

**LOCALIZATION OF SOURCES IN THE FINITE DISTANCE
USING MUSIC ALGORITHM WITH THE SPHERICAL MODE VECTOR**

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1. Introduction

An undesired electromagnetic radiation power from a printed circuit board (PCB) becomes more and more serious according to the uses of high speed and nonlinear electronic circuit elements. A frequency band tends to spread widely as well by the same reason. Therefore, it is highly probable that the unexpected portions on the PCB become the sources of the undesired radiation. In order to take measures for reducing the radiation from the PCB, it is necessary to localize these sources.

The MUSIC algorithm is widely used to estimate the direction-of-arrival (DOA) for the waves come from far field. Moreover it has been reported that the original MUSIC algorithm is sufficiently applicable for the waves whose sources separates from the observation antenna more than 5 wavelengths. However, considering actual situations such as the power level of the wave from the PCB and the dynamic range of the equipment to measure, the localization method for more closer sources should be investigated.

This paper investigates the localization of the electromagnetic sources in a finite distance using the MUSIC algorithm with a spherical mode vector [1][2]. The localization procedure is performed in 2-dimensional for applying the PCB.

2. Spherical mode vector MUSIC algorithm

The geometry of problem is shown in Fig.1 a)~c). We assumed that the sources on the PCB are placed in x-y plane at z=z₀. The planar array [3] to observe the wave is also in the x-y plane. The received signal x_{n_x,n_y} of (n_x,n_y)th array from L sources is expressed as

$$x_{n_x, n_y} = \sum_{l=1}^L \frac{F_l}{d_{n_x, n_y}} \exp(-j\Phi_{n_x, n_y}) + w_{n_x, n_y} \tag{1}$$

where, $d_{n_x, n_y} = \sqrt{(x_l - dx_{n_x})^2 + (y_l - dy_{n_y})^2 + z_0^2}$ $z_0 = const.$ 2)

$$\Phi_{n_x, n_y} = \frac{2\pi(d_{n_x, n_y} - d_{n_x, n_y})}{\lambda} \tag{3}$$

F_l is the emitted amplitude from lth source, d_{n_x,n_y} is distance from wave source to (n_x,n_y)th array, d_{n_x,n_y} is distance from wave source to base element, dx_{n_x} dy_{n_y} is distance from base element to n_xth n_yth array element, Φ_{n_x,n_y} is phase delay. z₀ is a distance from array plane to wave source plane which is known apriori. Amplitude vector **F**, mode vector matrix of planar array **A**, the observed data **X** can be expressed as

