A NEW REGULARIZATION METHOD FOR INVERSE SCATTERING OF DIELECTRIC CYLINDERS

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1. Introduction

Microwave imaging of internal properties, shape and location of a scattering object remains one of the most important and challenging problems in electromagnetics due to its practical applications in biomedical diagnostics, nondestructive testing of materials, and detection of buried objects. Many inversion methods have been proposed to solve the two- and three-dimensional inverse scattering problems [1]-[15]. In most of these techniques, a regularization method based on the minimization of a cost fuctional has been employed to circumvent the ill-posedness of the problem.

In this paper, we present a new regularization method of accelerating an iterative inversion algorithm of reconstructing the relative permittivity of a dielectric cylinder. The object located in a homogeneous background medium is assumed to be illuminated with multifrequency cylindrical waves. The inverse scattering problem of interest here can be formulated as the solution to a nonlinear integral equation for a contrast function, which is related to the relative permittivity of the object. A cost functional is defined as the sum of a residual error term in the scattered electric field and an additional regularization term. Thus the inverse scattering problem can be treated as an optimization problem where the contrast function is found by minimizing the functional. The conjugate gradient method and the frequency-hopping technique [3] are applied to the optimization problem. The regularization parameter is determined by minimizing the absolute value of the radius of curvature of the generalized cross-validation (GCV) function [10]. Numerical results are given for dielectric circular cylinders to show the comparison of the rates of convergence obtained for our regularization method and the conventional one.

2. Theory

Consider a dielectric cylinder of relative permittivity $\epsilon_s(\boldsymbol{\rho})$, which is located in a homogeneous background medium of relative permittivity ϵ_b. The object with cross section Ω is assumed to be infinitely long and its axis is in the z-direction. TM cylindrical waves with electric field $\mathbf{E}_p^i$ $(= \mathbf{u}_z E_p^i(\theta;\, \boldsymbol{\rho}))$ corresponding to frequency f_p illuminate the object, where $\mathbf{u}_z$ is the unit vector in the z-direction and $p = 1,\, 2,\, \cdots, P$. Line sources generating the incident waves are placed at points with polar coordinates $(\rho,\, \theta + \pi)$. For each illumination, measurements of the scattered electric field $\mathbf{E}_p^s$ $(= \mathbf{u}_z E_p^s(\theta;\, \boldsymbol{\rho}))$ are made at the observation points with polar coordinates $(\rho,\, \phi)$. The situation of the prob-

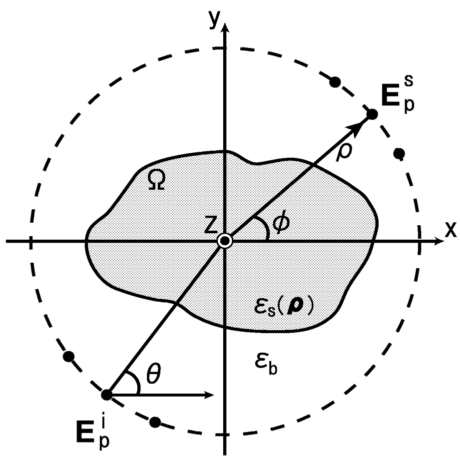

Fig. 1 Situation of the problem.

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lem is illustrated in Fig. 1. The material property of the object is characterized by a contrast function,

$$c(\boldsymbol{\rho}) = \epsilon_s(\boldsymbol{\rho}) - \epsilon_b. \tag{1}$$

As is well known [10], the inverse scattering problem can be formulated as the solution to the following nonlinear integral equation for the contrast function:

$$E_p^s(c;\, \theta;\, \boldsymbol{\rho}) = k_p^2 \iint_\Omega c(\boldsymbol{\rho}')\, E_p^t(c;\, \theta;\, \boldsymbol{\rho}') G_p(\boldsymbol{\rho};\, \boldsymbol{\rho}')\, d\boldsymbol{\rho}', \quad \boldsymbol{\rho} \in \overline{\Omega} \tag{2}$$

where $\overline{\Omega}$ denotes a domain outside of Ω, k_p is a free-space wavenumber for the frequency f_p, and $G_p(\boldsymbol{\rho};\, \boldsymbol{\rho}')$ represents the two-dimensional Green's function for the background medium given by

