Evaluation of Convolution Algorithms in the Erlang Multirate Loss Model under the Bandwidth Reservation Policy

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***Abstract*–In this paper, we study a single link that accommodates Poisson arriving calls of different service-classes with different bandwidth requirements. The link is modelled as a loss system while the call admission mechanism is based on the Bandwidth Reservation (BR) policy. We name this model Erlang Multirate Loss Model (EMLM) under the BR policy (EMLM/BR). The BR policy achieves Call Blocking Probabilities (CBP) equalization among calls of different service-classes by reserving bandwidth units in favor of calls of certain service-classes. The existence of the BR policy in the EMLM/BR destroys reversibility in the corresponding Markov chains and therefore the steady state probabilities cannot be described by a product form solution. To determine CBP, we study and evaluate two convolution-based teletraffic loss models that exist in the literature. Our study shows that both algorithms provide CBP results of decent accuracy when compared to the exact CBP results.**

I. Introduction

Teletraffic loss models based either on recursive formulas or on convolution algorithms provide an efficient way for the call-level Quality of Service (QoS) assessment in contemporary communication networks which accommodate multirate traffic. By the term "multirate traffic" we refer to calls of different service-classes with different bandwidth-per-call requirements. The simplest multirate loss model used to analyze a single link that accommodates Poisson traffic is the classical Erlang Multirate Loss Model (EMLM) [1], [2].

In the EMLM, a link of capacity C bandwidth units (b.u.) accommodates K different service-classes. Service-class k calls (k=1, …, K) arrive in the link according to a Poisson process with arrival rate λ_k and require b_k b.u. Calls compete for the available link b.u. under the Complete Sharing (CS) policy. According to the CS policy, new calls are accepted in the link if their required b.u. are available at the time of their arrival; otherwise they are blocked and lost without further affecting the link. Accepted calls remain in the link for an arbitrarily distributed service time [1]. The steady-state probabilities in the EMLM have a Product Form Solution (PFS). The latter leads to an accurate Call Blocking Probabilities (CBP) calculation via the classical Kaufman-Roberts (K-R) recursive formula [1], [2] which has led to various extensions of the EMLM (e.g., [3]-[26]). The K-R formula is used to calculate recursively the link occupancy distribution. A different and more complicated approach for the CBP determination in the EMLM is based on the convolution algorithm [27]. The latter exploits the PFS of the EMLM and the principle of independency among service-classes and therefore the link occupancy distribution can be determined by successively convolving the link occupancy distributions obtained for each service-class. Contrary to the macro-state K-R formula, the convolution algorithm keeps the micro-state information of the number of in-service calls in the link. Such information is necessary when studying more complicated (than the CS policy) call admission policies (e.g., [24], [27] - [32]).

In this paper, we consider the EMLM under the Bandwidth Reservation (BR) policy (EMLM/BR). The BR policy is used to reserve b.u. to benefit calls of high bandwidth requirements and is mainly applied in a link when CBP equalization is required among calls of different service-classes. The existence of the BR policy in the EMLM/BR destroys reversibility in the corresponding Markov chains and therefore the steady state probabilities cannot be described by a PFS. This means that the CBP determination in the EMLM/BR based on convolution algorithms can become a quite complex procedure. We study and evaluate two main convolution algorithms that exist in the literature, named the Asymmetric Convolution Algorithm (ACA) [33], [34] and the Permutational Convolutional Algorithm (PCA) [35], proposed for the CBP calculation in the EMLM/BR. The PCA requires more convolution operations and has a higher time complexity compared to the ACA (see e.g., Table 6, pg. 85 in [35]) but according to [35] it has a higher accuracy (in terms of CBP results), especially for many service-classes.

This paper is organized as follows: In Sections II and III, we review and provide insight to the ACA of [33], [34] and the PCA of [35], respectively. In Section IV, we provide analytical CBP results for both algorithms and compare them with the corresponding exact results. We conclude in Section V.

