EM Scattering from Gauss Rough Surface Through Wavelet Method

Lu Guizhen Feng Feng Li Yanjun and Ning Yuemin
Beijing Broadcasting Institute
#1,Ding Fuzhuang East Street, ChaoYang, Beijing, 100024, P.R.China
E-mail : luguizhen@bbi.edu.cn

Abstract: In this paper, the scattering from Gauss rough surface is studied through wavelet method. The matrix equation to discrete integral equation is reduced to small size matrix equation by using two-scale formulation. After solving the small size matrix equation , the solution of original equation can be constructed by using two-scale formulation. Through the method, the size of the matrix can be reduced to a half of original one. The computed results are compared with one that is solve by direct matrix inverse method. The results show that the agreement is very good.

Key Words: EM Scattering; Moment Method; Wavelet; Rough Surface

1. INTRODUCTION

In EM Scattering, integral equations are usually used in the study of the EM Scattering from objects. Generally, integral equations are more effective in reducing the number of unknowns than differential equations. And the solutions to integral equations are much more stable. The wavelet method has been widely paid attention to in recent ten years. At present, the wavelet method is mainly used in signal processing. In EM Scattering, it is used to reduce the size of the matrix. In this paper, the author develops a new method based on Wavelet. In the method, the matrix equation can be reduced to two small sized matrix equations, which are more easily to solve.

2. THEORY

Based on the theory of multiresolution, the orthogonal wavelet basic functions can be constructed through a scale functions. Here, the scale function is selected as

$$\varphi(t)=\begin{cases}1 & 0\le t<1\\ 0 & others\end{cases} \qquad 1••$$

Obviously, the function set of $\{\varphi(t-k)\}$, which is constituted by the displacement of the scale function, forms the orthogonal basis. The wavelet function and the scale function of the next scale can be obtained through the following formula by the two-scale equation.

$$\varphi_1(\frac{t}{2})=\sqrt{2}\cdot\sum_n h_0(n)\cdot\varphi(t-n)$$
$$\psi_1(\frac{t}{2})=\sqrt{2}\cdot\sum_n h_1(n)\cdot\varphi(t-n) \qquad 2••$$

where $h_0(n)$• •$h_1(n)$ are the coefficients of filter for wavelet functions.

In the study of EM scattering from one-dimensional random rough surfaces with Dirichlet boundary condition, the integral equation is:

$$E_{inc}(\vec{r})=\int_S dx' G(\vec{r},\vec{r}')\cdot u(\vec{r}') \qquad 3••$$

where $u(\vec{r}) = \sqrt{1+\left(\frac{df}{dx}\right)^2} \cdot \frac{\partial E}{\partial n}$• •$G(\vec{r},\vec{r}\,') = \frac{i}{4} H_0^1(k|\vec{r}-\vec{r}\,'|)$ 4• •

The unknown function in the integrand is expanded by a set of basis functions in order to transform the integral equation into matrix equation. If the discretization interval is Δ, the scale functions which has the form of formula 1• are used to be the expansion basis functions,
The matrix equation is obtained as

$$[Z][\vec{X}] = [\vec{C}] \quad 5\text{• •}$$

And the dimension of the matrix is N. Now by using the two-scale relationship in the scale functions and wavelet function, the matrix equation can be decoupled into several small sized matrix equations. For the case of the equation 8), let the scale of the expansion basis function in 8) be doubled and another wavelet basis function in the same scale be used as expansion basis functions. The two sets of basis functions have the dimension N/2. Using the two set of basis function as basis functions, the two matrix equation with dimension N/2 are obtained.

In order to gain the matrix equation of the moment-method , the • •function is taken as weight function. However, the scope of support of basis function is 2Δ with respect to the case of the scale 0 function as basis function ,the weight function is taken as

$$v_m(\vec{r}) = \frac{1}{2}\{\delta(\vec{r}-\vec{r}_{m,1}) + \delta(\vec{r}-\vec{r}_{m,2})\} \quad 6\text{• •}$$

In wavelet space, the weight function is taken as

$$w_m(\vec{r}) = \frac{1}{2}\{\delta(\vec{r}-\vec{r}_{m,1}) - \delta(\vec{r}-\vec{r}_{m,2})\} \quad 7\text{• •}$$

