

# Active non-Foster Metamaterials – State of the Art

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## Abstract

A concept of active ENZ non-Foster metamaterial, recently introduced at University of Zagreb is reviewed. This approach goes around the basic dispersion constraints of a passive material and, therefore, enables ultra-broadband operation. The problem of achieving stable operation and associated technological constraints are discussed. Several state-of-the-art examples of extremely broadband (a bandwidth of more than two octaves), 1D and 2D active RF ENZ metamaterials are presented.

**Keywords :** Active metamaterial, Non-Foster, Negative Capacitor, Negative Inductor

## 1. Introduction

All known passive materials (or metamaterials) that have either negative (ENG, MNG) or less-than-unity (ENZ, MNZ) real parts of permittivity or permeability suffer from narrow operating bandwidth. This issue is associated with the reactive energy stored within a material. Stored energy is non-negative quantity and (at the same time) it must be greater than the energy in vacuum [1]:

$$W = \frac{1}{2} \frac{\partial[\omega \cdot \varepsilon(\omega)]}{\partial \omega} |E|^2 + \frac{1}{2} \frac{\partial[\omega \cdot \mu(\omega)]}{\partial \omega} |H|^2, \quad W > \frac{1}{2} \varepsilon_0 |E|^2 + \frac{1}{2} \mu_0 |H|^2 \Rightarrow \frac{\partial[\varepsilon(\omega)]}{\partial \omega} > 0, \frac{\partial[\omega \cdot \mu(\omega)]}{\partial \omega} > 0. \quad (1)$$

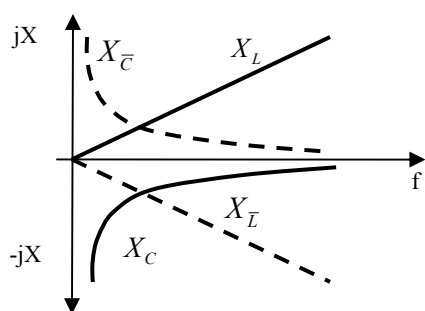
Here,  $\omega$  is the angular frequency while  $\varepsilon$  and  $\mu$  stand for permittivity and permeability, respectively. The most right part of (1) is well-known dispersion-energy constraint that applies for every passive material. In addition, it is an equivalent of the Foster reactance theorem in circuit theory [2] that requires  $(\partial X/\partial \omega) > 0$  and  $(\partial B/\partial \omega) > 0$  ( $X$  and  $Y$  being the reactance and susceptance, respectively). Very recently, it has been shown that, in some cases, it is possible to go around the basic dispersion constraints by the use of non-Foster active elements [4-11]. Here, the basic physics of non-Foster active metamaterials is reviewed and several illustrative examples of practical prototypes developed at University of Zagreb are presented.

## 2. Physics of Non-Foster-based metamaterials and State of the Art

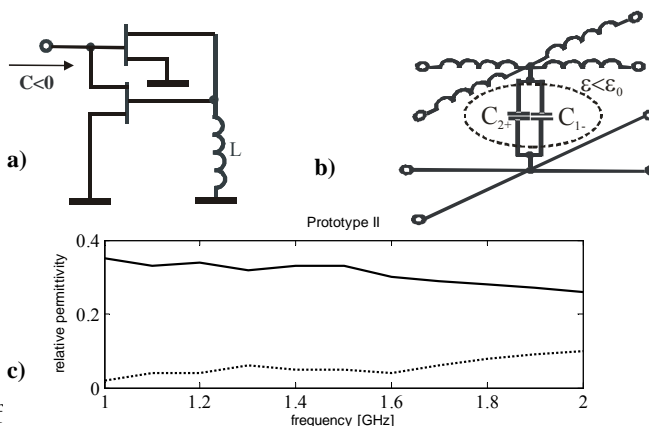
There are electronic circuits that behave as negative capacitors or negative inductors [3]. Negative capacitors and negative inductors have dispersion curves that are the exact inverse of the dispersion curves of ordinary ‘positive’ elements (Fig.1). Therefore, one could expect that the dispersion of ordinary passive metamaterials can be compensated for with the ‘inverse’ dispersion of non-Foster elements, resulting in broadband behavior [4-11]. The ‘non-Foster’ networks are based on Negative Impedance Converter (NIC), originally introduced back in the 1950’s [3]. Although this idea is indeed old, there are only a few papers in the open literature that report successful implementations of negative capacitors or inductors due to serious stability problems [12]. In our recent study [11] it was shown that the stability criteria are strongly dependent on the configuration of a passive network connected to the negative capacitor. Thus, assuring stability of non-Foster circuit is rather difficult task, but it is certainly feasible (for some predetermined topology of load network, as detailed in [11]). The first non-Foster-based active dispersionless ENZ metamaterials was presented in [5] and further improved in [6,7]. The negative capacitors were constructed using FET-based circuits (Fig. 2 a). They were employed as building blocks of 2D unit cells manufactured in microstrip technology (Fig. 2b). The basic idea deals with the parallel combination of a shunt (positive) distributed capacitance ( $C$ ) and a negative capacitor ( $C_N$ ). In this way, the overall capacitance is decreased below the free-space value yielding frequency-independent ENZ behavior:

$$\epsilon_r(\omega) = [C/\epsilon_0 - |C_N|/(\epsilon_0\Delta)] \quad (2)$$

Here  $\Delta$  stands for the unit cell dimension. It is important to try to understand this counter-intuitive behaviour. Essentially, the electronic circuit (that behaves as a negative capacitor) acts as ‘an active load’ that drains an excess of energy stored in the positive capacitor. A sample of measured results (Fig. 2 c) shows fairly constant ENZ behaviour within one octave (1GHz - 2GHz). These measured results were also used as input data for ADS<sup>TM</sup> simulation of proposed active 2D plasmonic cloak [7] and showed operating relative bandwidth of 100% comparing to relative bandwidth of 20% of passive cloak.

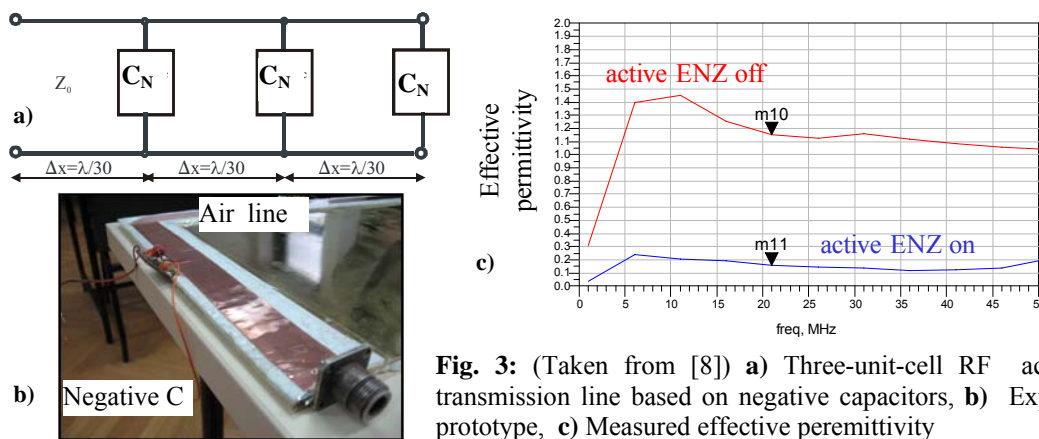


**Fig. 1:** (Taken from [7]), Reactance of positive (solid line) and negative (dashed line) reactive elements



**Fig. 2:** (Taken from [7]), a) Basic negative capacitor circuit, b) 2D active ENZ unit cell, c) Measurement results

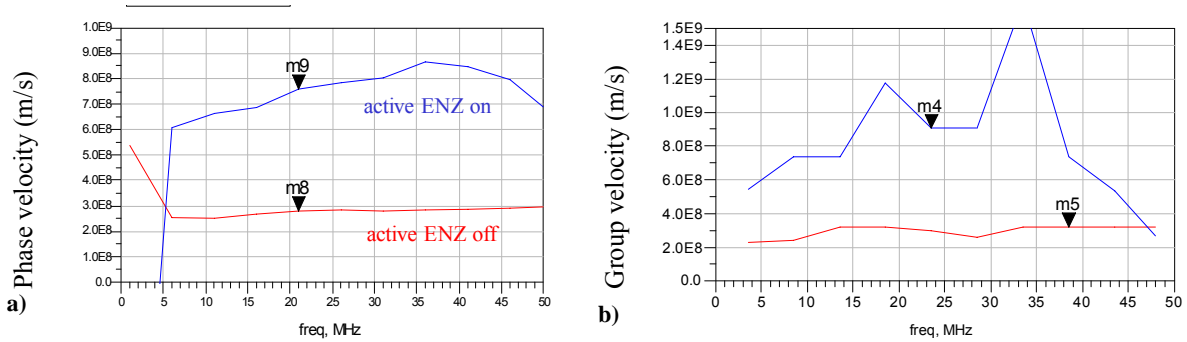
In the subsequent study [8] a complete 1D active RF ENZ metamaterial was analysed, built and tested. It was based on air transmission line loaded with three op-amp-based negative capacitors (Fig. 3 a and b).



**Fig. 3:** (Taken from [8]) a) Three-unit-cell RF active ENZ transmission line based on negative capacitors, b) Experimental prototype, c) Measured effective permittivity

Measured real part of effective permittivity (Fig. 3c) was rather constant (between 0.1 and 0.2), in the frequency range 10 MHz to 40 MHz (within two octaves). This bandwidth is significantly wider than a bandwidth of any passive ENZ metamaterial. On top of this, it was found that this type of line supports superluminal phase and group velocity (Fig. 4) [13]. Although these results look counter-intuitive, they are in a perfect agreement with causality. The transmission line in Fig. 3. is an active system, the group velocity of which is not equal to the energy velocity. Causality is preserved due to the fact that any realistic negative capacitor must be band-limited and the frequency components within this band travel with superluminal velocity. However, there is always a fraction of energy (whatever small it can be) that lies outside the operating band and it travels with the speed of light. These luminal components cause (whatever small) distortion of the signal and preserve causality. Thus, non-Foster transmission line is, in a sense, similar to active negative delay

circuit based on anomalous dispersion [14]. However, the operational principle is fundamentally different, which enables broadband ENZ behavior.



