

# Downlink Multichannel Sharing and Joint Detection for Cell-Edge Users

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**Abstract**—Since data rates for cell-edge users cannot be improved by merely increasing transmission power due to inter-cell interference (ICI), the interference management and alignment become crucial for cell-edge users. In this paper, we consider a simple approach where each base station (BS) in coordination transmits signals to a dedicated user through multiple subcarriers shared by adjacent BSs so that a higher diversity can be exploited for a better performance through joint detection.

**Keywords:** inter-cell interference, joint detection, coordination

## I. INTRODUCTION

In cellular systems, the overall performance is strongly dependent on inter-cell interference (ICI) and performance improvement can be achieved by dealing with ICI properly. While ICI can be managed by a higher layer's controller (e.g., through radio resource management) [1] [2], there are also various techniques that can effectively mitigate ICI at physical layer [3]. In long-term evolution (LTE) - advanced standards, coordinated multi-point (CoMP) transmission and reception techniques are considered to improve the data rate for cell-edge users who suffer from ICI. In [4], implementation issues for CoMP downlink transmission are studied. As noted in [4], a number of CoMP techniques are inspired by the notion of network multiple input multiple output (MIMO) [5], which is a generalization of MIMO [6], where multiple base stations (BSs) form an antenna array for multiple inputs. In this case, beamforming or precoding approaches can be applied [7] [8]. However, this kind of approaches requires *i*) channel state information (CSI) at transmitter; *ii*) unlimited backhaul transmission between BSs. Therefore, in some cases, beamforming approaches for CoMP downlink transmission could be impractical. Note that as in [9], backhaul constraints can be taken into account for a realistic coordination between BSs. However, the availability of CSI could be problematic when the channel variation is fast.

In this paper, we consider a simple technique for downlink transmission to cell-edge users over orthogonal frequency division multiple access (OFDMA). As in CoMP downlink transmission, multiple adjacent BSs are coordinated to support multiple cell-edge users. However, the level of coordination

is minimized to avoid extensive backhaul transmissions. In this simple technique, each BS in the coordination only needs to know the group of cell-edge users. Thus, it requires no CSI at BSs and the backhaul transmission can be limited only to exchange the information to identify the group of cell-edge users. Each BS transmits signals to its dedicated user in the group over the multiple subcarriers or multi-channels that are shared by the other BSs in the coordination. The resulting channel becomes an interference channel where multiple transmitters and multiple receivers exist.

*Notation:* Upper-case and lower-case boldface letters are used for matrices and vectors, respectively.  $\mathbf{A}^T$  and  $\mathbf{A}^H$  denote the transpose and Hermitian transpose of  $\mathbf{A}$ , respectively. The subspace generated by  $\mathbf{A}$  is denoted by  $\text{span}(\mathbf{A}) = \{\mathbf{A}\mathbf{x} | \mathbf{x} \in \mathbb{C}^m\}$ , where  $m$  is the number of column vectors of  $\mathbf{A}$ .  $\mathcal{CN}(\mathbf{a}, \mathbf{R})$  represents the distribution of circularly symmetric complex Gaussian (CSCG) random vectors with mean vector  $\mathbf{a}$  and covariance matrix  $\mathbf{R}$ .

## II. PRELIMINARIES

### A. System Models

In the conventional approach, each BS can transmit signals to its user through a dedicated subcarrier or multiple subcarriers. To avoid serious ICI, adjacent BSs do not transmit signals through these subcarriers (this is the case where the frequency reuse factor is greater than 1). Suppose that  $L$  subcarriers are used and there are  $K$  users. In this case, the received signal at user  $k$  is given by

$$\mathbf{r}_k = \sqrt{\frac{1}{L}} \mathbf{h}_k s_k + \mathbf{n}_k, \quad (1)$$

where  $\mathbf{h}_k$  denotes the  $L \times 1$  channel gain vector from BS  $k$  to user  $k$  over a dedicated channel,  $s_k$  is the data symbol to user  $k$ , and  $\mathbf{n}_k \sim \mathcal{CN}(0, N_0 \mathbf{I})$  is the background noise. Let  $s_k \in \mathcal{S}$ , where  $\mathcal{S}$  is the symbol alphabet. The detection performance at the user's receiver depends on the signal-to-noise ratio (SNR) and statistical characteristics of the channel gain,  $\mathbf{h}_k$ . For fading channels, the diversity gain depends on the number of subcarriers. In order to achieve a better performance by exploiting diversity gain, more subcarriers can be used.

