THEORY ANALYSIS OF ANTENNA SCATTERING

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1. Introduction

Antennas are important contributors to the overall radar cross-section (RCS) signature of an airborne vehicle, and engineers are generally required to study the antenna RCS and RCS reduction. Antenna is a special scatterer and its scattering is related to the feed load. When the feed port is match loaded, the scattering is structural mode scattering. If not, part of the energy is reradiated which is antenna mode scattering. There are various definitions of antenna scattering. A new equation is proposed and the antenna scattering is analyzed in detail.

2. Analysis Process:

2.1 The general scattering theory

The scattering matrix of antenna element is [1]:

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ \vdots \end{bmatrix} = \begin{bmatrix} S_{00} & S_{01} & S_{02} & \cdots & \cdots \\ S_{10} & S_{11} & S_{12} & \cdots & \cdots \\ S_{20} & S_{21} & S_{22} & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ \vdots \end{bmatrix} \tag{1}$$

The following can be got from the above equation.

$$b_0 = \sum_{j=0}^{\infty} S_{0j} a_j = S_{00} a_0 + \sum_{j=1}^{\infty} S_{0j} a_j \tag{2}$$

$$b_i = \sum_{j=0}^{\infty} S_{ij} a_j = S_{i0} a_0 + \sum_{j=1}^{\infty} S_{0j} a_j \tag{3}$$

For the antenna port,

$$a_0 = \Gamma_l b_0 \tag{4}$$

Where the reflection coefficient is $\Gamma_l = \dfrac{Z_l - Z_c}{Z_l - Z_c}$

Inserting (4) into (2) gives

$$b_0 = \frac{1}{1 - \Gamma_l S_{00}} \sum_{j=1}^{\infty} S_{0j} a_j = \frac{1}{1 - \Gamma_l S_{00}} b_0^m \tag{5}$$

Where $$b_0^m = \sum_{j=1}^{\infty} S_{0j} a_j \tag{6}$$

Also inserting (4) into (3) yields

$$b_i = b_i^m + \frac{\Gamma_l}{1-\Gamma_l S_{00}} b_0^m S_{i0} \tag{7}$$

Where $$b_i^m = \sum_{j=1}^{\infty} S_{ij} a_j \tag{8}$$

Equation (7) is expressed with field as

$$\overline{E}^s(Z_l) = \overline{E}^s(Z_c) + \frac{\Gamma_l}{1-\Gamma_l S_{00}} b_0^m \overline{E}_1^t(Z_c) \tag{9}$$

where $S_{00} = \Gamma_a = \frac{Z_{in} - Z_c}{Z_{in} + Z_c}$ is the reflection coefficient of antenna. From equation (9), we can see that the scattering field is composed of two components. The first part is the structural mode component under match load and the second antenna mode component, which is related to the antenna character.

2.2 The antenna mode scattering component

With some equations from reference [2], we can deduce and get

$$b_0^m = -j\lambda(\frac{c}{a_1})\left|\vec{A}(\vec{\kappa}_i)\right|(\vec{U} \cdot \vec{u}^*) \tag{10}$$

From the above process, we can see some ways to reduce the antenna mode RCS.

1) $\Gamma_L = 0$; 2) $b_0^m = 0$.

There are two kinds of situations for case 2.

(1) $\left|\vec{A}(\vec{\kappa}_i)\right| = 0$; This is the null points of radiation pattern.

(2) $\vec{U} \cdot \vec{u}^* = 0$; When the polarization of the incident wave is orthogonal to the polarization of the receiving antenna, the energy to enter the antenna feed is zero.

2.3 Other antenna scattering definitions

From equation (9), when the load is short, there is

$$\vec{E}^s(0) = \vec{E}^s(Z_c) - \frac{1}{1+\Gamma_a} b_0^m \vec{E}_1^t(Z_c) \tag{11}$$

For the situation under arbitrary load

$$\vec{E}^s(Z_l) = \vec{E}^s(0) + \{\frac{\Gamma_l}{1-\Gamma_l\Gamma_a} + \frac{1}{1+\Gamma_a}\} b_0^m \vec{E}_1^t(Z_c) \tag{12}$$

By use of the relation between unit amplitude source and unit current source

$$F_3 = \frac{\sqrt{Z_c}}{1-\Gamma} \frac{I_3}{a_1} F_1 \tag{13}$$

we can get

$$\vec{E}^s(Z_l) = \vec{E}^s(0) + \left\{ \frac{\Gamma_l}{1-\Gamma_l\Gamma_a} + \frac{1}{1+\Gamma_a} \right\} b_0^m (1-\Gamma_a)\sqrt{Y_c}\,\vec{E}_3^t \tag{14}$$

where $\vec{E}_3^t$ is the radiation field under unit current source.

