Efficient User Scheduling Algorithm for Zero-Forcing Beamforming in MIMO Broadcast Channels

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Abstract

This paper presents a novel user scheduling algorithm that provides a maximum sum-rate based on zero-forcing beamforming (ZFBF) in multiple-input multiple-output (MIMO) systems. The proposed technique determines primary user pairs of which the sum-rate exceeds the predetermined threshold. To determine the threshold, we define the maximum-sum-rate criterion (MSRC) derived from the extreme value theory (EVT). Applying the MSRC in ZFBF-based user scheduling, we found that the performance of the proposed method is comparable to the exhaustive searching scheme which requires greater computational load. Through computer simulations, we show that the proposed method outperforms the very well-known correlation-based method, semi-orthogonal user selection (SUS), yielding a sum rate that is about 0.56 bps/Hz higher when the transmit SNR is 10dB and the number of users and transmit antennas in a cell is 100 and four, respectively.

Index Terms

multiuser MIMO, multiuser diversity, user scheduling, zero-forcing beamforming, broadcast channel

I. Introduction

Multiple-input multiple-output (MIMO) is an essential technique to achieve high throughput in wireless communication systems. Especially, multiuser-MIMO (MU-MIMO), which can serve multiple users simultaneously, is currently a very active research focus because it can provide spatial multiplexing gain and multi-user diversity gain [1], [12]. In MU-MIMO schemes, dirty paper coding (DPC) [2] is a capacity achieving scheme for the MIMO broadcast channel [3]. However, DPC is difficult to implement in practical systems due to its high computational complexity. Alternatively, linear beamforming, which involves reduced complexity relative to DPC, is often investigated [4]-[10]. In this strategy, zero-forcing beamforming (ZFBF) has attracted the most interest due to the simplicity of finding beamforming vectors, which are chosen to avoid inter-user interference. The concept of ZFBF is that each user is assigned one column vector of the pseudo-inverse of the broadcast channel matrix. ZFBF combined with user scheduling can achieve a sum-rate that has the same scaling law as that of DPC [4]. Hence, efficient ZFBF strategies with user scheduling are proposed [4]-[8]. However, their performance suffers because they always select a primary user that provides the largest channel power. The user group that includes the primary user with the largest channel power does not always achieve the maximum sum-rate. In this paper, we propose a novel user scheduling algorithm that considers primary user pairs of which the sum-rate exceeds a predetermined threshold of the maximum-sum-rate criterion (MSRC) derived from the extreme value theory (EVT). Using the primary user pairs, a M-tuple user group, where the number of transmit antennas is M, who achieves the maximum sum-rate is selected efficiently. Applying the MSRC in ZFBF-based user scheduling, we found that the proposed algorithm provides comparable performance to the exhaustive search scheme which requires greater computational load and outperforms the very well-known correlation-based method, semi-orthogonal user selection (SUS). This paper is organized as follows. Section II introduces the system model defined in this paper. Section III explains the proposed user scheduling procedure. Section IV compares the performance of the proposed algorithm, the exhaustive search, and conventional SUS through computer simulation results. Section V concludes this paper.

Notation: We use lowercase boldface letters to denote vectors and uppercase bold letters to denote matrices. $\mathbf{A}^H$ and $\mathbf{a}^H$ stand for the conjugate transpose of matrix $\mathbf{A}$ and vector $\mathbf{a}$, respectively. $\mathbf{A}^T$ and $\mathbf{a}^T$ stand for the transpose of matrix $\mathbf{A}$ and vector $\mathbf{a}$, respectively. $[\mathbf{A}]_{i,j}$ denotes the (i, j)th element of matrix $\mathbf{A}$. $\|\mathbf{a}\|$ denotes the Euclidean norm of a vector $\mathbf{a}$. $\mathbf{I}_M$ is the $M \times M$ identity matrix. $|S|$ denotes the size of set S.

