Analysis of magnetic resonance wireless power transfer with multiple self resonators

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Abstract

This paper presents efficiency analysis of a magnetic resonance wireless power transfer (WPT) system with multiple self resonators. In the system, there are Tx and Rx self resonators and an additional coil, called intermediate self resonator. The intermediate resonant coil is arranged coaxially and perpendicular to both Tx and Rx resonant coils. The power efficiency is calculated using coupled mode theory (CMT). Impedance matching conditions are also shown using the CMT. Analysis results show that the intermediate coil properly improves efficiency and extends distance between Tx and Rx. The calculated efficiency is in good agreement with measurement.

Keywords : Magnetic coupling wireless power transfer, Coupled mode theory, Self resonator.

1. Introduction

Wireless power transfer based on magnetic resonance (MR WPT) in near field region is attracting more attention since the WPT via strongly coupled magnetic resonances was reported [1]-[3]. To extend the distance of power delivery or increase power transfer efficiency using the magnetic resonance WPT, an additional self resonator, called an intermediate resonant coil between Tx and Rx resonant coils is used [4]. In [4], the intermediate coil with the same resonant frequency as Tx and Rx resonant coils can be well applied to improve power transfer efficiency. However, detailed analysis or consideration such as power transfer efficiency depending on Tx and Rx resonant coils and the intermediate resonant coil are not available.

In this paper, power efficiency of a magnetic resonance wireless power transfer system with an intermediate coil is analyzed. In particular, the intermediate resonant coil is perpendicular to a Tx resonant coil or a Rx resonant coil. Some preliminary results of this investigation were given in [5].

2. Derivation of power transfer efficiency

In Fig.1, the configuration of a magnetic resonance wireless power transfer with an intermediate resonant coil is shown. The variables of κ_1, κ_2, κ_{m1}, κ_{m2}, and κ_{12} are denoted as coupling coefficients between coils. The intermediate resonant coil is placed between Tx and Rx resonant coils and central axis is misaligned with that of Tx and Rx resonant coils.

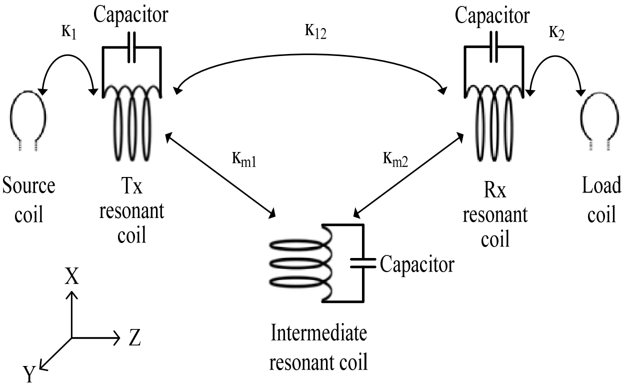

Figure 1: Configuration of a magnetic resonance wireless power transfer system with an intermediate resonant coil.

By referring to [6], a modified CMT formula of the intermediate system including the source coil and the load coil is presented as follows:

$$\begin{pmatrix} \dot{a}_1 \\ \dot{a}_m \\ \dot{a}_2 \\ S_{-1} \\ S_{-2} \end{pmatrix} = \begin{pmatrix} -(i\omega_1+\Gamma_1)-\kappa_1 & i\kappa_{1m} & i\kappa_{12} & \sqrt{2\kappa_1} & 0 \\ i\kappa_{m1} & -(i\omega_m+\Gamma_m) & i\kappa_{m2} & 0 & 0 \\ i\kappa_{21} & i\kappa_{2m} & -(i\omega_2+\Gamma_2)-\kappa_2 & 0 & 0 \\ \sqrt{2\kappa_1} & 0 & 0 & -1 & 0 \\ 0 & 0 & \sqrt{2\kappa_2} & 0 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_m \\ a_2 \\ S_{+1} \\ S_{+2} \end{pmatrix}. \quad (1)$$

Here, κ_1 are rates of energy transfer to the Tx coil, κ_2 is the same as the loading factor, and $S_{\pm 1}$ are field amplitudes of incident field (+) and reflect field (-) at the source and $S_{\pm 1}$ are field amplitudes at the load. The rates of field amplitudes (S11, S21, S12, S22) are scattering parameters.