II. The Asymmetric Convolution Algorithm

We consider a link of capacity C b.u. that accommodates K different service-classes. Service-class k calls (k=1, …, K) arrive in the link according to a Poisson process with arrival rate λ_k, require b_k b.u and have a BR parameter t_k. The latter refers to the b.u. reserved to benefit calls of all other service-classes apart from service-class k. We denote by j the occupied link b.u., j = 0, …, C. Then, a new service-class k call is accepted in the link if $j \le C - b_k - t_k$ at the time of its arrival. Otherwise, the call is blocked and lost. An accepted call remains in the link for a generally distributed service time with mean μ_k^{-1}.

To describe the ACA, let $a_k = \lambda_k / \mu_k$ be the offered traffic-load of service-class k and $q_k(j)$ (k=1, …, K) the link occupancy distribution assuming that only service-class k exists in the link. Then, the ACA is described according to the following steps:

Step 1: Determine $q_k(j)$ of each service-class k via:

$$q_k(j) = q_k(0)\frac{a_k^i}{i!}, \text{ for } 1 \le i \le \left\lfloor \frac{C - t_k}{b_k} \right\rfloor \text{ and } j = i \times b_k \quad (1)$$

Step 2: Let A and B be two subsets of all service-classes in the link. We have $A, B \in \{1, ..., K\}$ and $A \ne B$. Let also $Q_{(A)}$ and $Q_{(B)}$ be the normalized aggregated occupancy distribution of the service-classes that belong to A, B, respectively. Assuming that $b_1 \le b_2 \le ... \le b_K$, then the 1st time we execute the convolution operation, $Q_{(A,B)}$ (defined in (2)), the initial values of the subsets A, B are the following: A = {1}, B = {2}. The 2nd time, the new subsets are: A = {1, 2} and B = {3}. Finally, the (K - 1)th time the new subsets are: A={1, 2, 3, …, K-1} and B = {K}. The values of $Q_{(A,B)}$ are determined by:

$$Q_{(A,B)} = f_{(A,B)}^{(B)}\left(Q_{(A)} * Q_{(B)}\right) + f_{(A,B)}^{(A)}\left(Q_{(B)} * Q_{(A)}\right) \quad (2)$$

where $f_{(A,B)}^{(x)}$ is a weight factor which determines the proportion of $Q_{(r)} * Q_{(x)}$, $r \ne x$, that contributes to $Q_{(A,B)}$ and is calculated by:

$$f_{(A,B)}^{(x)} = \frac{\sum_{k \in \{x\}} a_k b_k}{\sum_{k \in \{A\}} a_k b_k + \sum_{k \in \{B\}} a_k b_k}, \; x = A, B \quad (3)$$

The rationale behind (2) is the following: The term $Q_{(A)} * Q_{(B)}$ refers to the case that the last accepted call is from a service-class that belongs to subset B. The opposite refers to the case of $Q_{(B)} * Q_{(A)}$. Such an approach is intuitively based on the fact that in the EMLM/BR, due to the selection of the BR parameters, a micro-state of the form $\boldsymbol{n} = (n_1, …, n_k, …, n_K)$, where n_k is the number of in-service calls of service-class k in state $\boldsymbol{n}$, may be reached from a new call of a certain service-class but not from a new call of another service-class.

The convolution operation $Q_{(A,B)} = Q_{(A)} * Q_{(B)}$ of (2) is defined as:

$$\begin{aligned} Q_{(A,B)} = Q_{(A)} * Q_{(B)} = \Big\{ & Q_{(A,B)}(j=0) = Q_{(A)}(0)Q_{(B)}(0), \\ & Q_{(A,B)}(j=1) = \sum_{x=0}^{1} Q_{(A)}(x)Q_{(B)}(1-x), ..., \\ & Q_{(A,B)}(j=tr+1) = \sum_{x=0}^{tr+1} \gamma(x, tr+1-x) Q_{(A)}(x) Q_{(B)}(tr+1-x), ..., \\ & Q_{(A,B)}(j=C) = \sum_{x=0}^{C} \gamma(x, C-x) Q_{(A)}(x) Q_{(B)}(C-x) \Big\} \end{aligned} \quad (4)$$

where: $tr = tr_1 = … = tr_k = ... = tr_K = C - b_{\max} = C - b_K$. The parameter tr_k denotes the last state of the link where it is still possible to accept a service-class k call. As an example, consider the case where K = 3, b_1=1, b_2=2, b_3=3 and C = 7 b.u. The assumption that $tr = tr_1 = tr_2 = tr_3 = C - b_3 = 4$, is equivalent to the assumption that CBP equalization is achieved by selecting the BR parameters t_1, t_2 and t_3 according to the rule: $b_1 + t_1 = b_2 + t_2 = b_3 = 3$.