II. System Model

We consider a ZFBF-based MU-MIMO downlink system combined with user scheduling. There are M transmit antennas and K $(K \geq M)$ single-antenna users. We denote the flat fading channel of user k as $\mathbf{h}_k \in C^{1\times M}$. Its entries are independent and identically distributed (i.i.d.) complex Gaussian values with zero mean and unit variance. Throughout this paper, the BS is assumed to have perfect channel state information of all users. We also assume a subset of M users $S = \{k_1, \cdots, k_M\} \subset \{1, \cdots, K\}$ is selected. Let $\mathbf{H}(S) = [\mathbf{h}_1^T \ \cdots \ \mathbf{h}_M^T]^T$ and $\mathbf{W}(S) = [\mathbf{w}_1^T \ \cdots \ \mathbf{w}_M^T]^T$ be, respectively, a channel matrix and a beamforming weight matrix for user set S. Since the proposed procedure is based on ZFBF, $\mathbf{W}(S)$ can be designed from the pseudoinverse of $\mathbf{H}(S)$, which is $\mathbf{H}(S)^H(\mathbf{H}(S)\mathbf{H}(S)^H)^{-1}$. $\mathbf{w}_k \in C^{M\times 1}$ is the unit-norm beamforming weight vector of the k-th user. For user $k \in S$, we have the following received signal:

$$\begin{aligned} y_k &= \sqrt{P_k}\mathbf{h}_k\mathbf{w}_k s_k + \mathbf{h}_k \sum_{j\in S, j\neq k}\sqrt{P_j}\mathbf{w}_j s_j + n_k \\ &= \sqrt{P_k}\mathbf{h}_k\mathbf{w}_k s_k + n_k \end{aligned}, \tag{1}$$

where P_k is the transmit power for user k, s_k is the normalized data symbol, and n_k is the additive white Gaussian noise (AWGN) at the k-th user. In effect, using ZFBF gives zero cross channel interference. The BS must satisfy the power constraint of $\sum_{k\in S} P_k = P$. For ease of explanation, we assume equal power allocation, i.e. $P_k = \frac{P}{M}, \forall k \in S$. Therefore, the sum rate achieved by this system is

$$R_{ZF}(S) = \sum_{k\in S}\log_2\left(1 + \frac{P}{M}|\mathbf{h}_k\mathbf{w}_k|^2\right). \tag{2}$$

III. Proposed Primary Pair User Scheduling Method

As mentioned earlier, an exhaustive search that achieves the maximum sum-rate requires high computational complexity, especially for large K, whereas the conventional SUS that requires low computational complexity has poor performance. In this section, we present a novel user scheduling method, which provides the maximum sum-rate with moderate complexity. The proposed algorithm considers the primary user pairs. A user pair of which the sum-rate exceeds a predetermined threshold is selected as the primary user pair. To determine the threshold, we define the MSRC as follows,

$$R_{ZF}^{\mathrm{MSRC}}(\mathrm{a} \cup \mathrm{b}) \tag{3}$$

$$\overset{(i)}{=} \log_2\left(1 + \mathrm{SNR}_{\mathrm{a}}^{\mathrm{MSRC}}\right) + \log_2\left(1 + \mathrm{SNR}_{\mathrm{b}}^{\mathrm{MSRC}}\right) \tag{4}$$

$$\overset{(ii)}{=} \log_2\left(1 + \frac{P}{M[(\mathbf{G}(a\cup b)\mathbf{G}(a\cup b)^H)^{-1}]_{1,1}}\right) + \log_2\left(1 + \frac{P}{M[(\mathbf{G}(a\cup b)\mathbf{G}(a\cup b)^H)^{-1}]_{2,2}}\right) \tag{5}$$

$$\overset{(iii)}{=} \log_2\left\{\left(1 + \frac{P}{M}\gamma_{\mathrm{a}}\right)\left(1 + \frac{P}{M}\gamma_{\mathrm{b}}\right)\right\} \tag{6}$$