In the proposed approach, we assume that  $Q$  BSs transmit signals to  $K$  users over  $M$  ( $\geq L$ ) subcarriers simultaneously

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and the received signal at user  $k$  is given by

$$\begin{aligned}\mathbf{r}_k &= \sqrt{\frac{1}{M}} \sum_{q=1}^Q \mathbf{h}_{k,q} s_q + \mathbf{n}_k \\ &= \sqrt{\frac{1}{M}} \mathbf{H}_k \mathbf{s} + \mathbf{n}_k,\end{aligned}\quad (2)$$

where  $\mathbf{h}_{k,q}$  is the  $M \times 1$  channel gain vector from BS  $q$  to user  $k$ ,  $\mathbf{H}_k = [\mathbf{h}_{k,1} \ \mathbf{h}_{k,2} \ \dots \ \mathbf{h}_{k,Q}]$ ,  $\mathbf{s} = [s_1 \ s_2 \ \dots \ s_K]^T$ , and  $\mathbf{n}_k \sim \mathcal{CN}(0, N_0 \mathbf{I})$ . The resulting approach is called the multichannel sharing approach in this paper, as the  $K$  signals share a group of multiple subcarriers. Note that the BSs in coordination only need to agree with a set of the subcarriers that will be used to transmit signals to a group of cell-edge users. Thus, the level of coordination is low and does not require an extensive backhaul transmission. Throughout the paper, for the sake of simplicity, we assume that  $Q = K$  (i.e., the number of cell-edge users of interest is equal to the number of BSs in cooperation).

In the conventional approach, since each BS uses a different set of subcarriers for downlink transmissions, there is no significant ICI. However, due to this orthogonal channel allocation, the number of subcarriers per user can be reduced. For the comparison between the conventional and multichannel sharing approaches, we can have the following relationship between  $L$  and  $M$ :

$$M = LK. \quad (3)$$

Note that in the conventional approach,  $L$  is an integer as shown in (1), while  $L = \frac{M}{K}$  can be a rational number in the proposed multichannel sharing approach when  $K > 1$ .

### B. LR-based Detectors for Joint Detection

While various joint detection algorithms are available for MIMO systems, the lattice reduction (LR)-based detectors [10], [11] are promising due to its excellent performance with relatively low complexity. In particular, they can exploit a full receive diversity gain [12], [13].

For the LR-based detection, we assume that the received signal vector in (2) is properly scaled and shifted so that the elements of  $\mathbf{s}$  can be considered as nonnegative integers (for details, see [12]). The lattice basis reduction of the channel matrix,  $\mathbf{H}_k$ , is performed as follows:

$$\mathbf{H}_k = \mathbf{G}_k \mathbf{U}_k, \quad (4)$$

where  $\mathbf{U}_k$  is (complex) integer unimodular and  $\mathbf{G}_k$  has nearly orthogonal column vectors. Then, the received signal vector is re-written as

$$\mathbf{r}_k = \sqrt{\frac{1}{M}} \mathbf{G}_k \underbrace{\mathbf{U}_k \mathbf{s}}_{=\mathbf{c}_k} + \mathbf{n}_k, \quad (5)$$

where  $\mathbf{c}_k$  is also an (complex) integer vector. Any linear detector can be applied to  $\mathbf{r}_k$  to detect  $\mathbf{c}_k$  (not  $\mathbf{s}$ ). Since the column vectors of  $\mathbf{G}_k$  are nearly orthogonal, even the zero-forcing (ZF) detector can provide a reasonably good performance as the background noise is not significantly enhanced.

