Comparison of Detection Probability for Conventional and Time-Reversal(TR) Radars

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Abstract

In this paper, we analyze the detection probability of the time-reversal radar system in both analytical and numerical ways. The comparison of the probabilities of the conventional and the TR systems is performed for a Rayleigh cluttered environment. Based on the comparison, we discuss the advantage and disadvantages of the TR detection scheme over the conventional system.

Keywords: Time-reversal, Detection Probability

1. Introduction

The acquisition of an image of the target hidden inside the cluttered environment has been the important issue especially for military purposes [1]. The performance of the conventional synthetic aperture radar(SAR) is not satisfactory to detect the target in a highly cluttered environment since the strong scattering of the background exists, which results in so-called spurious ghost images. To overcome this problem, several ways have been proposed. One of them is to use the time-reversal technique in a SAR system, known as TR-SAR [2]. The TR-SAR system can focus the transmitted energy at a specific spatial point with a help of the scattering of the background nearby the target. The TR-SAR images of the target and the clutter are overlapped at a point, which can be obtained after the image processing of the SAR raw data. We analyze the detection probability of the TR-SAR image and compare it with the probability of the conventional radar system.

2. Considered TR-SAR System

Unlike the conventional SAR system, the TR-SAR system receives the radar echoes, reverses them in time domain around the time-reversal reference point, and re-transmit them to the environment again. A TR-SAR image of the target can be obtained by applying a proper image processing algorithm such as RDA(Range Doppler Algorithm) to the finally received echoes. To simplify the SAR scattering problem, we regard the cluttered environment as a collection of point scatterers. Figure 1 shows the interaction of the transmitted signal with the point scatterers as the SAR system moves along the synthetic aperture line.

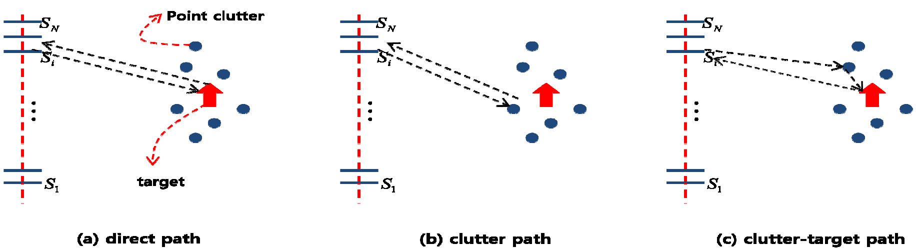


Figure 1: Interaction of signal with cluttered environment.

This TR-SAR system has one interesting property: the target-related components of the echo, (a) & (c) in Figure 1 and the clutter-related components of the echo, (b) in Figure 1 are overlapped at the time-reversal reference point. Hence, if we assume the LOS(Line-Of-Sight) component of the target, (a), i^{th} clutter LOS component, (b), and i^{th} clutter-target interaction, (c) as α , c_i and d_i , respectively. These scatterings can be modelled by complex random variables. The signal at the time-reversal reference point can be decomposed into three components and written as

$$|\alpha|^2 A(u - x_r) \cdot R\left(t - \frac{2}{c} R_r\right) \quad (1)$$

$$\sum_{i=1}^{N} |c_i|^2 A(u - x_r) \cdot R\left(t - \frac{2}{c} R_r\right) \quad (2)$$

$$\sum_{i=1}^{N} |d_i|^2 A(u - x_r) \cdot R\left(t - \frac{2}{c} R_r\right) \quad (3)$$

$A(\cdot)$ and $R(\cdot)$ are the point spread functions of the azimuth and the range directions, respectively. R_r and x_r are the coordinates of the time reversal reference point in the range and azimuth directions, respectively. *N* is the number of the point clutters in the environment. (1) is the signal component due to the target LOS component. (2) is the LOS component of the clutter. (3) is the signal component due to the clutter-target interaction. Thus, (1) and (3) can be considered as the signals from the target, while (2) is the component from the clutter (noise).

3. Detection Probability

3.1 **PDF of Signals for Conventional System**

In the conventional system, the signal components of Figure 1(b) & (c) can be considered as a noise. Hence, if we assume c_i & d_i are complex Gaussian random variables with zero mean and the variances, σ_c^2 & σ_t^2 ,respectively. The noise power can be calculated as $2\sigma_N^2$, and the noise power from the background can be written as below when no correlation between the real and the imaginary parts of the random variables is assumed.

$$2\sigma_N^2 = 2N\left(\sigma_c^2 + \sigma_t^2\right) \quad (4)$$

If we have a constant target signal, α , the pdfs of the signal amplitudes of the noise and the target plus the noise can be expressed as the Rayleigh and the Rician distributions, respectively, as

$$f_N(y) = \frac{y}{\sigma_N^2} \exp\left(-\frac{y^2}{2\sigma_N^2}\right) \quad (5)$$

$$f_{N+T}(y) = \frac{y}{\sigma_N^2} \exp\left(-\frac{y^2 + \alpha^2}{2\sigma_N^2}\right) I_0\left(\frac{y\alpha}{\sigma_N^2}\right) \quad (6)$$

where y is a random variable for the signal amplitude. $I_0(\cdot)$ is the 0^{th} order modified Bessel function.

