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INVESTIGATION OF THE DEPENDENCE OF EIGENFREQUENCIES OF THE IONOSPHERIC-MAGNETOSPHERIC ALFVEN RESONATOR ON GEOMAGNETIC LATITUDE

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1. Introduction

In this paper, a study of a new type of the object, an ionospheric-magnetospheric Alfven resonator (IMAR) [1] in the near-earth space, is continued. This object is generalization of the two resonators - the ionospheric Alfven resonator (IAR) and the magnetospheric Alfven resonator (MAR) that were examined separately in previous studies. The IMAR has a series of new properties related to the interaction between the IAR and MAR. In the present paper, we investigate the transformation of the IMAR eigenfrequencies as a geomagnetic latitude changes.

2. Statement of the problem

We will consider the low-frequency range (0.01-10 Hz). The solution of a problem is sought in a spherical coordinate system (r, θ, φ) with the origin in the centre of the Earth. The angle θ is counted from a direction connecting the northern and southern magnetic poles. We suggest that the properties of medium do not depend on an angle φ, and the propagation occurs in a plane of a geomagnetic meridian. The earth surface in the given frequency band is considered indefinitely conductive.

The initial system of equations can be obtained from the system of the Maxwell's equations for the waves of Alfven polarization and be written in the following form:

$$\frac{\partial \tilde{E}_\theta}{\partial r} = -\frac{\partial (h_\theta / h_r)}{r \partial \theta} \tilde{E}_\theta - \frac{h_\theta / h_r}{r} \frac{\partial \tilde{E}_\theta}{\partial \theta} + i\omega \tilde{B}_\varphi ,$$

$$-\frac{\partial \tilde{B}_\varphi}{\partial r} = \frac{k^2}{i\omega h_r^2} \tilde{E}_\theta + \frac{h_\theta / h_r}{r \sin\theta} \frac{\partial (\tilde{B}_\varphi \sin\theta)}{\partial \theta} .$$

Here, $\tilde{E}_\theta = rE_\theta$, $\tilde{B}_\varphi = rB_\varphi, k^2 = i\omega\mu_0\sigma_1 = \omega^2/c_A^2$; h_r, h_θ – are the components of unite vector of the geomagnetic field; σ_1- Pedersen component of conductivity tensor; E_θ, B_φ – are the components of the electromagnetic field..

The spherical admittance can be expressed as $-B_\varphi(r,\theta) = A_{11}(r,\theta) E_\theta(r,\theta) / (\omega L)$. Here L is a typical scale of inhomogeneity of Alfven velocity c_A in an ionosphere (about 300 km/s). Let's introduce a modified admittance as $A(r,\theta) = A_{11}(r,\theta)\cos\theta$, and assume $h_\theta/h_r=\tan\theta$. Then the equation for modified admittance in an ionosphere and magnetosphere can be reduced to

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$$i\frac{dA}{d\bar{h}} + A^2(h) - \bar{k}^2(h) = 0\,. \qquad (1)$$

Here h is the coordinate along a field- line of a geomagnetic field and $\bar{h} = h / L$.

3. Model of medium

Let's consider an indefinitely conductive spherical earth surface of radius a=6400 km covered by a spherical layer of radius b=7000 km (ionosphere). A wave number inside this layer is designated as k. Outside of this layer, a homogeneous medium is characterized by wave number k1 (magnetosphere). As an example we will use the condition of the night and maximum solar activity $(k/k1)^2 = (c_{A1}/c_A)^2 = M$. Applying boundary conditions, we obtain the dispersion equation for eigenfrequencies (ω_n) of the IMAR in the form:

$$M\sin(\frac{\omega}{c_{A1}}h1)\cos^2(\frac{\omega}{c_A}h) + \sqrt{M}\sin(2\frac{\omega}{c_A}h)\times$$
$$\cos(\frac{\omega}{c_{A1}}h1) - \sin^2(\frac{\omega}{c_A}h)\sin(\frac{\omega}{c_{A1}}h1) = 0. \qquad (2)$$

Here c_{A1} is the Alfven velocity in a magnetosphere, c_A - Alfven velocity in an ionosphere, h1 – length of a line of a geomagnetic field in a magnetosphere, h - length a line of a geomagnetic field in an ionosphere.

4. The analysis of the problem

For large M, the equation (2) has two obvious series of the roots. First of them represent a solution of

$\sin(\frac{\omega}{c_{A1}}h1) = 0$ and define the eigenfrequencies of the MAR, whereas the second one have double roots determined from

$\cos^2(\frac{\omega}{c_A}h) = 0$ and corresponds to eigenfrequencies of the IAR.

Although the parameter M is large, it is finite and it is necessary to analyze solutions of the equation (2), which correspond to a spectrum of eigentones of unified object, which is the ionospheric-magnetospheric Alfven resonator. In the paper [1] we have considered a dependence of the left-hand part of the dispersion equation (2) on frequency x=f=ω/2π.

Each eigenfrequencies of IAR obtained normally for planar- stratified model of medium, is the case of spherical model splitted in two [1, 2]. The spectrum of resonant frequencies of IAR is doubled.

Let's illustrate dependence of natural frequencies of the IAR and MAR from on angle θ. Fig 1 shows the dependence of the resonant frequencies of the first four MAR modes and first three IAR modes on an angle t (θ) within the range from 10^0 up to 60^0. An angle θ corresponds to the coordinate of intersection of the geomagnetic field-line and the Earth surface. θ is expressed in radians. The following features are evident from Fig. 1: a) the frequencies of the MAR modes increase with increasing angle; б) the frequencies of the IAR modes decrease as the angle increases. The frequencies are designated on vertical axes in Hz.