As a simple example, the case of $\kappa_{12} \approx 0$ and $\kappa_{m1}=\kappa_{m2}=\kappa_m$ is considered. It means that Tx and Rx resonant coils are identical and the intrinsic decay rates of Tx and Rx coils Γ_1, Γ_2 are different from that of the intermediate resonant coil, Γ_m while all resonant frequencies are identical. Coupling coefficient between Tx and intermediate resonant coil is not the same as the coupling one between Rx and intermediate coil. Distance between Tx and Rx is large enough to neglect the direct coupling coefficient between Tx and Rx resonant coils.

Then, the field amplitude transmitted to the load from source (S21) is obtained as

$$S_{21} = \frac{-2U_m{}^2 U_0}{[1+U_0-iD_1][1+U_0-iD_2][1-iD_m]+U_m{}^2[2(1+U_0)-i(D_1+D_2)]} \qquad (2)$$

where $U_0 = \kappa_1/\Gamma_1 = \kappa_2/\Gamma_2$, $D_1 = (\omega_1-\omega)/\Gamma_1$, $D_2 = (\omega_2-\omega)/\Gamma_2$, $D_m = (\omega_m-\omega)/\Gamma_m$, $U_m = k_m/(\Gamma \cdot \Gamma m)^{1/2}$. Power transfer efficiency is also obtained as follows

$$\eta \equiv \frac{P_{\text{transfer}}}{P_{\text{incidence}}} = |S_{21}|^2. \qquad (3)$$

The matching condition is $U_0^{opt} = (1+2U_m{}^2)^{1/2}$ when $D_1 = D_2 = D_m = 0$. The derived efficiency formula in the matching condition is obtained as follows.

$$\eta = \left(\frac{\kappa_m{}^2}{\Gamma\Gamma_m}\right)^2 \Bigg/ \left[\sqrt{1+\frac{2\kappa_m{}^2}{\Gamma\Gamma_m}}+1+\frac{\kappa_m{}^2}{\Gamma\Gamma_m}\right]^2. \qquad (4)$$

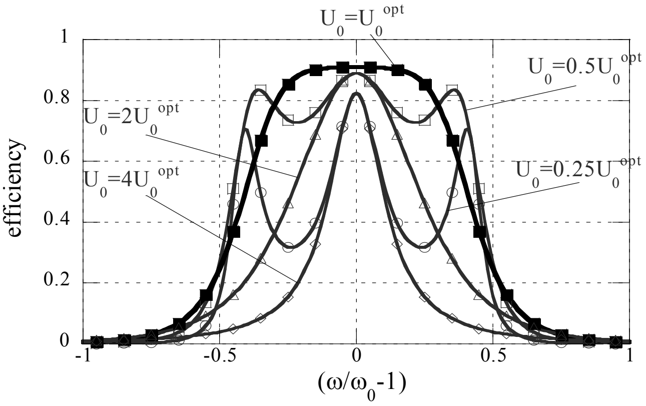

Figure 2: Efficiencies according to normalized frequency for three different matching conditions ($U_m=30$).

Figure 2 shows efficiencies according to normalized frequency for three different matching conditions with $U_m = 30$ and $\Gamma_1 = \Gamma_2 = 104$. In the case of under coupling condition ($U_0 < U_0^{opt}$), three peak frequencies are observed. Maximum power transfer is obtained at the center frequency. In the case of over coupling condition ($U_0 > U_0^{opt}$), peak frequency is observed at the center frequency. In critical coupling ($U_0 = U_0^{opt}$), the system have a maximum efficiency at the center frequency and a wideband characteristic. In the under and over coupling conditions, maximum power transfer is unable to be achieved. Therefore, satisfying the impedance matching condition is very important for higher efficiency.

3. Fabrication and measurement

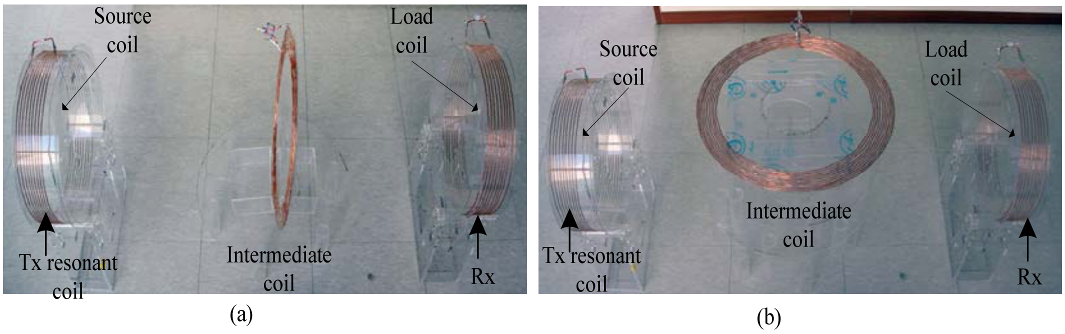

Figure 3: Measurement setup for a MR WPT system with intermediate coils (a) coaxially arranged (b) perpendicular arranged.