3.2 **PDF of Signals for TR-SAR System**

In the TR-SAR system, as seen in Figure 1, the target-related amplitude, which is the sum of the amplitudes of (1) and (3), can be written as

$$y_1 = \alpha^2 + \sum_{i=1}^{2N} N_i^2\left(0, \sigma_t^2\right) \tag{7}$$

where $N(\cdot,\cdot)$ is the Gaussian random variable. The clutter-related amplitude, the amplitude of (2) in Figure 1, can be written as

$$y_2 = \sum_{i=1}^{2N} N_i^2\left(0, \sigma_c^2\right) \tag{8}$$

Since (7) and (8) is the sum of the squared Gaussian random variables, (7) and (8) become the χ^2 distribution. Therefore, the pdfs of y_1 and y_2 are explicitly given as

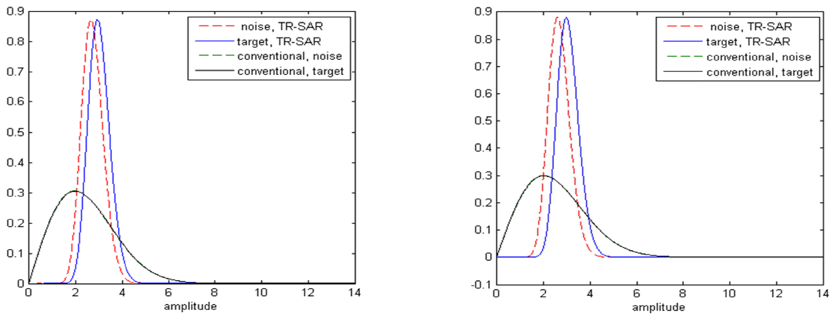
$$f_T(y_1) = \frac{1}{\sigma_t^2 2^N \Gamma(N)} \left(\frac{y_1 - \alpha^2}{\sigma_t^2}\right)^{N-1} e^{-\frac{y_1-\alpha^2}{2\sigma_t^2}}, \qquad y_1 \geq \alpha^2 \tag{9}$$

$$f_N(y_2) = \frac{1}{\sigma_c^2 2^N \Gamma(N)} \left(\frac{y_2}{\sigma_c^2}\right)^{N-1} e^{-\frac{y_2}{2\sigma_c^2}}, \qquad y_2 \geq 0 \tag{10}$$

In order to calculate the detection probability, the pdf of the signal plus the noise($y_1 + y_2$) is required. The pdf can be directly obtained by convolving (9) and (10). The calculation of that convolution is not easy. So, we numerically calculate the convolution in this paper.

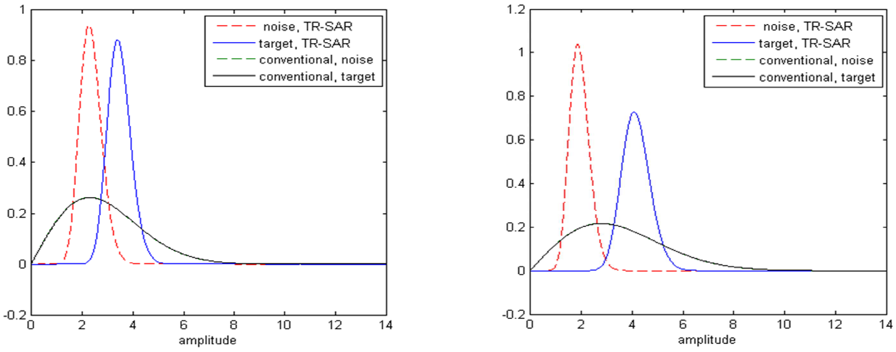
3.3 Comparison of Detection Probability

The time-reversal effect of the TR-SAR scheme mainly comes from the clutter-target interaction in Figure 1(c). Hence, by varying the ratio of the clutter-target interaction to the clutter LOS component, we can analyze the advantage of the TR-SAR system over the conventional system with respect to the detection probability. We calculate the pdf for the signal plus the noise of the TR-SAR system in four cases, $\sigma_t = 0$, $\sigma_t = 0.2\sigma_c$, $\sigma_t = 0.6\sigma_c$ and $\sigma_t = 1.0\sigma_c$. The powers of the conventional system and the TR-SAR system are set to be equal. σ_c is calculated from the RCS value of a forest environment [3].



(a) $\sigma_t = 0$ (b) $\sigma_t = 0.2\sigma_c$

Figure 2: Pdf of TR-SAR system vs. conventional system



(a) $\sigma_t = 0.6\sigma_c$ (b) $\sigma_t = 1.0\sigma_c$

Figure 3: Pdf of TR-SAR system vs. conventional system

For numerical calculations, the number of point clutter, N, is set to be 100. The SNR(Signal to Noise Ratio) of the conventional system is assumed as 0.01. As shown in Figure 2 and 3, the overlapping region of the pdf for the target plus the noise and the pdf for the noise only is decreased as σ_t increases. With a proper choice of the threshold amplitude, we can calculate the detection probability and the false alarm probability of the system. Here, we choose the intersection points of the two pdfs as the threshold amplitude. Table 1 shows the calculated the detection and false alarm probabilities. As observed in Table 1, the TR-SAR system can detect the target better than the conventional system.

Table 1: Detection and false alarm probabilities.

	TR-SAR system		Conventional system		
	P_D	P_{FA}	P_D	P_{FA}	SNR
$\sigma_t = 0.2\sigma_c$	0.7021	0.3794	0	0	0.0096
$\sigma_t = 0.6\sigma_c$	0.9136	0.1177	0	0	0.0074
$\sigma_t = 1.0\sigma_c$	0.9913	0.0099	0	0	0.0050

4. Conclusion

We analyze the detection probability and the false alarm probability of the TR-SAR system, and compare them with those of the conventional system in an analytical and a numerical way. We show that the TR-SAR system performs better than the conventional one in terms of detecting a target inside a cluttered background. Hence, the TR-SAR system can drastically increase the detection probability.

References

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