$$G_p(\boldsymbol{\rho};\, \boldsymbol{\rho}') = -\frac{j}{4}\, H_0^{(2)}(\sqrt{\epsilon_b}\, k_p\, |\boldsymbol{\rho} - \boldsymbol{\rho}'|) \tag{3}$$

where $H_0^{(2)}$ is the zeroth-order Hankel function of the second kind. The total electric field $E_p^t(c;\, \theta;\, \boldsymbol{\rho})$ inside the object, which is written as the sum of the incident electric field $E_p^i(\theta;\, \boldsymbol{\rho})$ and the resultant scattered electric field $E_p^s(c;\, \theta;\, \boldsymbol{\rho})$, satisfies the linear integral equation,

$$E_p^t(c;\, \theta;\, \boldsymbol{\rho}) = E_p^i(\theta;\, \boldsymbol{\rho}) + k_p^2 \iint_\Omega c(\boldsymbol{\rho}') E_p^t(c;\, \theta;\, \boldsymbol{\rho}')\, G_p(\boldsymbol{\rho};\, \boldsymbol{\rho}')\, d\boldsymbol{\rho}', \quad \boldsymbol{\rho} \in \Omega. \tag{4}$$

The method of moments with pulse-basis functions and point matching [16] is employed to discretize Eqs. (2) and (4).

Now the line sources generating the incident electric field and the observation points of the scattered electric field are placed at the positions with polar angles $\theta = \theta_l$ and $\phi = \phi_m$, where $l = 1, 2, \cdots, L$ and $m = 1, 2, \cdots, M$. Let us define the cost functional,

$$F(c) = \sum_{l=1}^{L} \sum_{m=1}^{M} \left|\, E_p^s(c;\, \theta_l;\, \phi_m) - \tilde{E}_p^s(\theta_l;\, \phi_m)\, \right|^2 + \alpha \iint_\Omega \left|\, c(\boldsymbol{\rho}) - c_-(\boldsymbol{\rho})\, \right|^2 d\boldsymbol{\rho} \tag{5}$$

where $\tilde{E}_p^s(\theta_l;\, \phi_m)$ and $E_p^s(c;\, \theta_l;\, \phi_m)$ denote the scattered electric fields measured and calculated for an estimated contrast function, respectively. Note that the scattered electric field measured is simulated from the true contrast function. Furthermore, α is a regularization parameter, and $c_-(\boldsymbol{\rho})$ is the function which is updated when a reduction in the residual error in the scattered electric field becomes very small. The regularization parameter is now determined by minimizing the absolute value of the radius of curvature of the GCV function $g(\alpha)$ [10],

$$R_g(\alpha) = \frac{[\,1 + g'(\alpha)^2\,]^{\frac{3}{2}}}{|\, g''(\alpha)\,|}. \tag{6}$$

Introducing the functional $F(c)$, the inverse scattering problem is reduced to an optimization problem where $c(\boldsymbol{\rho})$ is found by minimizing $F(c)$. The conjugate gradient method and the frequency-hopping technique [3] are applied to the minimization of $F(c)$. The gradient of the functional can be derived from the Fréchet derivative of $F(c)$ [4]. Then the $(q+1)$-th estimate $c^{(q+1)}(\boldsymbol{\rho})$ of the contrast function is iteratively obtained from

$$c^{(q+1)}(\boldsymbol{\rho}) = c^{(q)}(\boldsymbol{\rho}) + \lambda^{(q)} d^{(q)}(\boldsymbol{\rho}). \tag{7}$$

The direction $d^{(q)}(\boldsymbol{\rho})$ may be obtained from the gradient of $F(c)$ through the Polak-Ribière-Polyak method [17], and the step size $\lambda^{(q)}$ is determined by using the Davies-Swann-Campey method [17]. The iteration terminates if the relative residual error δ in the scattered electric field is finally less than a prescribed convergence criterion, where δ is defined by the first term of the right side of Eq. (5) normalized by the norm of the scattered electric field measured.