### III. JOINT DETECTION OVER SUBSPACE

In this section, we assume that each element of  $\mathbf{h}_k$  or  $\mathbf{H}_k$  is an independent zero-mean CSCG random variable, which results in Rayleigh fading. In OFDMA, if multiple subcarriers with sufficient spacing (in the spectrum) are allocated, we can assume that they are independent. In this case, the full diversity order that a user can achieve in the conventional approach is  $L$ , while that in the multichannel sharing approach is  $M = LK$ . Thus, it is clear that the multichannel sharing approach can provide a better performance. However, this result becomes valid only when the ML or near ML detection is employed. Since the ML detection requires a high computational complexity, a low complexity suboptimal detection method is desirable (e.g., the LR-MMSE-SIC detector). In this section, by noting that we only need to detect the desired signal reliably from the received signal in (2), we derive low complexity detection methods.

A candidate for such a low complexity detector is a linear detector. The output of the linear detector, which is an estimate of  $s_k$ , is given by

$$\begin{aligned}\hat{s}_k &= \mathbf{w}_k^H \mathbf{r}_k \\ &= \sqrt{\frac{1}{M}} \mathbf{w}_k^H \mathbf{h}_{k,k} s_k + \sqrt{\frac{1}{M}} \bar{\mathbf{w}}_k^H \bar{\mathbf{H}}_k \bar{\mathbf{s}}_k + \mathbf{w}_k^H \mathbf{n}_k,\end{aligned}\quad (6)$$

where  $\bar{\mathbf{H}}_k$  and  $\bar{\mathbf{s}}_k$  are the submatrix of  $\mathbf{H}_k$  and subvector of  $\mathbf{s}$  obtained by deleting  $\mathbf{h}_{k,k}$  and  $s_k$ , respectively. Here,  $\mathbf{w}_k$  represents a linear filtering vector. The ZF or MMSE criterion can be used to find  $\mathbf{w}_k$  which can suppress the interfering signals from the other BSs. However, from (3), the diversity order due the linear suppression is reduced to  $M - (K - 1) = (L - 1)K + 1$ . Note that this diversity gain is greater than or equal to the diversity gain,  $L$ , that the conventional approach can achieve. That is, the difference between the two diversity orders is  $M - (K - 1) - L = (L - 1)(K - 1) \geq 0$ . This implies that if  $K > 1$  and  $L > 1$ , the performance of the multichannel sharing approach can be better than that of the conventional approach even if a low complexity linear detector is employed. In summary, we have the diversity order bounds for the multichannel sharing approach as follows:

$$DO_{\text{conv}} = L \leq \underbrace{(L - 1)K + 1}_{\text{with linear detection}} \leq DO_{\text{prop}} \leq \underbrace{LK}_{\text{with ML detection}}, \quad (7)$$

where  $DO_{\text{conv}}$  and  $DO_{\text{prop}}$  represent diversity orders of the conventional and proposed multichannel sharing approaches, respectively.

Note that the full diversity order can be achieved using relatively low complexity detectors. For example, various LR-based detectors can achieve a full diversity order [13] [12]. The complexity of the LR-based detectors depends on the lattice basis reduction. The best algorithm in terms of the complexity is the LLL algorithm [14], which has a polynomial time complexity. However, this complexity could be still high when  $K$  is large.

In order to select the signals to be mitigated, we consider the orthogonality deficiency (OD) [15] that is defined as

$$\zeta(\mathbf{A}) = 1 - \frac{\det(\mathbf{A}^H \mathbf{A})}{\prod_{k=1}^n \|\mathbf{a}_k\|^2}, \quad (8)$$