The parameter $\gamma(x_A, x_B)$ in (4), shows if the individual operations $Q_{(A)}(x_A)Q_{(B)}(x_B)$ are permitted from the BR policy point of view. Depending on the definition of $\gamma(x_A, x_B)$, we have two variations of the ACA, the MaxR and the MinR, determined by (5) and (6), respectively [34], [33]:

$$\gamma(x_A, x_B) = \begin{cases} 0 \; for\left(x_A > tr + \max(b_A)\right) \cup \left(x_A + x_B > tr + \max(b_B)\right) \\ 1 \; otherwise \end{cases} \quad (5)$$

$$\gamma(x_A, x_B) = \begin{cases} 0 \; for\left(x_A > tr + \min(b_A)\right) \cup \left(x_A + x_B > tr + \min(b_B)\right) \\ 1 \; otherwise \end{cases} \quad (6)$$

By adopting MaxR/MinR we choose as the last accepted call the one with the maximum/minimum bandwidth requirement in the subset, respectively, when the convolution process enters the reservation space (i.e., states tr + 1, tr + 2, …, C).

Step 2 is executed K - 1 times and based on (2) we have a total number of 2(K - 1) convolutions in this step. The output of step 2 is the aggregated occupancy distribution of all service-classes, $Q_{(1,2,...,K)}$, defined as:

$$Q_{(1,2,...,K)} = \text{ACA}(\text{ACA}(...(\text{ACA}(\{1\},\{2\}),\{3\}),...,\{K\text{-}1\}),\{K\}) \quad (7)$$

Step 3: Determine the CBP of service-class k, B_k, via the formula:

$$B_k = \sum_{j=C-b_k-t_k+1}^{C} Q_{(1,2,...,K)}(j) = \sum_{j=tr+1}^{C} Q_{(1,2,...,K)}(j) \quad (8)$$

A drawback of the ACA is that it does not consider all possible K! sequences in the aggregation process of step 2, but only 2^{K-1} sequences. As an example, consider a link that accommodates K = 3 service-classes. Then, according to step 2, the ACA starts the aggregation process by letting A = {1} and B = {2}. Thus:

$$Q_{(1,2)} = f_{(1,2)}^{(2)}\left(Q_{(1)} * Q_{(2)}\right) + f_{(1,2)}^{(1)}\left(Q_{(2)} * Q_{(1)}\right) \quad (9)$$

where:

$$f_{(1,2)}^{(1)} = \frac{a_1 b_1}{a_1 b_1 + a_2 b_2}, \; f_{(1,2)}^{(2)} = \frac{a_2 b_2}{a_1 b_1 + a_2 b_2} \quad (10)$$

The aggregation process continues by letting A = {1, 2} and B = {3}. Thus:

$$Q_{(1,2,3)} = f_{(1,2,3)}^{(3)}\left(Q_{(1,2)} * Q_{(3)}\right) + f_{(1,2,3)}^{(1,2)}\left(Q_{(3)} * Q_{(1,2)}\right) \quad (11)$$

where:

$$f_{(1,2,3)}^{(3)} = \frac{a_3 b_3}{a_1 b_1 + a_2 b_2 + a_3 b_3}, f_{(1,2,3)}^{(1,2)} = \frac{a_1 b_1 + a_2 b_2}{a_1 b_1 + a_2 b_2 + a_3 b_3} \quad (12)$$