3. Numerical examples

Numerical results are obtained for dielectric circular cylinders using the multifrequency scattering data in microwave region. The numerical simulations are performed for the parameters normalized by the wavelength λ for the highest frequency in the background medium, which is now assumed to be free space. 36 positions of line sources and 36 measurement points for each illumination are uniformly distributed along a circle of radius 2λ. We employ four frequencies, f_1=1.5GHz, f_2=3.0GHz, f_3=4.5GHz, and f_4=6.0GHz, in the frequency-hopping technique. The current frequency is changed to the next higher-frequency when the difference between δs obtained at the current and the next iterations takes the values less than 10^{-2} two times in succession. The $2\lambda \times 2\lambda$ square domain containing the object and the background medium is uniformly subdivided into 32×32 elementary square cells. Now the initial guess of the contrast function is zero, i.e., the relative permittivity of the object is the same as that of the background medium.

Let us consider the reconstruction of an object with the relative permittivity of 5.0 and the radius of 0.8λ. Figure 2 illustrates the value of δ versus the number of iterations. The solid and the dotted lines present the results obtained for our regularization method (method I) and the conventional one (method II). Figure 3 shows the reconstructed results of the relative permittivity after 189 and 228 iterations for the method I and the method II, where $\delta = 10^{-4}$. For reference, the true profile of the relative permittivity is also depicted by the thin solid line in Fig. 3.

Figures 4 and 5 present the results for an object with the relative permittivity of 4.0 and the radius of 0.8λ. These results are obtained from the scattering data with signal-to-noise ratio (SNR) of -20dB. The final convergent solutions for $\delta = 10^{-2}$ in Fig. 5 are the results after 64 and 78 iterations corresponding to the method I and the method II.

It is seen from Figs. 2$-$5 that the rate of convergence obtained for our regularization method is faster than that for the conventional one to achieve the same accuracy in the reconstructed profile.

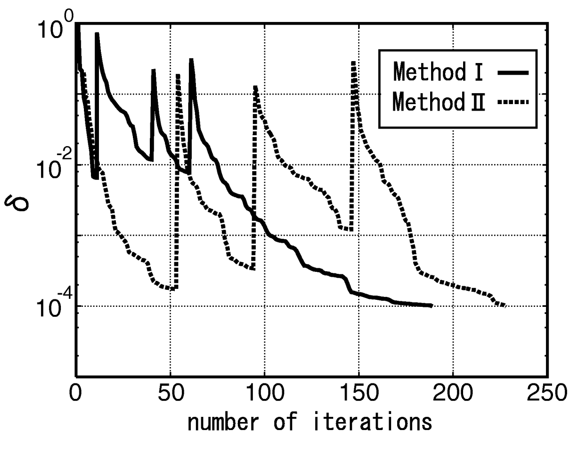

Fig. 2 Relative residual errors in the scattered electric field obtained for noise-free case.

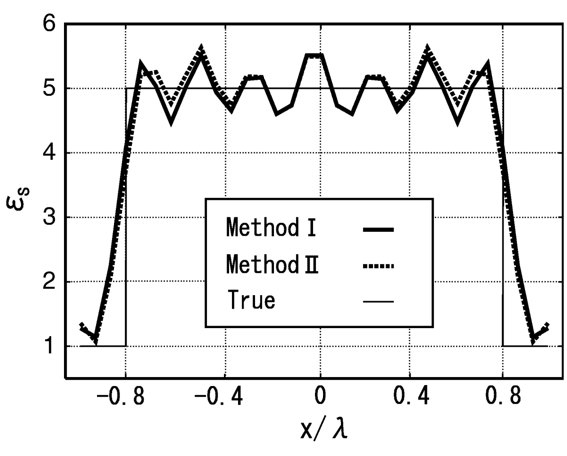

Fig. 3 Reconstructed results of the relative permittivity obtained for noise-free case.

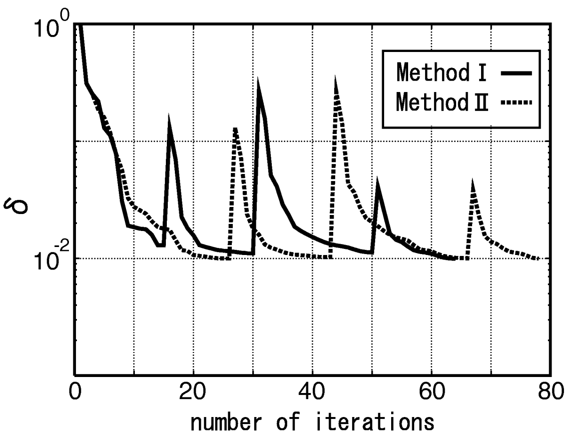

Fig. 4 Relative residual errors in the scattered electric field obtained for SNR=−20dB.