$$\mathbf{X} = \mathbf{A}\mathbf{F} + \mathbf{W} \tag{4}$$

W: noise vector of N_x × N_y-dimension

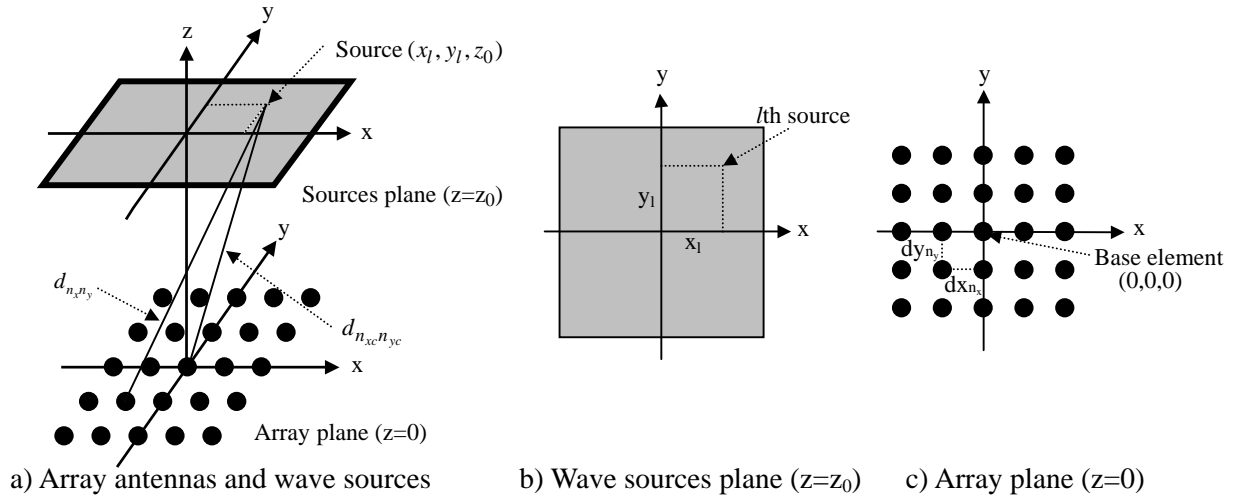


Fig.1 Planer array antenna and wave source

3. Spatial Smoothing for Spherical wave

The spatial smoothing is effectively used for the decorrelated coherent signals when the plane waves are incident to the uniformly spaced array. However, the original spatial smoothing technique can not be applied to the spherical waves studied in this paper by reason of the non-uniformity of the phase delay. In other words, the signals received by the uniform array correspond to the signals received by the non-uniform array. In this paper, the phase delays at all antenna are corrected so as to use the uniform array by using an interpolation matrix B [4][5] which is determined by $\Psi = \|BA - \bar{A}\|_F^2 \rightarrow 0$, where A is the mode matrix for the spherical waves, \bar{A} correspond to the mode matrix for plane wave, $\|D\|_F^2$ denotes Frobenius norm. The correlation matrix is interpolated by the matrix B as

$$R_{xx} = E[\mathbf{X}\mathbf{X}^H] = \mathbf{A}\mathbf{S}\mathbf{A}^H + \sigma^2\mathbf{I} \quad (5)$$

$$R_{xxl} = \mathbf{B}R_{xx}\mathbf{B}^H = \bar{\mathbf{A}}\mathbf{S}\bar{\mathbf{A}}^H + \sigma^2\mathbf{B}\mathbf{B}^H \quad (6)$$

where, $E[\bullet]$ is a statistical expectation, $[\bullet]^H$ is the Hermitian transpose, σ^2 is the thermal noise power and \mathbf{I} is a unit matrix. R_{xxl} can adapt spatial smoothing.

4. Incoherent spherical wave estimation

In this section we discuss the localization of two sources for the case of the incoherent waves. The antenna array was set 5×5 whose space antenna is $\lambda/2$ for the frequency at 600MHz. Fig.2 shows the MUSIC spectrum when the distances between the array plane and the PCB z_0 are λ and 2.5λ . In the examples SNR is 30dB. It is found that the locations of the sources are detected correctly. Fig.3 shows the estimated resolution between two sources in which one source is fixed at (0,0) and other sources is placed at the coordinate (x, y) on the PCB. It is found that those sources are estimated as one source when two sources are close each other. Its region is indicated as an inestimable-area in Fig.3. The maximum radii of the inestimable-area (simply indicated as DE hereafter) for $z_0=\lambda$ and 2.5λ is about 7 cm ($\lambda/7$) and about 10 cm ($\lambda/5$), respectively. Thus, the closer the array approaches the PCB, the higher the resolution is achieved. The concrete relation between the distances of the SNR is shown in Fig.4, which is indicated as DE.

