

SCATTERING OF E OR H POLARIZED WAVES FROM 2D CONCAVE-CONVEX
SCREENS AND BOUNDARY

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1. Introduction.

The problem of scattering whispering-gallery mode from concave or concave-convex boundary is known to be under wide scientific discussion within middle part of 20th but it is important to underline that all achieved results were obtained by asymptotic methods only: the method of geometrical theory of diffraction (GTD), its uniform or local modifications, the method of physical optics, the method of parabolic equation or Kirchhoff approximation [1-6] and deal with describing a field nearby a scattering surface. In real practice all these methods could be effectively used for interpretation results but not for its obtaining due to many restrictions or runs against huge difficulties.

In this work we present a strict numerical results for diffraction of E and H polarized waves by perfect conducting concave-convex finite screens or cylindrical structure when whispering-gallery mode could be effectively launched. The results were obtained by applying developed numerical method [8-10] for currents integral equations (CIE) [7] in high frequency domain ($kD \gg 1$, k - wave number, D - maximum size of the scatterer).

2. Scattering of E or H polarized waves from concave-convex finite screens or cylinders.

Let at first consider the scattering problem for incident cylindrical wave $U_0(\vec{r})$:

$$U_0(\vec{r}) = H_0^{(2)}(k|\vec{r} - \vec{r}_0|) \tag{1}$$

by perfectly conducting finite concave-convex cylindrical surface S with cross-section function $\rho_0 = \rho(\varphi)$, $\varphi \in [\varphi_B, \varphi_E]$ in cylindrical coordinate system. In (1) $\vec{r} = \{r, \varphi\}$, $\vec{r}_0 = \{R_0, \varphi_0\}$ - coordinates of the point of view and source of cylindrical wave accordingly; $|\vec{r} - \vec{r}_0|$ - is a distance between these points. The scattering field $U_1(\vec{r})$ has to satisfy a wave equation

$$\Delta U_1 + k^2 U_1 = 0 \tag{2}$$

outside of S , boundary condition on S for E polarized field:

$$U_0(\vec{r})|_S + U_1(\vec{r})|_S = 0 \tag{3}$$

or for H polarized field:

$$\frac{\partial}{\partial n} U_0(\vec{r})|_S + \frac{\partial}{\partial n} U_1(\vec{r})|_S = 0 \tag{4}$$

and Sommerfield's radiation condition :

$$\frac{\partial u^1(\vec{r})}{\partial r} + iku^1(\vec{r}) = o(r^{-1/2}), |r| \rightarrow \infty$$

It is known that in these cases the scattering problem could be reduced to Dirichlet or Neumann value boundary problems and following integral equations with singular kernel could be obtained:

$$U_0(\vec{r}_S) = \int_S \mu(\vec{r}_\sigma) H_0^{(2)}(k|\vec{r}_S - \vec{r}_\sigma|) d\sigma, \tag{5}$$

$$\frac{\partial}{\partial n_S} U_0(\vec{r}_S) = \int_S \mu(\vec{r}_\sigma) \frac{\partial}{\partial n_S} \frac{\partial}{\partial n_\sigma} H_0^{(2)}(k|\vec{r}_S - \vec{r}_\sigma|) d\sigma, \tag{6}$$

In (5),(6) - unknown function $\mu(\vec{r})$ - is a current on surface S , $\vec{r}_S \in S$, $\vec{r}_\sigma \in S$, $H_0^{(2)}(k|\vec{r} - \vec{r}_\Sigma|)$ - fundamental solution to Helmholtz equation (2).

In case of closed concave-convex cylindrical surface S with cross-section contour $\rho_0 = \rho(\varphi)$, $\varphi \in [0, 2\pi]$ we had applied an integral equations with smooth kernel of the method of auxiliary current integral equation (ACIE) [9] which is very similar to (5),(6) but points $\vec{r}_\sigma \in \Sigma$ and Σ - is an auxiliary surface was constructed as indicated in [9].