And there are

$$E_{inc}(\vec{r}_m) = \sum_1^{N/2} c_n^1 G(\vec{r}_m,\vec{r}_n)\cdot(2\Delta) \quad 8\text{• •}$$

And,

$$[E_{inc}(\vec{r}_{m,1}) - E_{inc}(\vec{r}_{m,2})] = \sum_1^{N/2} d_n^1 [G(\vec{r}_{m,1},\vec{r}_n) - G(\vec{r}_{m,2},\vec{r}_n)]\cdot(2\Delta) \quad 9\text{• •}$$

From 8) and 9), it can be seen that $\{c_n^1\}$ is a solution for scale function with scale 1 as expansion basis and $\{d_n^1\}$ is a solution for wavelet function with scale 1as expansion basis. Since the solution $\{c_n^0\}$ with scale 0 is more precise than solution $\{c_n^1\}$ with scale 1, it is hoped that $\{c_n^0\}$ can be obtained from the solution $\{c_n^1\}$ with scale 1 and the solution $\{d_n^1\}$ with scale 1. In wavelet function, there are two scale relationship .For the Haar Wavelet, there exist

$$c_{2m}^0 = \frac{1}{\sqrt{2}}\left(c_m^1 + d_m^1\right) \qquad m = 1,\cdots,\frac{N}{2}$$
$$c_{2m+1}^0 = \frac{1}{\sqrt{2}}\left(c_m^1 - d_m^1\right) \qquad m = 1,\cdots,\frac{N}{2} \quad 10\text{• •}$$

So that, the solution $\{c_n^0\}$ with scale 0 can be obtained from $\{c_n^1\}$ with scale 1 and the solution $\{d_n^1\}$ with scale 1. And the size of solution of matrix equation is only a half of matrix equation for $\{c_n^0\}$.

3. RESULTS AND DISCUSSION

Using the formula developed in the above section, the numerical computing of EM scattering from Guass Rough Surface is performed. In numerical calculation, the correlation length of Guass Rough Surface is $1\ \lambda$ (λ denotes wavelength), and RMS height is 0.1λ. In the process of calculating, the total length of Guass Rough Surface is $25\lambda'$ .The Gauss Random Rough Surface is shown in Fig.1

In order to compare, a conventional MM solution is computed. In MM computing, the rough surface is discretized into 512Δ (Δ denotes discretization interval). Each interval is equal to 0.05λ approximately, that meets the discretization requirement provided by moment-method. In the proposed method of this paper, the surface is discretized into 256Δ, and each interval is equal to 0.1λ that meets the discretization requirement as well. It can be seen the size of matrix equation is only a half of original one. So the computing time and computing cost are saved.

The result of the computing of EM scattering from Guass Rough Surface is shown as Fig.2 and Fig.3. In Fig.2, the incidence angel between incidence direction and surface normal is 20°. In Fig.3, the incidence angel between incidence direction and surface normal is 70°. From the two figures, the calculated results show the agreement between wavelet method and moment-method.

From formula 23), it can be seen that the results from moment-method scale 0 can be viewed as the results from moment-method with scale 1 plus with detail modification with scale 1. Hence, the numerical solution of scattering with large scale can be used to construct to form solution with small scale. This method is very effective when the number of variable is very large.

REFERENCES

[1] Lu Gui-zhen, *The Scattering of EM From Rough Surfaces [D]*, Beijing: Beijing University of Aeronautics and Astronautics, 2001.

[2] Peng Yu-hua, *Wavelet Transformation and Appliance in Engineering [M],* Beijing: Science Press, 2002.

[3] D.J.Donohue, etc, *Application of Iterative Moment-Method Solutions to Ocean Surface Radar Scattering [J], IEEE Trans. Antenna & Propagation, 1998*,46, (1), pp121-131

[4] Kyungjung Kim, etc *Solution of Large Dense Complex Matrix Equations Using a Fast Fourier Transform (FFT)-Based Wavelet-Like Methodology [J], IEEE Trans. Antenna & Propagation,* 2002, 50 (30), pp277-283

[5] Lu Gui-zhen, *EM scattering from Guass Rough Surface [J], Acta Electronical Sinica,* 2002,30(6)

