Physical Concept in the Derivation of Boundary Conditions for $\frac{\partial \mathbf{B}}{\partial t}$ and $\frac{\partial \mathbf{D}}{\partial t}$ on a Perfect Conductor

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Abstract

In this paper, typical mathematical derivation processes of the boundary conditions presented in most undergraduate electromagnetic textbooks for time varying fields of **D** and **B** in Faraday's law and Ampere's law on a perfect conductor are physically reviewed. We notice that the derivation processes do not give physical meanings to learners. Therefore, this paper suggests that different derivation processes providing some physical insights or explanations are required.

Keywords : Boundary Condition   Education   Time Varying Fields   Physical Concept

1. Introduction

Various technological advances in the fields of RF/Microwave communication, radar, high speed electronics/computing, optics, robots, biomedicine, household appliances, and EMI/C (electromagnetic interference/compatibility) have continuously increased the necessity for a deep understanding of basic physical concepts in the field of electromagnetics (EM) [1],[2]. However, the introductory EM course is considered one of the most difficult courses to many electrical engineering undergraduates, because the subject is usually taught with intense mathematical manipulation and its treatment is often abstract [3],[4]. On the other hand, when most electrical engineers practice after graduation, they have very little use of intricate mathematics. Most of our practical problems are solved with a little more than algebra and by field simulators. To use simulation tools for some practical problems, physical principles of the EM boundary conditions are strongly recommended rather than simply applying the conditions mechanically. However, for analysis, derivation, and research of new and theoretical EM problems, mathematical abilities are also required. To satisfy the two aspects electromagnetics faces, we have to provide both of mathematical and physical backgrounds to students. However, students who do not like intense mathematical treatment from the outset of the subject easily drop out of the course. Because high quality comes from numbers, we should retain as many students as possible in the introductory EM course. To achieve this goal, some strong motivation should be given to students who "hate electromagnetics." [5].

When a difficult subject like electromagnetics is taught, if too much mathematics is introduced from the beginning, many students cannot grasp the physical meaning related to the subject and become haters of the subject. Usually, complete concepts are developed from qualitative Gestalts to quantitative examples and electromagnetic laws. Thus, when electromagnetics is first introduced, a good teaching method may be to present a mathematical derivation or treatment of a subject after the introduction of a physical concept of the subject. In this case, even though a student has some difficulty following a mathematical treatment of the EM subject, if he/she is interested in the physical concept, then, more effort can be devoted to understand the mathematical treatment of the subject. Then, the student can be an active and heuristic learner.

Maxwell's equations form the basic foundation of electromagnetics. The equations in differential form govern the interrelationships between the spatial field vectors and the associated source densities at points in a given medium. In practice we encounter EM problems with two or more different media. Although a complete and unique solution of an EM problem should include information about both the differential form of Maxwell's equations and boundary conditions, many introductory EM textbooks provide a good physical explanation only for the equations without the boundary conditions. Before the conditions are mathematically applied, if we have a physical concept providing insight on how an EM phenomenon occurs between boundaries, the problem can be easily solved while invoking interest.

When undergraduate textbooks introduce the boundary conditions, the conditions are usually mathematically derived from Maxwell's equations in integral forms. Some physical background and meaning on why the two time varying field terms in Faraday's and Ampere's Laws become zero in the derivation of the conditions are not physically explained even in the chapters that explain time varying fields. Especially, the physical reason why the two time varying field terms on the conductor surface between a dielectric and a perfect conductor become "zero" is not explained. Usually, many text books present that the fields become zero only in mathematical terms without any physical explanation. They provide the conditions based on the assumption that adopts the limitation $\Delta S \rightarrow 0$ in the derivation. Some textbooks present the boundary conditions for the static cases without explanation even in the chapters dealing with time varying fields.

In Section 2, the typical mathematical derivation processes for the boundary conditions - the tangential components **E** and **H** on a perfect conductor surface - presented in most undergraduate EM textbooks are introduced. In Section 3, we try to raise awareness of the problem in which the derivation has been carried out in purely mathematical ways to explain why the time varying terms vanish without physical meanings or explanations and to look for problems in the derivation process and physical meanings or concept to explain why the time varying field components become zero on a perfect conductor surface.

2. Typical Textbook Presentation for the Boundary Conditions

In this section, the typical derivation processes of the boundary conditions presented in most undergraduate EM textbooks are introduced to show how the two time varying field terms in the two Maxwell's curl equations are treated for the tangential components on a perfect conductor boundary. The normal components of the boundary conditions will not be covered in this paper.