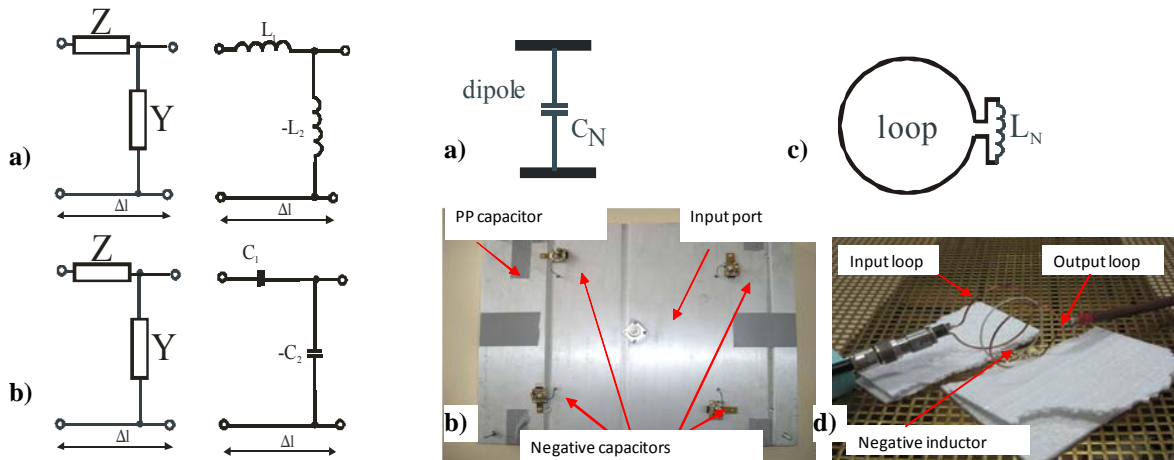
**Fig. 4:** (Taken from [13]) Measured superluminal behaviour of prototyped RF active ENZ transmission line, **a)** phase velocity, **b)** group velocity

### 3. New Research Directions

One of the issues that are currently being investigated in our group is a concept is frequency independent active transmission line (Fig. 5). These lines contain either positive and negative inductors or positive and negative capacitors, which yields counter-intuitive phase factors:

$$\beta_{FW} = \sqrt{L_1/L_2}, \quad \beta_{BW} = \sqrt{C_2/C_1}. \quad (3)$$

These two phase factors do not depend on frequency at all and they represent forward-wave mode and a backward-wave mode, respectively. This phenomenon may be used for construction of very broadband phase shifters. A negative inductor needed for the line in Fig.5 a) has been developed recently using modified FET-based circuit from Fig 3.a (loading inductor is replaced by loading capacitor). Experimental investigation of the whole line from Fig. 5a is under way and the results will be reported at the conference. Finally, we are attempting to extend explained approach to volumetric 2 D metamaterials. In order to achieve the ENZ behavior, one might use an array of the short dipoles loaded with negative capacitors (Fig. 6a). We developed an 2 x2 array of dipoles in RF frequency range (up to 50 MHz) , which was mounted within a parallel-plate capacitor (Fig. 6b). Measurement of the reflection coefficient revealed decrease of relative effective permittivity below the free-space value ( $\epsilon_r=0,3$ ), which is a direct proof of proposed concept. Similarly, one might think of an array of small loops (magnetic dipoles) loaded with the negative inductors (Fig. 6c).



**Fig. 5:** New concept of active frequency independent TL lines, **a)** active forward-wave transmission line, **b)** active backward-wave transmission line

**Fig. 6:** New concept of volumetric active metamaterials **a)** short dipole loaded with negative capacitor (ENZ) **b)** 2D experimental prototype with four dipoles, **c)** small loop loaded with negative inductor (MNZ unit cell) **d)** Measurement of the MNZ unit cell

So far we developed one unit cell based on this principle, which is actually an active replica of a SRR. An additional positive capacitor was connected in a parallel with negative inductor in order to achieve the MNZ behavior. The 'active SRR' was put between two loops that were connected to the ports 1 and 2 of a network analyzer and the transmission coefficient ( $S_{21}$ ) was measured (Fig 6 d). Preliminary results revealed that the achieved effective permeability is strongly dependent on the coupling with excitation loops. Nevertheless, the basic principle of a broadband decrease of the permeability was proven. Proposed volumetric active ENZ and MNZ metamaterials might find application in ultra-broadband active antennas.

## 4. Conclusions

A concept of (almost) dispersionless active non-Foster metamaterial was reviewed and a couple of representative examples were given. It was shown that this novel concept enables construction of ultra-broad-band RF devices, which might find application in antennas, communications and cloaking technology.

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