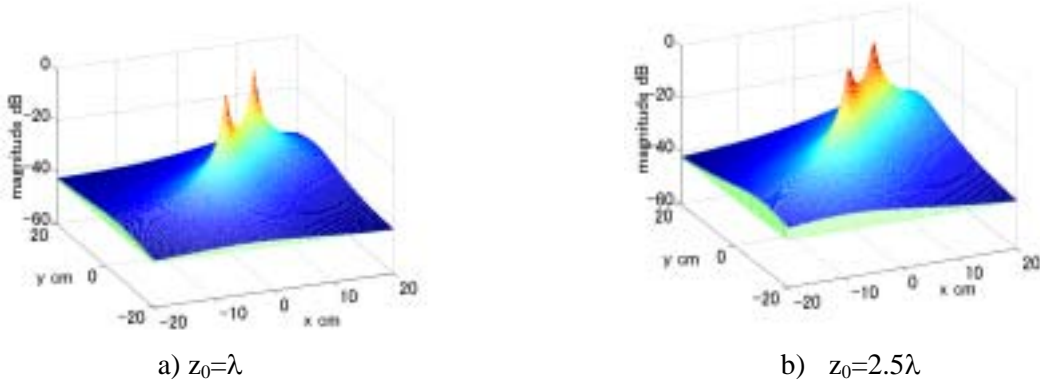


Fig.2 Example of estimation when the sources are located at (0,0) and (8,8)