Based on (9), (11) takes the form:

$$Q_{(1,2,3)} = f^{(3)}_{(1,2,3)}\left(f^{(2)}_{(1,2)}\left(Q_{(1)} * Q_{(2)}\right) * Q_{(3)} + f^{(1)}_{(1,2)}\left(Q_{(2)} * Q_{(1)}\right) * Q_{(3)}\right) + f^{(1,2)}_{(1,2,3)}\left(f^{(2)}_{(1,2)} Q_{(3)} * \left(Q_{(1)} * Q_{(2)}\right) + f^{(1)}_{(1,2)} Q_{(3)} * \left(Q_{(2)} * Q_{(1)}\right)\right) \quad (13)$$

According to (13), it is obvious that the ACA considers the sequences {1, 2, 3}, {2, 1, 3}, {3, 1, 2} and {3, 2, 1} and omits two sequences, namely {1, 3, 2} and {2, 3, 1}. The sequences {1, 2, 3} and {2, 1, 3} refer to the case where the last accepted call is from the service-class 3. On the other hand, the sequences {3, 1, 2} and {3, 2, 1} refer to the case where the last accepted call is either from service-class 1 or from service-class 2.

The fact that $K! - 2^{K-1}$ sequences are not taken into account in the ACA, may lead to CBP results that are not quite close to the exact (or simulation) CBP results especially for many service-classes ($K > 3$) [35]. This drawback has been tackled in the PCA, at the cost of higher complexity, where all $K!$ forms of aggregations have been considered [35].

III. The Permutational Convolution Algorithm

Similar to Section II, we consider the EMLM/BR with same characteristics as in the case of the ACA.

Then, the PCA can be described according to the following three steps:

Step 1: Determine $q_k(j)$ of each service-class k via (1).

Step 2: To represent the $K!$ different sequences, let $\boldsymbol{O}$ be a $K!$ x K matrix defined as:

$$\boldsymbol{O} = \begin{bmatrix} O_{1,1} & O_{1,2} & \cdots & O_{1,K} \\ \vdots & \vdots & & \vdots \\ O_{K!,1} & O_{K!,2} & \cdots & O_{K!,K} \end{bmatrix} \quad (14)$$

Each row s of $\boldsymbol{O}$, denoted as $\vec{O}_s$, represents a different sequence and is expressed as:

$$\vec{O}_s = \{O_{s,1}, O_{s,2}, \ldots, O_{s,K}\}, \text{ for } 1 \le s \le K! \quad (15)$$

where: $O_{s,x} \in \{1, 2, ..., K\}$, $x = 1, \ldots, K$ and $O_{s,x} \neq O_{s,x'}$ for $x \neq x'$.

As an example, in the case of $K = 3$ service-classes, (14) can be written as:

$$\boldsymbol{O} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix} \quad (16)$$

Step 2a: For each row of the $\boldsymbol{O}$ matrix we have to calculate the occupancy distribution according to the 2^{K-2} possible aggregation patterns. These patterns are dictated by the possible sequence of the arriving calls of each service class. Let m, $1 \le m \le 2^{K-2}$, be the m^{th} aggregation of sequence s. Define now by $\vec{Q}^{\vec{O}_{s,m}}$ a row vector of C+1 elements with $\vec{Q}^{\vec{O}_{s,m}} = \left\{\vec{Q}^{\vec{O}_{s,m}}(j)\right\}, j = 0, 1, ..., C$. Each element of $\vec{Q}^{\vec{O}_{s,m}}$ expresses the steady state probability of the link occupancy state j which is determined under the m^{th} aggregation pattern of sequence s. E.g., assuming a system of $K = 3$ service-classes and based on (16), we have two different aggregation patterns for each row of $\boldsymbol{O}$. Considering the first row of $\boldsymbol{O}$ in (16), we may first convolve service-classes 1, 2 and then convolve (1, 2) with service-class 3 (see (17) below, for m=1) or we may first choose to convolve 2 and 3 and then convolve (2, 3) with service-class 1 (see (18) below, for m=2):

$$\vec{Q}^{\vec{O}_{1,1}} \overset{(1*2)*3}{=} \left\{\vec{Q}^{\vec{O}_{1,1}}(0), \vec{Q}^{\vec{O}_{1,1}}(1), ..., \vec{Q}^{\vec{O}_{1,1}}(C)\right\}, \; s = 1, \; m = 1 \quad (17)$$

$$\vec{Q}^{\vec{O}_{1,2}} \overset{1*(2*3)}{=} \left\{\vec{Q}^{\vec{O}_{1,2}}(0), \vec{Q}^{\vec{O}_{1,2}}(1), ..., \vec{Q}^{\vec{O}_{1,2}}(C)\right\}, \; s = 1, \; m = 2 \quad (18)$$

