by about 2 dB and better than the one obtained for the 2x1 system by about 3dB. We observe that a slight

improvement in BER is achieved for the systems using binary encoded STBC CPM signals in all cases. We can see that better results of BER can be obtained when concatenation STBC and non-binary CE was used.



Fig. 2. BER for STBC system (CPM: h = 1/2, M = 4, L = 1,  $\Delta_f T = 3/4$ ), 8users, with ICI cancelation and different numbers of receiver antennas

# V. CONCLUSION

In the paper, a multiuser STBC FDMA-CPM system with non-binary encoding has been proposed. Through MMSEbased multiuser detection and low-complexity iterative ICI cancellation, considerable improvements in both BER are achieved with respect to single antenna systems, while the multiuser receiver complexity is kept low. A performance evaluation has been presented to demonstrate the superiority of the proposed multiuser FDM-CPM MIMO system. The study shows that it is possible to increase transmission efficiency by using CPM modulation and concatenation non-binary convolutional encoding with STBC coding for MU transmissions.

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