Fig. 1 shows the typical geometry used for the derivation of the boundary conditions between two different media. The typical derivation process of the boundary conditions considers one integral equation at a time and applies it to a closed path for the tangential components and a closed surface encompassing the boundary for the time varying field terms, as shown in Fig. 1 for a plane boundary. For the time varying term to vanish, most undergraduate textbooks adopt the limit that the area enclosed by the closed path goes to zero and conclude that the time varying terms disappear.

In the derivation of the boundary condition of the tangential **E** fields, the time varying term $\frac{\partial \boldsymbol{B}}{\partial t}$ in Faraday's law is considered as in (1). Referring to Fig. 1, we apply the integral form of Faraday's law to the path in the limit where ad = bc = Δh goes to 0 by making the area abcd = ΔS converge to zero, but with ab = cd =Δℓ remaining on either side of the boundary. Therefore, tangential components of an electric field will be continuous between two different dielectric media. And, if either medium of the two dielectric media is a perfect conductor, then, an E_t will be zero on the perfect conductor without physical explanation. In this derivation the time derivative of the **B** field disappears under the condition of the integration area vanishing.

For the second step, we consider the time varying term $\frac{\partial \boldsymbol{D}}{\partial t}$ in Ampere's law. Typical undergraduate textbooks present a similar derivation process as the tangential electric field case mentioned above. They describe that H_t is continuous between two different dielectrics without physical explanation especially if one medium is a perfect conductor, where the H_t induces surface current on the perfect conductor like (2).

$$\lim_{\substack{ad\to 0\\ bc\to 0}} \oint_{\ell=abcda} \mathbf{E}\cdot d\boldsymbol{\ell} = -\lim_{\substack{ad\to 0\\ bc\to 0}} \int_{\substack{Area\\ abcd}} \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S} \;\rightarrow\; E_{t2}\Delta\ell - E_{t1}\Delta\ell = -\frac{\partial \mathbf{B}}{\partial t}\cdot\Delta\mathbf{S} = \mathbf{0} \;\rightarrow\; E_{t1} = E_{t2} \;\rightarrow\; \mathbf{n}\times(\mathbf{E}_1 - \mathbf{E}_2) = 0 \quad (1)$$

And

$$\lim_{\substack{ad\to 0\\ bc\to 0}} \oint_{\ell=abcda} \mathbf{H}\cdot d\boldsymbol{\ell} = -\lim_{\substack{ad\to 0\\ bc\to 0}} \int_{\substack{Area\\ abcd}} \mathbf{J}\cdot d\mathbf{S} + \lim_{\substack{ad\to 0\\ bc\to 0}} \int_{\substack{Area\\ abcd}} \frac{\partial \mathbf{D}}{\partial t}\cdot d\mathbf{S} \rightarrow \mathrm{H}_{t2}\Delta\ell - H_{t1}\Delta\ell = \mathbf{J}\cdot\Delta\mathbf{S} \rightarrow H_{t1} - H_{t2} = \mathrm{J}_S \rightarrow \mathbf{n}\times(\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_S \quad (2)$$

where the subscripts 1 and 2 represent media 1 and 2 and **n** is the unit normal vector to the surface directed into medium 1. $\mathbf{J}_S$ is the surface charge density (A/m).

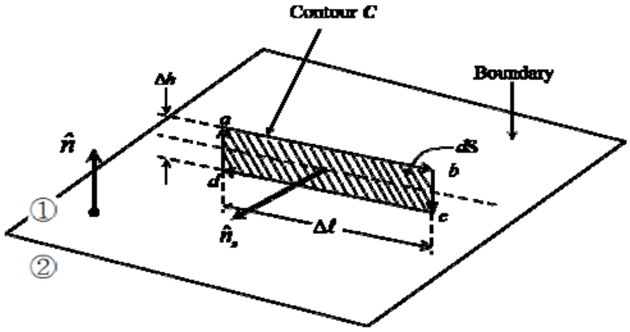

Fig. 1 A typical geometry used in the derivation of the boundary conditions for the tangential components based on Faraday's law and Ampere's circuital law.

3. Problems in the Typical Derivation Processes and Physical Concepts for the Time Varying Fields on a Perfect Conductor Boundary

The typical derivation processes of the boundary conditions on a perfect conductor adopted in most undergraduate textbooks was presented in Section 2. The books do not include any explanations on why the tangential component of **E** vanishes and the tangential component of **H** induces the surface current on a perfect conductor without considering physical natures of the vanishing two time varying terms on a perfect conductor surface.