As an example, to obtain $\vec{Q}^{\vec{O}_{1,1}}(0)$ of (17) we multiply the normalized value of $q_{1,2}(0)$ (obtained after the convolution of service-classes 1, 2) with the normalized value of $q_3(0)$ (obtained in step 1 of the PCA), i.e., $\vec{Q}^{\vec{O}_{1,1}}(0) = q_{1,2}(0) \times q_3(0)$. Similarly, to obtain $\vec{Q}^{\vec{O}_{1,2}}(0)$ of (18) we multiply the normalized value of $q_{2,3}(0)$ (obtained after the convolution of service-classes 2, 3) with the normalized value of $q_1(0)$ (obtained in step 1 of the PCA), i.e., $\vec{Q}^{\vec{O}_{1,2}}(0) = q_{2,3}(0) \times q_1(0)$.

Considering the second row of $\boldsymbol{O}$ in (16), we have:

$$\vec{Q}^{\vec{O}_{2,1}} \overset{(2*1)*3}{=} \left\{\vec{Q}^{\vec{O}_{2,1}}(0), \vec{Q}^{\vec{O}_{2,1}}(1), ..., \vec{Q}^{\vec{O}_{2,1}}(C)\right\}, \; s = 2, \; m = 1 \quad (19)$$

$$\vec{Q}^{\vec{O}_{2,2}} \overset{2*(1*3)}{=} \left\{\vec{Q}^{\vec{O}_{2,2}}(0), \vec{Q}^{\vec{O}_{2,2}}(1), ..., \vec{Q}^{\vec{O}_{2,2}}(C)\right\}, \; s = 2, \; m = 2 \quad (20)$$

A similar procedure is required for the remaining four rows of $\boldsymbol{O}$ in (16).

Step 2b: At this point, let $\boldsymbol{V}$ be a $K!$ x (C+1) matrix defined as:

$$\boldsymbol{V} = \begin{bmatrix} V_1(0) & V_1(1) & \cdots & V_1(C) \\ \vdots & \vdots & & \vdots \\ V_{K!}(0) & V_{K!}(1) & \cdots & V_{K!}(C) \end{bmatrix} \quad (21)$$

Each row s of $\boldsymbol{V}$, denoted as $\vec{V}_s$, expresses the link occupancy distribution obtained according to the sequence of the K service-classes determined by $\vec{O}_s$ [35]:

$$\vec{V}_s = \sum_{m=1}^{2^{K-2}} \alpha^{\vec{O}_{s,m}} \times \vec{Q}^{\vec{O}_{s,m}} \quad (22)$$

where $\alpha^{\vec{O}_{s,m}}$ is a weight factor that expresses the contribution of the normalized values of $\vec{Q}^{\vec{O}_{s,m}}$ in the value of $\vec{V}_s$.

Each weight factor $\alpha^{\vec{O}_{s,m}}$ is determined as the proportion of the product $a_k b_k$ of the service-class k involved in the left-hand side of an aggregation operation divided by the total number of service-classes involved in that aggregation operation.