where  $\mathbf{A} = [\mathbf{a}_1 \dots \mathbf{a}_n]$ . If the column vectors of  $\mathbf{A}$  are orthogonal,  $\zeta(\mathbf{A}) = 0$ , while  $\zeta(\mathbf{A}) = 1$  when the column vectors are linearly dependent. If  $n = 2$ ,  $\zeta(\mathbf{A}) = \frac{|\mathbf{a}_1^H \mathbf{a}_2|^2}{\|\mathbf{a}_1\|^2 \|\mathbf{a}_2\|^2}$ . Let  $\mathbf{A} = [\mathbf{h}_{k,k} \ \mathbf{h}_{k,q}]$ ,  $q \neq k$ . If  $\zeta(\mathbf{A}) = 0$  or small (i.e.,  $\zeta(\mathbf{A}) \ll 1$ ), then  $\mathbf{h}_{k,q}$  is nearly orthogonal to  $\mathbf{h}_{k,k}$ . In this case, the joint detection of  $s_k$  and  $s_q$  has an insignificant performance gain over individual detection of  $s_k$  and  $s_q$ . This means that  $q \in \mathcal{K}_2$ . On the other hand, if  $\zeta(\mathbf{A})$  is close to 1,  $s_q$  should be jointly detected with  $s_k$ , i.e.,  $q \in \mathcal{K}_1$ .

Based on the properties of OD, we can propose the following selection criterion for  $\mathcal{K}_2$ :

$$\mathcal{K}_2^* = \arg \min_{\mathcal{K}_2} \max_{m \in \mathcal{K}_2} \zeta([\mathbf{h}_{k,k} \ \mathbf{h}_{k,m}]), \quad (9)$$

Note that  $k \notin \mathcal{K}_2$ . There are  $\binom{K-1}{K_2}$  possible index sets for  $\mathcal{K}_2$ . Thus, an exhaustive search requires a high computational complexity. However, if we have the following inequalities:

$$\zeta([\mathbf{h}_{k,k} \ \mathbf{h}_{k,k_1}]) \leq \zeta([\mathbf{h}_{k,k} \ \mathbf{h}_{k,k_2}]) \leq \dots \leq \zeta([\mathbf{h}_{k,k} \ \mathbf{h}_{k,k_{K-1}}]), \quad (10)$$

where  $k_i \in \{1, 2, \dots, K\} \setminus k$ , we can easily show that

$$\mathcal{K}_2^* = \{k_1, k_2, \dots, k_{K_2}\}.$$

Since the inequalities in (10) can be found by performing  $K-1$  inner products (each OD in (10) can be found by the inner product of two vectors), the resulting complexity is low.

Let  $\mathbf{H}_{k,(1)}$  and  $\mathbf{H}_{k,(2)}$  be the submatrices of  $\mathbf{H}_k$  that have the column vectors with the indices in  $\mathcal{K}_1$  and  $\mathcal{K}_2$ , respectively. Define  $\mathbf{s}_{(1)}$  and  $\mathbf{s}_{(2)}$  accordingly. Then,  $\mathbf{r}_k$  in (2) is given by

$$\mathbf{r}_k = \sqrt{\frac{1}{M}} \mathbf{H}_{k,(1)} \mathbf{s}_{(1)} + \sqrt{\frac{1}{M}} \mathbf{H}_{k,(2)} \mathbf{s}_{(2)} + \mathbf{n}_k. \quad (11)$$

Using a linear filtering operation, the interfering signal,  $\sqrt{\frac{1}{M}} \mathbf{H}_{k,(2)} \mathbf{s}_{(2)}$ , can be suppressed. To completely suppress, we can use the orthogonal projection:

$$\begin{aligned} \mathbf{y}_k &= \mathbf{P}_2^\perp \mathbf{r}_k \\ &= \sqrt{\frac{1}{M}} \mathbf{P}_2^\perp \mathbf{H}_{k,(1)} \mathbf{s}_{(1)} + \mathbf{P}_2^\perp \mathbf{n}_k, \end{aligned} \quad (12)$$

where  $\mathbf{P}_2^\perp = \mathbf{I} - \mathbf{H}_{k,(2)} (\mathbf{H}_{k,(2)}^H \mathbf{H}_{k,(2)})^{-1} \mathbf{H}_{k,(2)}^H$ . Other approaches (e.g., the MMSE-based approach) are also available. From  $\mathbf{y}_k$ , we can detect  $\mathbf{s}_{(1)}$  using the LR-MMSE-SIC detector. This detection can be seen as joint detection over subspace as  $\mathbf{y}_k$  in (12) is a vector in the orthogonal subspace of  $\text{span}(\mathbf{H}_{k,(2)})$ . Note that if  $K_1 = K$ , the resulting detector becomes the full LR-MMSE-SIC detector, while if  $K_1 = 1$ , it becomes the ZF detector. Thus, by adjusting  $K_1$ , we can enjoy the tradeoff between performance and complexity.