The $\mu(\vec{r})$ is known in (5),(6), the scattering pattern $g(\varphi)$ could be calculated as follows

$$g(\varphi) = \int_S \exp[ik\rho_0(\theta)\cos(\theta - \varphi)]\mu(\theta)d\theta \quad (7)$$

Numerical solution of the this 2D scattering problems on the base of CIE or ACIE (5), (6) by applying technique deal with extracting singularities or applying such system of basis function as piece-wise function, any kind of orthogonal polynomial approximations, spline functions or Fourier leads to many difficulties for making accurate and stable numerical results in high frequency domain ($kD \gg 1$) Therefore we had applied the method “prolonged” boundary conditions [8-10] and Haar wavelet functions as a system of basis functions when a procedure of algebraisation for integral equation was made. As it was shown in [8-10] the method “prolonged” boundary conditions uses analytical properties for presentation of scattering field in simple potential layer form [7,10]. It allows making displacement for points r_s from S into Σ' surface ($\Sigma' = S + i\Delta$, $\Delta \ll 1$, S - is original surface; Δ - is a value of displacement from original surface). So, as a result we will have an integral equation of the first kind with smooth kernel and Haar wavelet functions (as a system of basis functions) could be effective applied for its stable numerical solution in $kD \gg 1$ region.

The error in the numerical solution of the problem we estimate as the residual Δ of the boundary condition on S : $\tilde{\Delta} = |u_0 - u|_{S'}, \varphi = \varphi_m \equiv 2m\tilde{\Delta}_\varphi / M1$; $m = 1, 2, \dots, M1$; $\tilde{\Delta}_\varphi = \varphi_B - \varphi_E$, $\varphi_{B,E}$ - determines the beginning and the ending angles of the scatterer's surface S .

3. Numerical results.

We had applied the described procedure for numerical solution of scattering whispering-gallery mode by concave finite convex boundary S as a part of circular cylinder or part of cylinder with cross-section as apart of sinus function in Cartesian coordinate system:

$$\rho_0(\varphi) = a, \varphi \in [\varphi_B, \varphi_E] \quad \text{or} \quad (8)$$

$$y = A \sin(\pi x / 2T), x \in [x_B, x_E] \quad (9)$$

or of cylinder with cross-section as oval Cassini or multifoil :

$$\rho_0^2(\varphi) = a^2 \{ \cos(2\varphi) + \sqrt{\cos^2(2\varphi) + [b^4 / a^4 - 1]} \} \quad \text{or} \quad (10)$$

$$\rho_0(\varphi) = a + b \cos(2\varphi) . \quad (11)$$

We had considered a case of cylindrical incident wave (1) or plane gauss beam:

$$U_0(x, y) = \exp\{-ikx - k^2(y - Y_0)^2 / k^2\sigma^2\} \quad (12)$$

It is well known that these scattering surfaces formed different types of diffracted rays structures (families) [6].

The relative amplitude of scattering pattern $g(\varphi)$ ($g(\varphi) \equiv |g(\varphi) / \max\{g(\varphi)\}|$) for gauss beam (12) with parameters: $k\sigma = 1$, $kY_0 = -98$ or $k\sigma = 5$, $kY_0 = -95$; surface as a part of cylinder with parameters: $ka = 120$, $\varphi_{B,E} = \mp\pi/2$ are illustrated by Fig.1,2. It is seen that the process of beam's interaction with finite reflector (8) is accompany by complicate oscillating of scattering field's amplitude.

The relative amplitude of scattering pattern $g(\varphi)$ for cylindrical E and H polarized cylindrical wave (1) and cylindrical surface (8) presents at Fig.3, 4. Parameters of surface (8) were $ka = 80$; $\varphi_{B,E} = \mp\pi/2$ and coordinates of the source were $kR_0 = 78$; $\varphi_0 = -\pi/2$. It demonstrates the influence of wave polarization on scattering process.