As an example, consider again the first row of $\boldsymbol{O}$ in (16) and the values of $\vec{Q}^{\vec{O}_{1,1}}$ and $\vec{Q}^{\vec{O}_{1,2}}$ obtained by (17) and (18), respectively. Then, according to (22), $\vec{V}_1$ can be calculated by:

$$\vec{V}_1 = \alpha^{\vec{O}_{1,1}} \times \vec{Q}^{\vec{O}_{1,1}} + \alpha^{\vec{O}_{1,2}} \times \vec{Q}^{\vec{O}_{1,2}} = \\ = \frac{a_1b_1}{a_1b_1 + a_2b_2} \times \frac{a_1b_1 + a_2b_2}{a_1b_1 + a_2b_2 + a_3b_3} \times \vec{Q}^{\vec{O}_{1,1}} \\ + \frac{a_2b_2}{a_2b_2 + a_3b_3} \times \frac{a_1b_1}{a_1b_1 + a_2b_2 + a_3b_3} \times \vec{Q}^{\vec{O}_{1,2}} \quad (23)$$

Similarly, consider the second row of $\boldsymbol{O}$ in (16) and the values of $\vec{Q}^{\vec{O}_{2,1}}$ and $\vec{Q}^{\vec{O}_{2,2}}$ obtained by (19) and (20), respectively. Then, according to (22), $\vec{V}_2$ is given by:

$$\vec{V}_2 = \alpha^{\vec{O}_{2,1}} \times \vec{Q}^{\vec{O}_{2,1}} + \alpha^{\vec{O}_{2,2}} \times \vec{Q}^{\vec{O}_{2,2}} = \\ = \frac{a_2b_2}{a_1b_1 + a_2b_2} \times \frac{a_1b_1 + a_2b_2}{a_1b_1 + a_2b_2 + a_3b_3} \times \vec{Q}^{\vec{O}_{2,1}} \\ + \frac{a_1b_1}{a_1b_1 + a_3b_3} \times \frac{a_2b_2}{a_1b_1 + a_2b_2 + a_3b_3} \times \vec{Q}^{\vec{O}_{2,2}} \quad (24)$$

A similar procedure is required for the remaining four rows of $\boldsymbol{V}$ in (21).

Step 3: In order to obtain the CBP of each service-class k via (8), we sum the corresponding unnormalized elements of $\vec{V}_s$ for each value of s and j = 0,1, …, C and then normalize the retrieved results:

$$Q(j) = \sum_{s=1}^{K!} V_s(j), \text{ for } j = 0,1, \ \ldots, \ C \quad (25)$$

$$Q_{\{1,2,\ldots K\}}(j) = Q(j) \Big/ \sum_{j=0}^{C} Q(j) \quad (26)$$

IV. Numerical Examples - Evaluation

In this section, we present an application example and provide analytical CBP results of the ACA minR [33], the ACA maxR [34] and the PCA of [35]. These approximate CBP results (which are due to the non-reversible Markov chains that exist in the EMLM/BR) are compared with the corresponding exact CBP results.

As an application example, we consider a link of C = 30 b.u. that accommodates calls of K = 3 service-classes, with the following bandwidth-per-call requirements: b_1 = 1, b_2 = 2 and b_3 = 4 b.u. To achieve CBP equalization, let tr = 4 b.u. In addition, let $a_1b_1 : a_2b_2 : a_3b_3 = 1:1:1$. This example, has already been presented in [35] (see Table 3 of [35]) but we decided to present it herein since: 1) the PCA CBP results we obtained are not similar to those presented in [35] and, 2) the ACA minR and maxR CBP results presented in [35] are different to those obtained by the algorithms of [33] and [34], respectively. The main difference relies on the fact that compared to (2), the calculation of $Q_{(A,B)}$ is given by the formula $Q_{(A,B)} = f_{(A,B)}^{(A)} \left(Q_{(A)} * Q_{(B)}\right) + f_{(A,B)}^{(B)} \left(Q_{(B)} * Q_{(A)}\right)$, i.e., each term is multiplied by the opposite values of the weight factor f compared to (2).

In Table I, we present for various values of $a = \sum_{k=1}^{K} a_k b_k / C$, the exact CBP results (2nd column), the PCA CBP results of [35] (3rd column), our PCA CBP results (4th column), the ACA minR CBP results of [35] (5th column), the ACA minR CBP results according to [33] which are verified in this paper (6th column), the ACA maxR CBP results of [35] (7th column) and the ACA maxR CBP results according to [34] which are verified in this paper (8th column).