$$\overset{(iv)}{\geq} 2\log_2\left\{1+\frac{P}{M}\mu\right\} \tag{7}$$

$$\overset{(v)}{=} \alpha\ , \tag{8}$$

where 'a' and 'b' are random users in a cell, and α is a threshold of the MSRC. (i) follows from (2), where $\mathrm{SNR}_{\mathrm{k}}^{\mathrm{crit}} = (P/M)\,|\,\mathbf{h}_k\mathbf{w}_k\,|^2 = (P/M)\,|\,\mathbf{g}_k\,|^2$, $k \in \{1,\cdots,K\}$ and $\mathbf{g}_{\mathrm{k}}$ is the effective channel gain which is orthogonal to the subspace spanned by $\{\mathbf{g}_1,\mathbf{g}_2,\cdots,\mathbf{g}_{k-1}\}$. In (ii), we use the effective channel gains of a user pair in the ZFBF provided using $\mathbf{g}_{\mathrm{k}}$, where $\mathbf{G}(a\cup b) = [\mathbf{g}_a^T \quad \mathbf{g}_b^T]^T$. In (iii), $\gamma_a = 1/[(\mathbf{G}(a\cup b)\,\mathbf{G}(a\cup b)^H)^{-1}]_{1,1}$ and $\gamma_{\mathrm{b}} = 1/[(\mathbf{G}(a\cup b)\,\mathbf{G}(a\cup b)^H)^{-1}]_{2,2}$ denote the effective channel gains of 'a' and 'b', respectively. (iv) is assumed that users whose channel directions are matched to zero-forcing beam directions are supposed to be selected, which means $\gamma_a = \|\mathbf{g}_a\|^2$, $\gamma_b = \|\mathbf{g}_b\|^2$ [14] and we use the maximum EVT with $\|\mathbf{g}_a\|^2$ and $\|\mathbf{g}_b\|^2$ [11], where $|S| = n-1$ and $\mu = \ln(K-\mathrm{n}+1) + (M-2)\ln\ln(K-\mathrm{n}+1) + O(\ln\ln\ln(K-n+1))$. Given that the K channel power values are chi-square distributed random variables with $2M$ degrees of freedom, we can compute the maximum extreme value boundary of the K channel powers [11]. We claim that μ is determined to be the lower bound of the maximum extreme value, which means that the primary user pairs who satisfies (7) will consist of users who achieve the promising sum-rate. In (v), we can obtain the threshold, α, through calculating the sum-rate with μ. Using the MSRC in user scheduling, we can select a M-tuple user group that achieves the maximum sum-rate with low complexity, where the number of transmit antennas is M. The details of our proposed user scheduling method are described in TABLE I.

In TABLE II, the complexity of ZFBF-PPUS is compared to that of the exhaustive search and conventional SUS. The complexity of ZFBF-PPUS is calculated as follows. First, we compute the probability [11] that the channel power becomes larger than μ as

$$P\left(\|\mathbf{h}_i\|^2 > \mu\right) = 1 - F(\mu) = \mu^{M-1}e^{-\alpha}\ . \tag{9}$$

We can see that $\mu^{M-1}e^{-\mu}K$ primary user pairs are considered at the first user pairing, where K is the total number of users in a cell. Subsequently, it is necessary to search $\mu^{M-1}e^{-\mu}(K-1)$, $\mu^{M-1}e^{-\mu}(K-2)$, ···, $\mu^{M-1}e^{-\mu}(K-M+1)$ primary user pairs for each user pairing when we select M users out of the K users in a cell. Thus, it can be found that the number of possible user sets in ZFBF-PPUS is

$$\{\mu^{M-1}\mathrm{e}^{-\mu}\cdot K\}\{\mu^{M-1}\mathrm{e}^{-\mu}\cdot(K-1)\}\cdots\{\mu^{M-1}\mathrm{e}^{-\mu}\cdot(K-M+1)\} < \left(\mu^{M-1}\mathrm{e}^{-\mu}\cdot K\right)^M\ . \tag{10}$$

The exhaustive search considers all ${}_K C_M$ possible user sets, and the case of the conventional SUS is shown in [4]. TABLE II shows the complexity of each algorithm, especially for $K = 100$ and $M = 4$. The complexity of ZFBF-PPUS is significantly smaller than that of the exhaustive search.

IV. Simulation Results

This section describes the simulation results obtained using the proposed method of user scheduling. For computer simulations, we assume that the channel information is perfectly fed from each subscriber to the BS without any distortion during the transmission. The performance of the proposed technique, ZFBF-PPUS (primary pair user scheduling) is compared to the exhaustive search scheme in ZFBF, which finds the maximum sum-rate user pair exhaustively from the entire user set, and to the conventional ZFBF-SUS (semi-orthogonal user scheduling) method, which pre-selects the primary user as the one producing the largest channel power. Figure 1 illustrates the sum-rate as a function of the total number of users provided using three methods: ZFBF-PPUS, the exhaustive search, and ZFBF-SUS when the number of BS antennas is four. We can see that ZFBF-PPUS outperforms ZFBF-SUS. ZFBF-PPUS improves the system capacity by about 0.56 bps/Hz compared to ZFBF-SUS, where the number of users is 100. Moreover, ZFBF-PPUS has slightly degraded performance compared to the exhaustive search in all range of K.