The relative amplitude of scattering pattern $g(\varphi)$ for cylindrical E and H polarized cylindrical wave

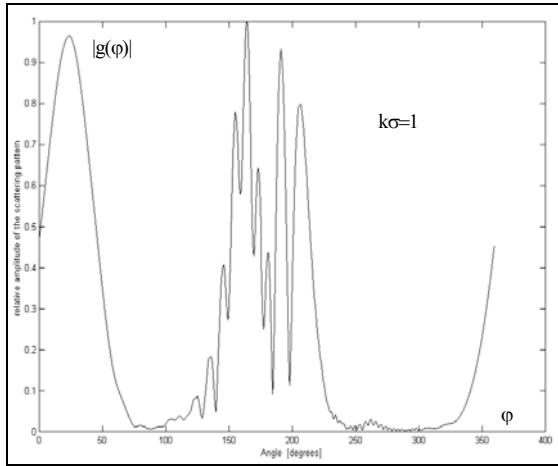


Fig.1

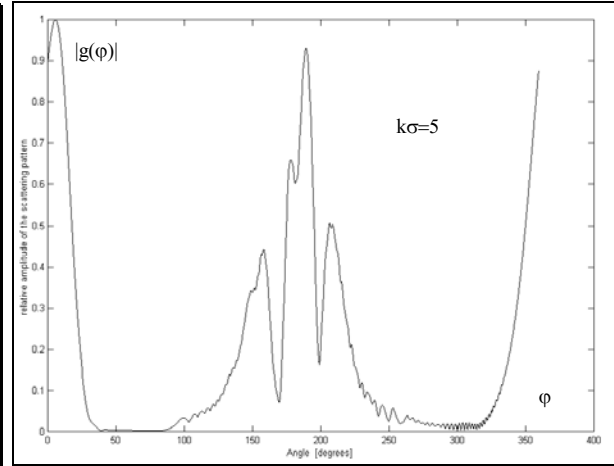


Fig.2

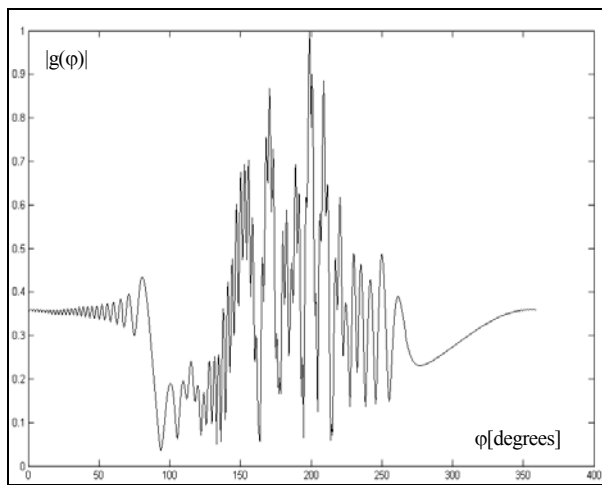


Fig.3

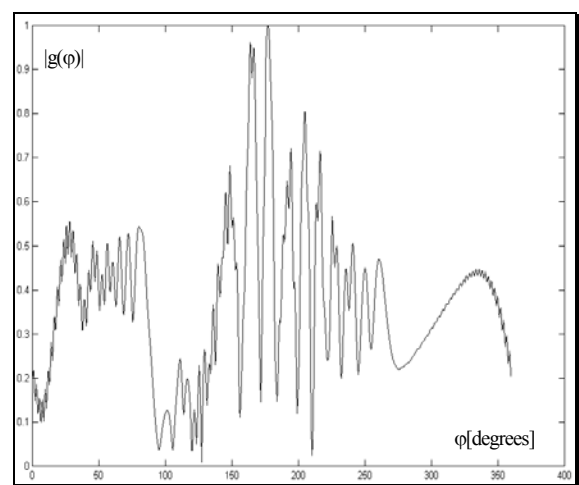


Fig.4

The relative amplitude of scattering pattern $g(\varphi)$ for cylindrical E polarized wave (the source of wave has coordinates: $kx_0 = -100$, $y_0 = -98$) and reflector with cross-section as apart of sinus function with $kx_B = -100, kx_E = 0$ (without point of inflection) or $kx_B = -100, kx_E = 100$,(with point of inflection) $kA = 100, T = A$ presented at Fig.5 or at Fig. 6 respectively. In this case we also see a complicated oscillation for the scattering fields and its structure depends on contour's shape.