Based on the results of Table I, we conclude that:

a) The difference between our implementation of the PCA and that of [35], is minor for low values of α, while our implementation achieves better accuracy compared to the exact values for low CBP values (<5%). The reason behind the difference between the two implementations is not yet clear. Based on our experience on these algorithms (both the ACA and the PCA), we conclude that they are sensitive (in terms of the obtained CBP results) when normalizations of the various distributions are included or omitted in each step. In addition, both algorithms are sensitive when different weight factors are considered.

b) The ACA maxR algorithm of [34] provides much better results than the ACA maxR algorithm as presented in [35]. The opposite behavior is observed in the case of the ACA minR, i.e., a better CBP approximation is achieved by the ACA minR of [35] compared to that of [33].

c) Our implementation of the PCA does not reveal a superiority of the PCA compared to the ACA maxR of [34], especially for K = 3 service-classes and medium to heavy traffic load conditions (CBP > 10%).

As a general conclusion, our study shows that for low CBP values (<5%) both the ACA maxR of [34] and the ACA minR as presented in [35] as well as the PCA behave quite satisfactory. In addition, both types of algorithms (ACA and PCA) keep the micro-state information of the number of in-service calls in the link which is quite important when more complicated (than the BR policy) call admission policies should be studied. On the other hand, in the EMLM/BR it is still questionable (at least to the authors) whether the complex convolution algorithms should be preferred instead of the efficient yet recursive Roberts' formula [36].

V. Conclusion

In this paper, we study and evaluate two convolution algorithms, the ACA and the PCA, that exist in the literature for the CBP calculation in the EMLM/BR. Our study shows that both algorithms provide quite satisfactory CBP results compared to the exact values for light traffic load conditions. As a future work, we intend to study convolution algorithms under the case of quasi-random traffic, i.e., traffic generated by a finite number of users and a combination of the BR policy with the threshold policy. The latter is used to block a new call of a service-class when the number of in-service calls (of that service-class) plus the new call extends a threshold.

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Table I: CBP results for the application example.

a	**Exact solutions**	**PCA [35]**	**PCA (this paper)**	**ACA MinR [35]**	**ACA MinR [33]**	**ACA MaxR [35]**	**ACA MaxR [34]**
0.2	5.000e−5	4.500e−5	4.612e−5	4.600e−5	3.634e−5	5.100e−5	4.645e−5
0.3	9.470e−4	8.540e−4	8.890e−4	8.900e−4	6.827e−4	9.920e−4	8.864e−4
0.4	6.076e−3	5.611e−3	5.883e−3	5.920e−3	4.429e−3	6.616e−3	5.802e−3
0.5	2.109e−2	2.002e−2	2.110e−2	2.136e−2	1.568e−2	2.385e−2	2.062e−2
0.6	4.962e−2	4.834e−2	5.115e−2	5.195e−2	3.789e−2	5.799e−2	4.968e−2
0.7	9.029e−2	9.004e−2	9.550e−2	9.713e−2	7.114e−2	1.084e−1	9.258e−2
0.8	1.385e−1	1.408e−1	1.494e−1	1.519e−1	1.128e−1	1.694e−1	1.451e−1
0.9	1.895e−1	1.954e−1	2.073e−1	2.105e−1	1.593e−1	2.346e−1	2.022e−1
1.0	2.397e−1	2.497e−1	2.647e−1	2.681e−1	2.076e−1	2.988e−1	2.597e−1
1.1	2.872e−1	3.014e−1	3.190e−1	3.222e−1	2.555e−1	3.589e−1	3.149e−1
1.2	3.312e−1	3.492e−1	3.689e−1	3.715e−1	3.016e−1	4.136e−1	3.666e−1
1.3	3.714e−1	3.927e−1	4.141e−1	4.156e−1	3.451e−1	4.624e−1	4.139e−1
1.4	4.079e−1	4.320e−1	4.546e−1	4.548e−1	3.856e−1	5.057e−1	4.570e−1
1.5	4.409e−1	4.674e−1	4.909e−1	4.896e−1	4.230e−1	5.438e−1	4.958e−1