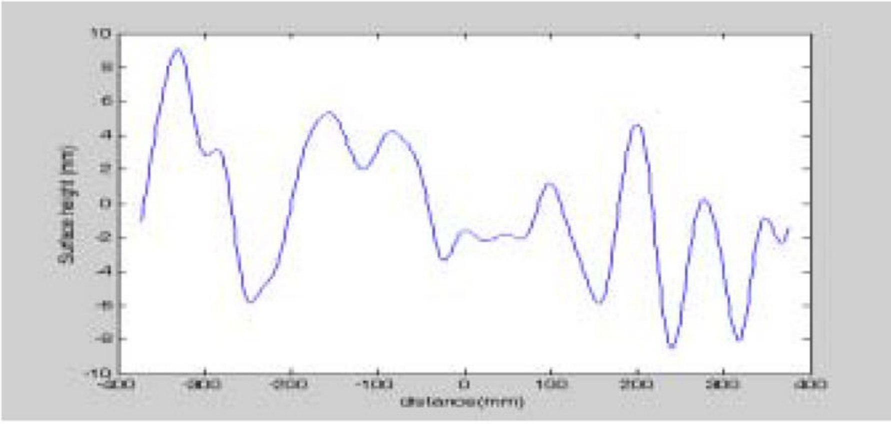

Fig.1. Guass Rough Surface. Correlation length is 1λ and rms height is 0.1λ

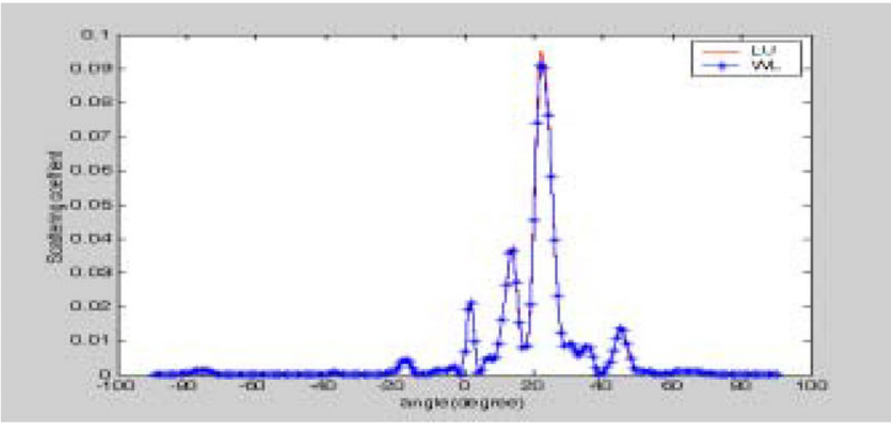

Fig.2. The result of the coefficient of EM scattering from Guass Rough Surface when the angel between incidence and normal is 20°. '----LU' denotes the solution of Moment-Method of small scale; '---WL' denotes the solution of Moment-Method using Wavelet Method

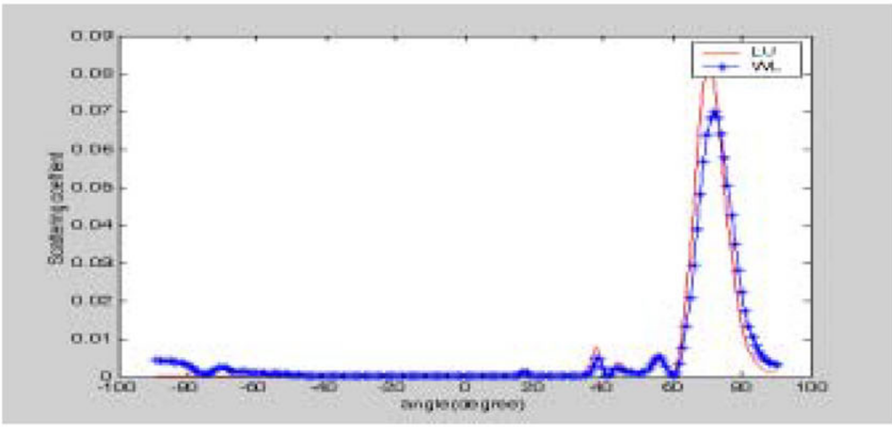

Fig.3. The result of the coefficient of EM scattering from Guass Rough Surface when the angel between incidence and normal is 70°. '----LU' denotes the solution of Moment-Method of small scale; '---WL' denotes the solution of Moment-Method using Wavelet Method