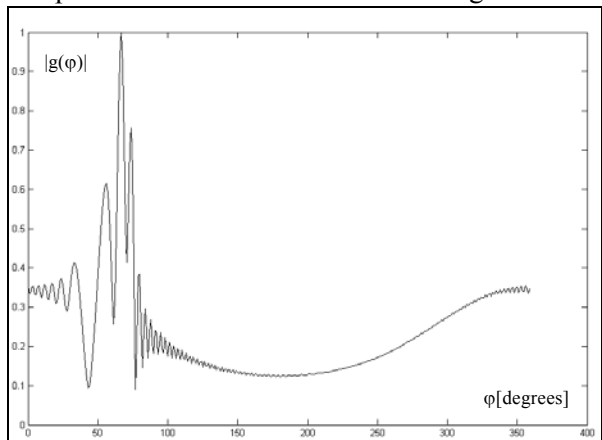


Fig.5

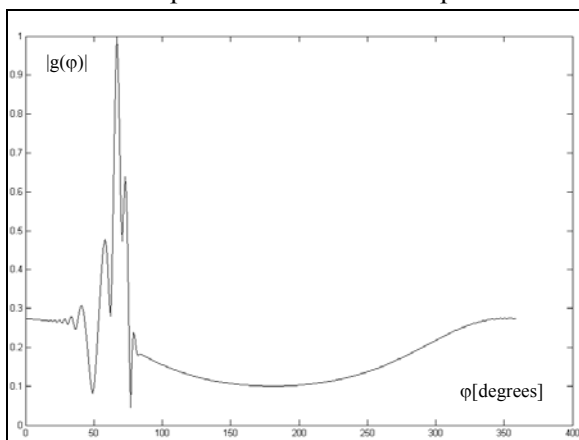


Fig.6

Example of calculated relative amplitude of the scattering pattern $g(\varphi)$ for contour (10) with parameters: $ka = 100$, $kb = 101$ and cylindrical E or H polarized wave (its coordinates

were $kR_0 = 15$; $\varphi_0 = -\pi/2$) one can see at Fig.7,8 respectively. It is clearly seen the differences in process of scattering E and H polarized waves. We had also calculated a case when $ka = 350$, $ka = 354$, ($kD \approx 1000$) with $\max(\Delta) = 10^{-8}$.

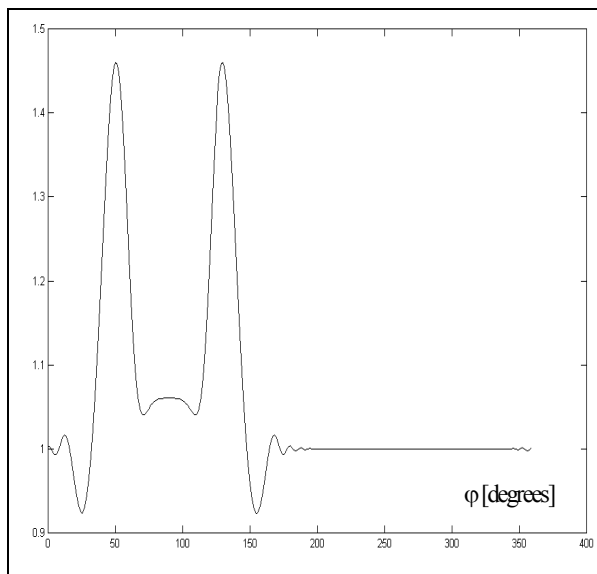


Fig.7

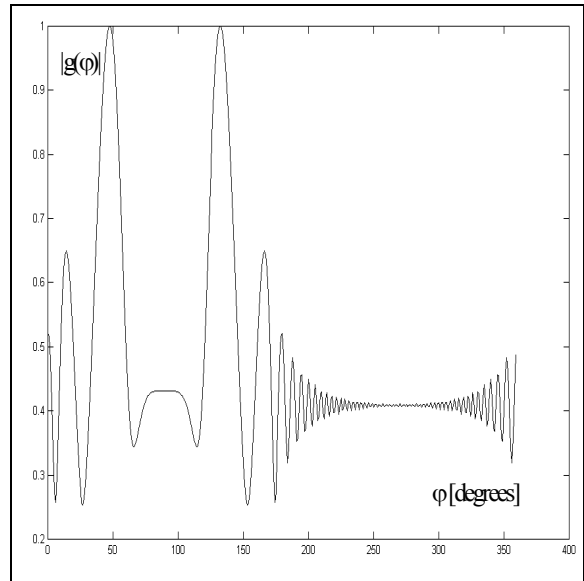


Fig.8

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