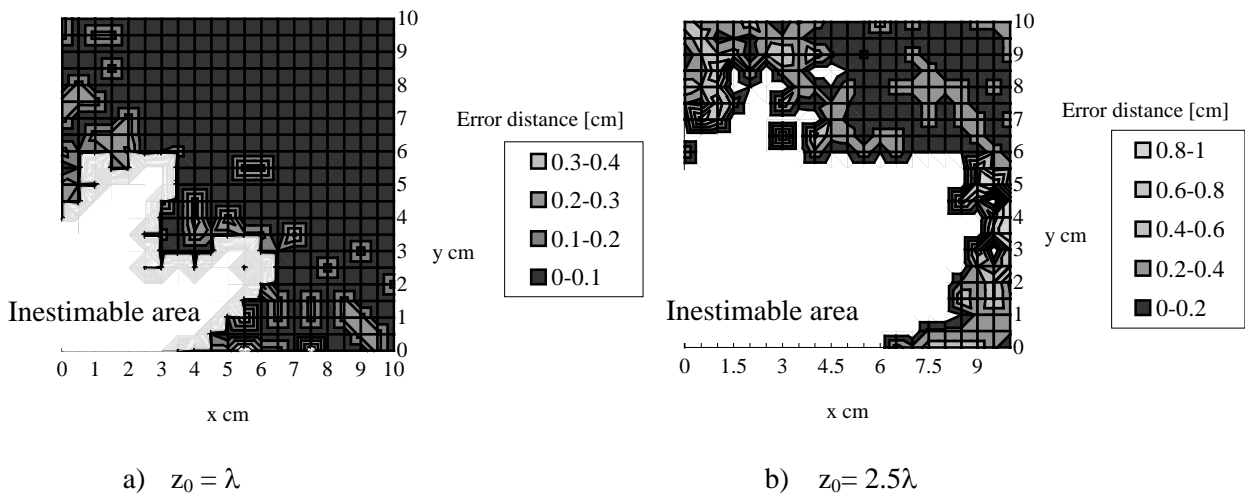


Fig.3 Estimation error with SNR 30dB

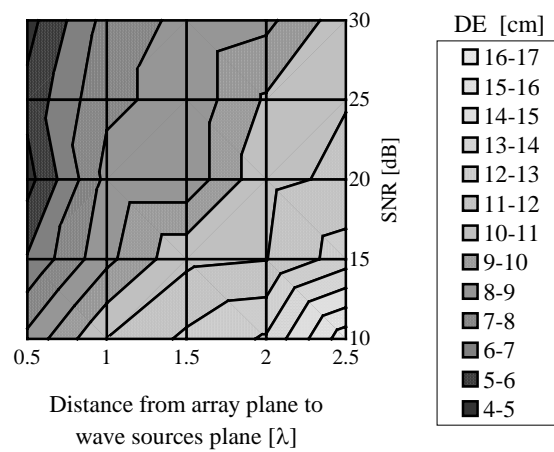


Fig.4 SNR vs. distance from array plane to wave sources plane

5. Coherent spherical wave estimation

In this section we discuss the localization of two sources for the case of the coherent waves. The antenna array was set 7×7 and the sub array element was set 5×5 whose space antenna is $\lambda/2$ for the frequency at 600MHz. Fig.5 shows the MUSIC spectrum when the distances between the array plane and the PCB z_0 are λ . In the examples there is no noise. It is found that the locations of the sources are detected correctly. Fig.5 shows the estimated resolution between two sources in which one source is fixed at (0,0) and other sources is placed at the coordinate (x, y) on the PCB. The DE for $z_0=\lambda$ is about 5cm ($\lambda/10$).

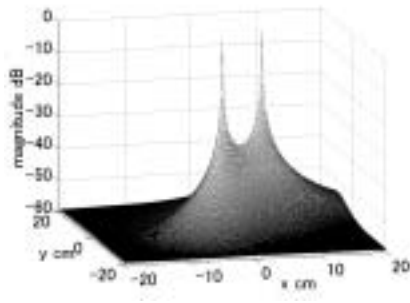


Fig5 Example of estimation where location of waves is (0,0), (8,8)

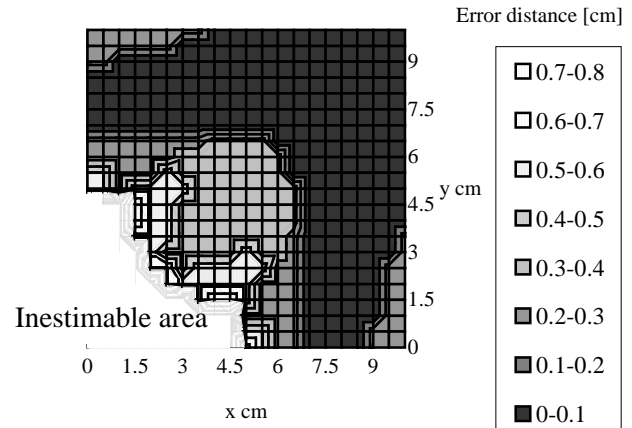


Fig6 Estimation error with no-noise

6. Conclusion

In this paper, we have shown that MUSIC algorithm is applicable for near-field estimation by using the spherical mode vector and interpolation. For the incoherent spherical wave estimation, the higher SNR and the closer z_0 give the higher resolution. Furthermore the coherent spherical wave estimation, it has been shown that the resolution of two sources is less than $\lambda/10$ when there is no noise. However, further investigation should be continued for noisy data multi-frequency estimation and so on.

References

- [1] H. Ohara, N. Kikuma, N. Inagaki, "3-Dimensional Localization of Near-Field Multiple Sources Using MUSIC with Circular Array," Proc. ISAP, pp.1085-1088, Aug. 1992
- [2] H. Akimoto, M. Takahashi, T. Uno, "Fundamental study for Estimation of sources in the finite distance by MUSIC Algorithm Using the Spherical mode vector," IEICE Technical Report, AP-2002-135, pp.7-12, Sep.2002
- [3] S. Sekizawa, "Study on Estimation of Arrival Direction Using MUSIC Algorithm with Planar Array," IEICE Technical Report, AP97-117, pp.21-28, Oct. 1997
- [4] B. Friedlander, A. J. Weiss, "Direction Finding Using Spatial Smoothing with Interpolated Arrays," IEEE Trans. Aerospace & Electronic Systems, vol.28, no.2, pp.574-587, Apr. 1992
- [5] A. Okamura, R. Yonezawa, T. Kirimoto, "Direction Finding by Spatial Smoothing Super Resolution Algorithm Using Window Function for Array Interpolation," IEICE Trans., vol.J82-B, no.6, pp.1185-1192, Jun. 1999