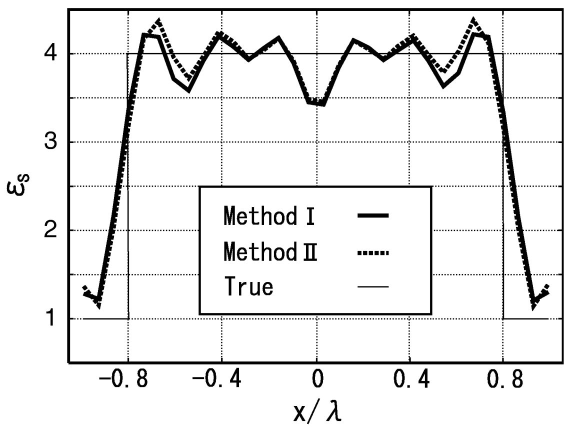

Fig. 5 Reconstructed results of the relative permittivity obtained for SNR=−20dB.

4. Conclusion

A new regularization method, which accelerates an iterative inversion algorithm of reconstructing the relative permittivity of a dielectric cylinder, has been presented. The algorithm is based on the conjugate gradient method and the frequency-hopping technique. The regularization parameter has been determined by minimizing the absolute value of the radius of curvature of the GCV function. The effect of measurement error in the scattering data on the reconstructed result has been also considered. It is confirmed from the numerical results for dielectric circular cylinders that the rate of convergence obtained for our regularization method is faster than that for the conventional one.

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References

[1] Y. M. Wang et al., *Radio Sci.*, vol. 27, no. 2, pp. 109–116, 1992.
[2] R. E. Kleinman et al., *J. Comp. Appl. Math.*, vol. 42, pp. 17–35, 1992.
[3] W. C. Chew et al., *IEEE Microwave Guided Wave Lett.*, vol. 5, no. 12, pp. 439–441, 1995.
[4] H. Harada et al., *IEEE Trans. Antennas Propagat.*, vol. 43, no. 8, pp. 784–792, 1995.
[5] S. Caorsi et al., *IEEE Trans. Microwave Theory Tech.*, vol. 43, no. 3, pp. 632–640, 1995.
[6] C.-S. Park et al., *Radio Sci.*, vol. 31, no. 6, pp. 1877–1886, 1996.
[7] A. Franchois et al., *IEEE Trans. Antennas Propagat.*, vol. 45, no. 2, pp. 203–215, 1997.
[8] P. Lobel et al., *Int. J. Imag. Syst. Technol.*, vol. 8, pp. 337–342, 1997.
[9] Z.-Q. Meng et al., *J. Electromag. Waves Appl.*, vol. 13, no. 1, pp. 95–118, 1999.
[10] M. Tanaka et al, *IEICE Trans. Commun.*, vol. E84-B, no. 9, pp. 2560–2565, 2001.
[11] H. Harada et al., *Microwave Opt. Technol. Lett.*, vol. 29, no. 5, pp. 332–336, 2001.
[12] A. Qing, *IEEE Trans. Antennas Propagat.*, vol. 50, no. 12, pp. 1786–1794, 2002.
[13] A. Abubakar et al., *IEEE Trans. Microwave Theory Tech.*, vol. 50, no. 7, pp. 1761–1770, 2002.
[14] S. Caorsi et al., *IEEE Trans. Microwave Theory Tech.*, vol. 51, no. 4, pp. 1162–1173, 2003.
[15] R. Ferrayé et al., *IEEE Trans. Antennas Propagat.*, vol. 51, no. 5, pp. 1100–1113, 2003.
[16] R. F. Harrington, *Field Computation by Moment Methods*, Macmillan, New York, 1968.
[17] S. L. S. Jacoby et al., *Iterative Methods for Nonlinear Optimization Problems*, Prentice-Hall, Englewood Cliffs, 1972.