Figure 2 illustrates the sum-rate as a function of the transmit SNR provided using the three methods, where the total number of users is 30. We can see that the respective performances of the exhaustive search and ZFBF-PPUS are very close to each other. We can also see that ZFBF-PPUS has a substantial performance improvement compared to ZFBF-SUS in the entire SNR region considered in Figure 2. The sum rate gap between ZFBF-PPUS and ZFBF-SUS is about 0.54 bps/Hz, where the transmit SNR is 5 dB.

V. Conclusions

In this paper, we propose a new user scheduling algorithm for improving the sum-rate in a MU-MIMO system. The sum-rate obtained using the conventional SUS in general is far poorer than that obtained using the exhaustive search even though it has the lowest complexity, mainly because a user with the largest channel power, which does not necessarily provide the maximum sum-rate, is always selected as the primary user with the SUS method.

The proposed user scheduling algorithm, ZFBF-PPUS, defines the maximum-sum-rate criterion (MSRC). A pair of users, which satisfies the MSRC, is determined as the primary user pair. Considering the primary user pairs in user scheduling, a M-tuple user group that achieves the maximum sum-rate is selected with low complexity, where the number of transmit antennas is M. Through computer simulations, we have found that the sum-rate provided using ZFBF-PPUS is comparable to that of the exhaustive search, but with much lower complexity and better performance than SUS by about 0.56 bps/Hz when the transmit SNR is 10dB and the total number of users and transmit antennas in a cell is 100 and four, respectively. Both theoretic analysis and simulation results show that the ZFBF-PPUS algorithm achieves a sum-rate close to the maximum rate achievable by ZFBF with a moderate complexity.

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FIGURES AND TABLES

TABLE I. CONSTRUCTION ALGORITHM OF PRIMARY PAIR USER SCHEDULING (PPUS)

Algorithm: Primary Pair User Scheduling
Step 1: $\mathrm{i} \to 1$, $\mathrm{S_T} = \mathrm{S_0} = \mathrm{S_{-1}} = \mathrm{r} \to \phi$, and $\mathrm{U} \to \{1, \cdots, \mathrm{K}\}$
Step 2: Calculate $\mathbf{g}_\mathrm{k}$ such as
$\mathbf{g}_\mathrm{k} \to \mathbf{h}_\mathrm{k}\left(\mathbf{I}_{M\times M} - \sum_{l=1}^{i-2} \frac{\mathbf{g}_{(l)}^\mathrm{H}\mathbf{g}_{(l)}}{\lVert\mathbf{g}_{(l)}\rVert^2}\right) \ \forall \mathrm{k} \in \mathrm{U} \setminus \mathrm{S_{i-2}}$ and
Choose any users such as follow
$\mathrm{P_i} \to \left\{\mathrm{q} \mid R_{ZF,\ \mathrm{q}\in\mathrm{U}\setminus\mathrm{S_{i-1}}}^{\mathrm{MSRC}}(\mathrm{r} \cup \{\mathrm{q}\}) > \alpha\right\}$.
Step 3: If $\lvert\mathrm{P_i}\rvert = 0$, then
$\mathrm{i} \to \mathrm{i} - 1$ and go to Step 5.
else,
- select a user, $\mathrm{q} \in \mathrm{P_i}$,
- $\mathrm{r} \to \{\mathrm{q}\}$, $\mathrm{S_i} \to \mathrm{S_{i-1}} \cup \mathrm{r}$, and $\mathrm{P_i} \to \mathrm{P_i} - \mathrm{r}$,
- $\mathbf{g}_{(\mathrm{i})} \to \mathbf{g}_\mathrm{q}$,
- if $\lvert\mathrm{S_i}\rvert < \mathrm{M}$, then
$\mathrm{i} \to \mathrm{i} + 1$ and go to Step 2.
Step 4: $\mathrm{S_T} \to \mathrm{S_T} \cup \{\mathrm{S_i}\}$
Step 5: If $\mathrm{i} \to 1$ and $\lvert\mathrm{P_i}\rvert \to 0$, then
go to Step 6.
else,
go to Step 3.
Step 6: Find $\mathrm{S_{FINAL}}$ as follow
- $\mathrm{S_{FINAL}} \to \arg\max_{\mathrm{S_{tmp}} \in \mathrm{S_T}} R_{ZF}(\mathrm{S_{tmp}})$