Figure 3 shows measurement setup for two MR WPT systems with an intermediate self resonator system. As is shown, the intermediate coil is a spiral coil and coaxially arranged regarding Tx and Rx self resonators in Figure 3b while the intermediate coil is perpendicular placed. The intermediate coil is placed in the middle of two coils. For the perpendicular intermediate system, the spiral intermediate coil is placed at x = 230 mm. The matching conditions are changed according to movement of the source and load coils. Therefore, the source and load coils are set up properly to get the critical coupling for the maximum power transfer efficiency.

Two helical coils ($r = 252$ mm, $H = 90$ mm, $N_{1,2} = 9$ turns, $a = 2.2$ mm) are made for Tx and Rx resonant coils. The intermediate coil is a spiral coil ($r_{in} = 230$ mm, $r_{out} = 300$ mm, $N = 10$ turns, $a = 3.2$ mm). The coils are made of copper pipe ($\sigma=5.8\times10^7$). To adjust the target resonant frequency of 1.25 MHz, a lumped high-Q capacitor is connected. Single loop coils are used for a source coil and a load coil.

To measure resonant frequency and transmission behaviour between Tx and Rx, a vector network analyzer (Agilent 4395A) is used. The measured intrinsic decay rates, Q-factors, lumped capacitances loaded, and resonant frequencies of the resonant coils are illustrated in Table I. By measuring splitting frequencies between Tx (or Rx) coil and the intermediate coil, the coupling coefficient, κ_m, is achieved.

TABLE Ⅰ: Specific parameters of self-resonant coils

	Γ	lumped C	Q	f_r
Tx	8168.14	221.10 pF	483.08	1.2560 MHz
Rx	8168.14	224.40 pF	481.73	1.2525 MHz
Intermediate	8325.22	233.60 pF	471.47	1.2494 MHz

Figure 4 shows the comparison between theoretical and measured efficiency results. Both the intrinsic decay rate of each coil and coupling coefficients measured are used in theoretical calculation. First, the calculated and measured results of the non-intermediate system are circular-marked solid line and the gray circles, respectively. Second, the square-marked and triangular-marked solid lines are theoretical results for the cases of the coaxially and perpendicular arranged intermediate systems, respectively while the gray squares and triangles are measured results. The comparison results for three cases show that the calculations are in good agreement with the measurements. Also, the efficiency of the coaxial intermediate system is the best at the same distance. The reason is that the coupling coefficient is the highest since the strongest magnetic field exists at the normal direction of the helical coil.

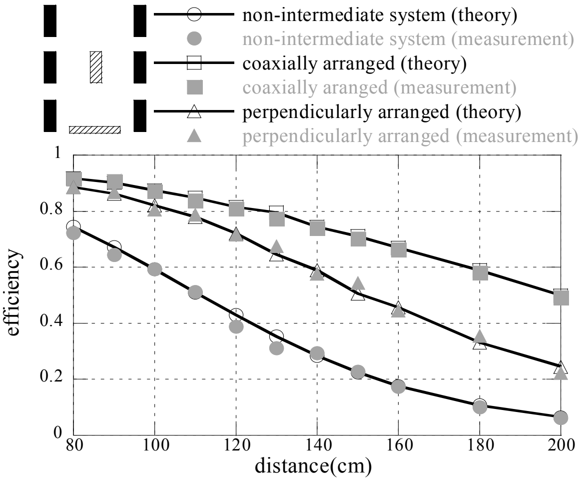

Figure 4: Efficiencies in cases of the coaxially and perpendicularly arranged intermediate systems and non-intermediate system.

4. Conclusion

The efficiency formula of magnetic resonance wireless power transfer with an intermediate resonant coil including a source coil and a load coil was derived and analyzed using a modified CMT. The theoretical calculations have a good agreement with measured results. It is shown that the efficiency is improved considerably in the cases of not only a coaxially arranged intermediate system but also a perpendicularly arranged intermediate one, comparing the non-intermediate system. It is also shown that the intermediate resonant system has a good efficiency and is superior to non-intermediate systems. Additionally, from a practical viewpoint, the perpendicularly arranged intermediate system can be widely used to extend the range of wireless power transfer and enhance the efficiency since the intermediate coil can be implemented adaptively in the space. The derived formulas can be also used for optimization of the efficiency in the intermediate system.

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