In general, it is necessary to derive detailed performance and complexity analysis to see the tradeoff. However, a detailed performance analysis of the LR-based MIMO detection is not easy although the diversity gain analysis would be relatively straightforward (e.g., [12] [16]). It is known that an LR-based MIMO detector<sup>2</sup> can achieve a full receive diversity gain. This means that the diversity gain is  $\text{rank}(\mathbf{P}_2^\perp \mathbf{H}_{k,(1)}) \leq M - K_2 = M - K + K_1$ <sup>3</sup> if an LR-based MIMO detector is used to detect  $\mathbf{s}_{(1)}$  from  $\mathbf{y}_k$  in (12). The complexity analysis is also subtle as the complexity of the LLL algorithm depends on  $\mathbf{P}_2^\perp \mathbf{H}_{k,(1)}$ , which means the complexity is random [17]. In general, the overall complexity depends on the number of column vectors of  $\mathbf{P}_2^\perp \mathbf{H}_{k,(1)}$  or  $K_1$ . In summary, we can see that as  $K_1$  increases the complexity increases, while the performance (i.e., the diversity order) is improved.

Previously, we assumed that  $M = LK$  for a fair comparison with the conventional approach. As mentioned earlier, while  $L$  is a positive integer in the conventional approach, it could be a rational number in the multichannel sharing approach. Since a single data symbol can be transmitted through  $L$  subcarriers,  $L$  is referred to as the spectral expansion factor. In the multichannel sharing approach,  $L = \frac{4}{3}$  if  $M = 4$  and  $K = 3$ . In this case, the diversity order that can be achieved by a linear detector is  $M - (K - 1) = 4 - 2 = 2$ . This diversity order can be achieved in the conventional approach only when  $L = 2$ , which results in a much higher spectral expansion factor. From this, we can see that the multichannel sharing approach is more flexible than the conventional one. In the next section, we will discuss the performance issues.

#### IV. SIMULATION RESULTS

For simulations, we assume that each element of  $\mathbf{h}_k$  and  $\mathbf{H}_k$  is an independent CSCG random variable with mean zero and unit variance. For signal modulation, 16 quadrature amplitude modulation (16-QAM) is employed with Gray mapping. The LR-MMSE-SIC detector is employed with the LLL algorithm for lattice basis reduction.

Fig. 1 shows the bit error rate (BER) simulation results for the conventional and proposed multichannel sharing approaches when  $K = 3$ ,  $L = 1$ , and  $M = 3$ . In this case,  $K_1$  can be 1 (a linear detector is applied), 2, or 3 (in this case, full joint detection is carried out) in the multichannel sharing approach. The multichannel sharing approach without the joint detection (i.e.,  $K_1 = 1$ ) performs worse than the conventional one, although the diversity gain is the same. However, when the joint detection is used, the multichannel sharing approach can provide a better performance when the SNR (here, the SNR is represented by  $E_b/N_0$ , where  $E_b$  is the bit energy) is sufficiently high. At a BER of  $10^{-3}$ , there is more than 3 dB SNR gain with  $K_1 = 2$  and 8 dB gain with  $K_1 = 3$ . Since the multichannel sharing approach has a higher diversity gain with joint detection than the conventional one, the SNR gain increases when the target BER becomes lower. Note that the

<sup>2</sup>When the LLL algorithm is used to perform the lattice basis reduction.