The books conclude that the time varying fields cannot be exist by assuming that the area $\Delta \boldsymbol{s} \rightarrow 0$ in the limit as $\Delta h \rightarrow 0$. And, since they use the shaded area in Fig. 1 for the surface integral of the time varying field **B,** the **B** should be considered as a tangential component with respect to the boundary as shown in Fig. 2 where the accompanying **E** fields are circulating fields around **B**. Thus, the **E** fields crossing the boundary at point B should be considered as normal components. In this context, even though the typical derivation in most textbooks uses Faraday's law mathematically, the derivation does not follow physical nature. To satisfy Faraday's law physically, the time varying field **B** should be a normal component to the plane boundary in deriving the boundary conditions. If a normal component of a time varying field **B** is assumed for the derivation of the boundary condition on a perfect conductor, the circulating current, like the infinite eddy current by a non zero electromotive force, is induced on the surface by the Faraday's law. Since the eddy current generates a magnetic field following Lenz's law, the inducing field is cancelled completely on the perfect conductor surface. As a result, the eddy current is a cause for the vanishing time varying **B** field on the conductor. The result follows the typical boundary condition for the normal component of **B**. This type of physical insight should be included in the derivation process of the boundary condition.

Next, we consider the typical derivation process of the boundary condition used for the tangential **H** field component on a perfect conductor, and consider the area $\Delta \boldsymbol{s}$ and the closed contour abcda for the surface and line integrals, respectively. The time varying **D** field in Ampere's

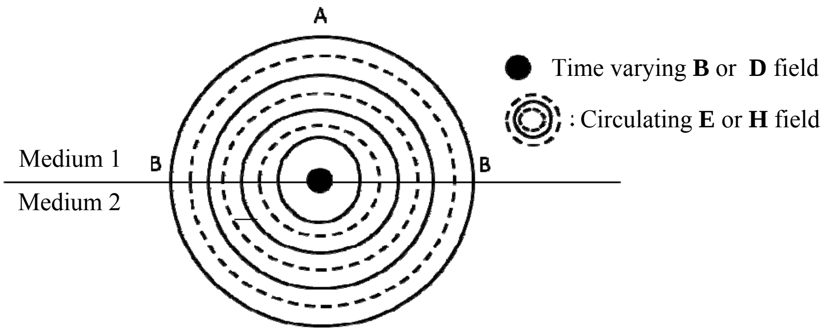

Fig. 2 Conceptual picture showing a circulating **E** field around time varying **B** field.

law is also considered as a tangential component to the boundary in the typical derivation. This is the same situation as in the previous explanation on the boundary condition for the tangential **E** field. The time varying **D** fields induce circulating **H** fields around the **D**. The circulating **H** fields will have normal components at the point B in Fig. 2 when the fields cross the boundary. Therefore, as in the previous case, we have to consider the normal component of time varying **D** with respect to the plane boundary if we want to derive the boundary condition for tangential **H** components. Since the normal component of the time varying **D** is a displacement current as shown in Fig. 3, circulating **H** around **D** is induced on the perfect conductors of the capacitor if we assume that the plates, at point c, of the capacitor is a perfect conductor. The **H** is circulating on the boundary surface which is a tangential component on the boundary. The line integral of **H** generates magnetomotive force (mmf) and current flows as shown in Fig. 3. On a perfect conductor, the time varying **D** induces a time varying charge which is a surface charge due to polarization changes. The time varying charge $\frac{\partial \rho}{\partial t}$ becomes a conduction current **J**=σ**E** along the conductor line.

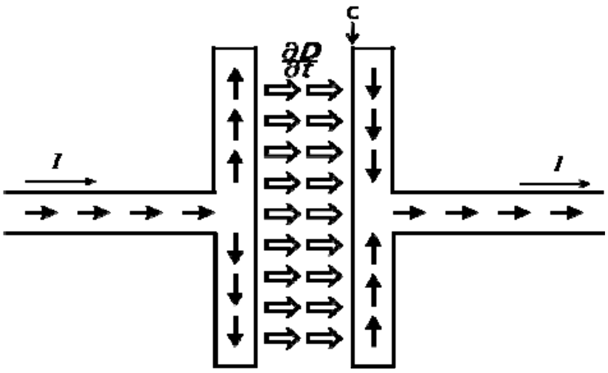

Fig. 3 A part of an AC circuit with a capacitor showing conduction current (black arrows) and time varying **D** (displacement current : empty arrows) in a dielectric of capacitor [6].

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