TABLE II. COMPLEXITY COMPARISON

	Complexity	**# of Possible user sets (M=4, K=100)**
ZFBF-PPUS	$\left(\mu^{M-1} e^{-\mu} K\right)^{M}$	201,586
The exhaustive search	$\frac{K!}{M!(K-M)!}$	3,921,225
ZFBF-SUS(β = 0.4)	$\sum_{p=1}^{M} \sum_{k=p-1}^{M-1} a\beta^{2k}(1-\beta^{2})^{(M-k-1)}$ $where \quad a = \frac{(M-1)!}{k!(M-k-1)!}$	148

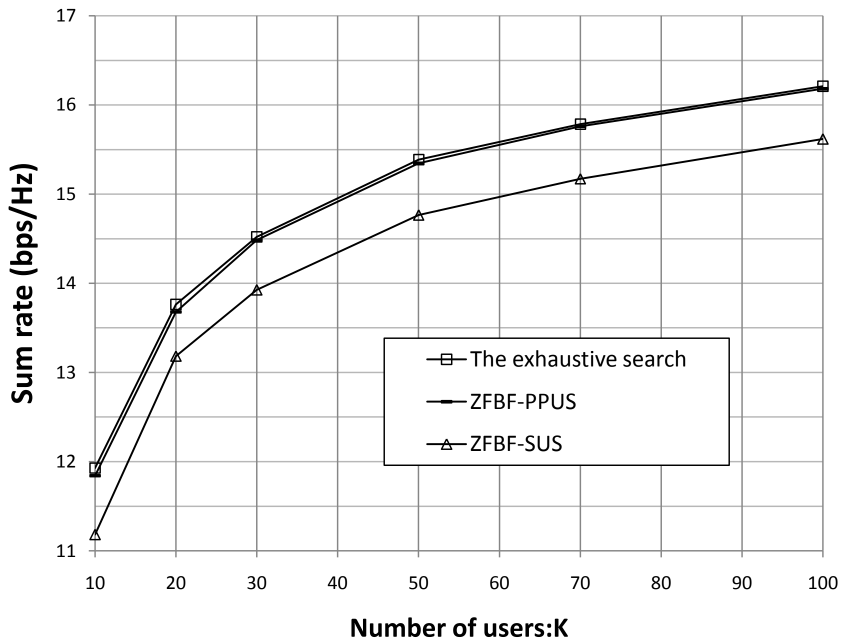

Figure 1. Sum-rates of ZFBF-PPUS, the exhaustive search, and ZFBF-SUS for different numbers of users. (M=4 and transmit SNR=10dB)

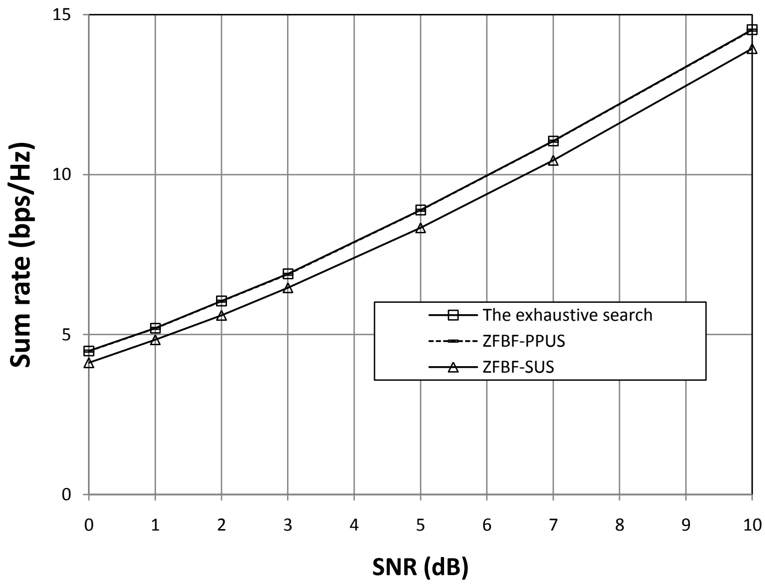

Figure 2. Sum-rates of ZFBF-PPUS, the exhaustive search, and ZFBF-SUS for different transmit SNR. (M=4 and K=30)