<sup>3</sup>If  $\mathbf{H}_k$  is a random matrix (each element is an independent CSCG random variable),  $\text{rank}(\mathbf{P}_2^\perp \mathbf{H}_{k,(1)}) = M - K_2$  with probability 1.

target BER should be lower if a higher data rate or throughput is required (possibly with a less powerful channel code).

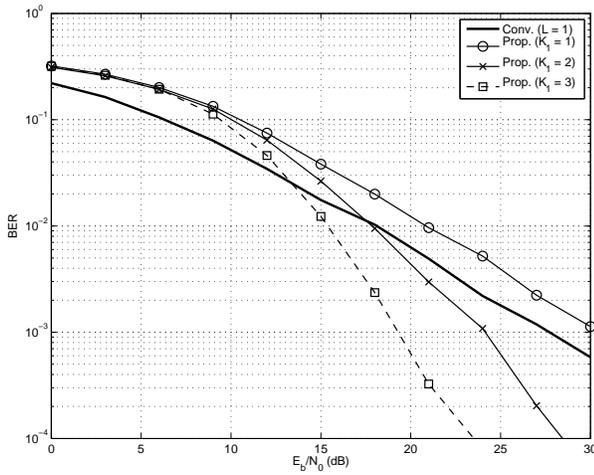


Fig. 1. BER results of the conventional and proposed multichannel sharing approaches with  $K = 3$ ,  $L = 1$ , and  $M = 3$ .

In Fig. 2, we show the BER results when the multichannel sharing approach has different values of the spectral expansion factor,  $L$ , with  $K = 3$  ( $L = \frac{M}{K} = 1, \frac{4}{3}, \text{ and } \frac{6}{3}$ ) and compare it with the conventional approach when  $L = 1$  and 2). At a BER of  $10^{-4}$ , the required SNR value for the multichannel sharing approach with  $L = \frac{4}{3}$  is smaller than that for the conventional approach with  $L = 2$ . Thus, as expected, the multichannel sharing approach can be more spectrally efficient when the required BER is relatively low (say  $10^{-4}$ ). In the case of  $L = 2$ , we can see that the multichannel sharing approach outperforms the conventional approach for a wide range of BER (say  $\leq 10^{-2}$ ). Therefore, we can see that the multichannel sharing approach is more suitable for cell-edge users when a lower uncoded BER is required (which may result in a high data rate or throughput).

## V. CONCLUDING REMARKS

In order to improve the data rate for cell-edge users, we considered a simple approach called the multichannel sharing approach, where each BS in coordination transmits signals to a dedicated user through shared multiple subcarriers with the other BSs in coordination. While the diversity gain can be improved by using more subcarriers, the ICI due to the transmission from other BSs should be mitigated. We derived a flexible detection method that suppresses insignificant ICI, while performs joint detection with significant ICI using an LR-based MIMO detector. Through simulation results, it was shown that the multichannel sharing approach can provide a reasonable performance with the derived flexible joint detection method.

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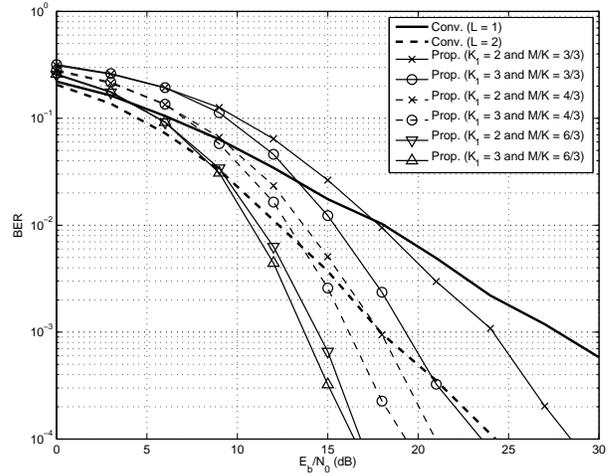


Fig. 2. BER results of the conventional and proposed multichannel sharing approaches for different values of  $L$  and  $M$ , respectively when  $K = 3$ .

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