According to $$b_0^m = -\frac{1}{2}(1+\Gamma_a)\sqrt{Z_c}\,I(0) \tag{15}$$

where $I(0)$ is the receiving current under short load, we can get

$$\vec{E}^s(Z_l) = \vec{E}^s(0) - \frac{1}{2}\frac{(1+\Gamma_l)(1-\Gamma_a)}{1-\Gamma_l\Gamma_a} I(0)\vec{E}_3^t \tag{16}$$

and $$\vec{E}^s(Z_l) = \vec{E}^s(0) - \frac{Z_l}{Z_{in}+Z_l} I(0)\vec{E}_3^t \tag{17}$$

This is one of the fundamental scattering equations defined by Collin[3] under short load.
From above, there is

$$\vec{E}^s(0) = \vec{E}^s(Z_c) + \frac{Z_c}{Z_{in}+Z_c} I(0)\vec{E}_3^t \tag{18}$$

$$\vec{E}^s(Z_{in}) = \vec{E}^s(0) - \frac{1}{2} I(0)\vec{E}_3^t \tag{19}$$

Then we can get

$$\vec{E}^s(Z_l) = \vec{E}^s(Z_{in}) + \Gamma_A\ I_m\vec{E}_3^t \tag{20}$$

where $$\Gamma_A = \frac{Z_{in}-Z_l}{Z_{in}+Z_l} \tag{21}$$

$$I_m = \frac{1}{2} I(0) \tag{22}$$

This is the classical scattering definition by Hansen [4] under match load.
In a similar way, we can get

$$\vec{E}^s(Z_l) = \vec{E}^s(Z_{in}^*) + \tilde{\Gamma}_A\ \tilde{I}_m\vec{E}_3^t \tag{23}$$

where $$\tilde{\Gamma}_A = \frac{Z_{in}^* - Z_l}{Z_{in}+Z_l} \tag{24}$$

$$\tilde{I}_m = \frac{Z_{in}}{2R_a} I(0) \tag{25}$$

That is the scattering definition by Green [5] under complex conjugate match.
From the above derivation, we can see that the classical definitions can be deduced from the proposed equation, which proves that they are all valid. But the proposed equation here is more apparent for understanding the antenna scattering fundamentals, from which some methods to reduce antenna RCS are

obtained.

2.4 Antenna array scattering

When the above process is used to analyze antenna array, we give the expression of antenna mode component with antenna efficient length. For incident wave of $\vec{k}_1$ and scattering wave of $\vec{k}_2$, the antenna mode is expressed as follows

$$\vec{E}^a(Z_l) = \frac{-j\lambda\Gamma_l}{1-\Gamma_l\Gamma_a} C\left[\vec{A}(-\vec{k}_1)\cdot\vec{u}^*\right]\sqrt{Z}\vec{A}(\vec{k}_2)\frac{e^{-jkr}}{r}$$

$$= \frac{-j\lambda\Gamma_l}{1-\Gamma_l\Gamma_a}\left[\vec{A}(-\vec{k}_1)\cdot\vec{E}^i\right]\vec{A}(\vec{k}_2)\frac{e^{-jkr}}{r} \qquad (26)$$

The effective length of antenna is $\vec{h}(\vec{k}) = \sqrt{\frac{Z_c}{Z}}(1+\frac{Z_{in}}{Z_c})(-j\lambda)\vec{A}(\vec{k})$ (27)

Substitute (27) into (26), we get

$$\vec{E}^a(Z_l) = \frac{j\Gamma_l}{1-\Gamma_l\Gamma_a}\frac{Z}{\lambda Z_c}(1+\frac{Z_{in}}{Z_c})^{-2}\left[\vec{h}(-k_1)\cdot E^i\right]\vec{h}(\vec{k}_2)\frac{e^{-jkr}}{r} \qquad (28)$$

If $Z_{in} = Z_c = R_a$, then $\Gamma_a = \frac{Z_{in}-Z_c}{Z_{in}+Z_c} = 0$, and

$$\vec{E}^a(Z_l) = \frac{jZ}{4\lambda R_a}\Gamma_l\left[\vec{h}(-\vec{k}_1)\cdot\vec{E}^i\right]\vec{h}(\vec{k}_2)\frac{e^{-jkr}}{r} \qquad (29)$$

It is similar to the expression in reference [6] but our result is under the match load condition.

3. Conclusion

An antenna scattering theory is proposed, from which previous definitions are deduced. From the theory, some methods to reduce the antenna mode scattering are illustrated. The work is useful for the analysis of antenna RCS and RCS reduction.

References

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