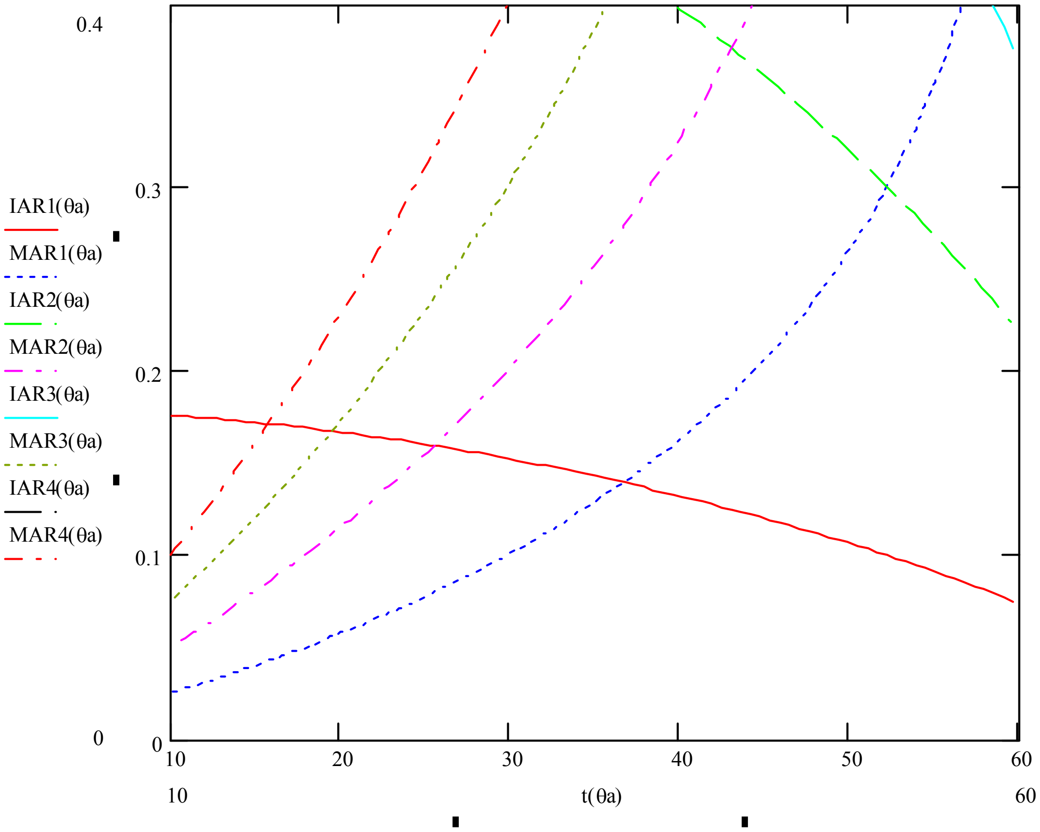
0.4
0.3
0.2
0.1
0
IAR1(θa)
MAR1(θa)
IAR2(θa)
MAR2(θa)
IAR3(θa)
MAR3(θa)
IAR4(θa)
MAR4(θa)
0
10
20
30
40
50
60
10
t(θa)
60

Fig. 1

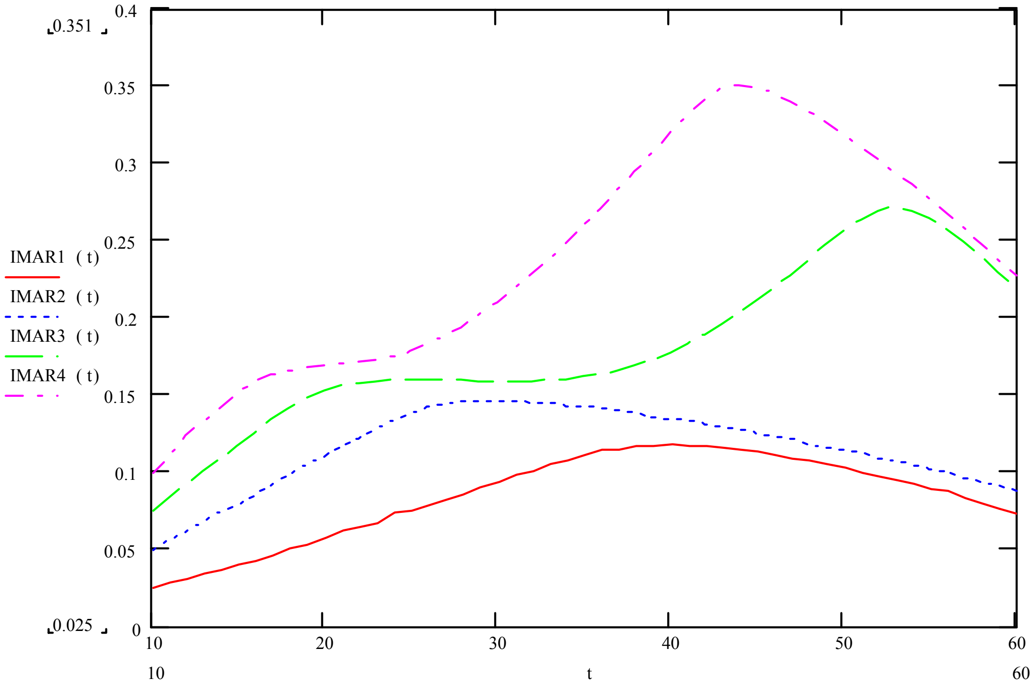
0.351
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0.025
0
IMAR1 (t)
IMAR2 (t)
IMAR3 (t)
IMAR4 (t)
10
20
30
40
50
60
10
t
60

Fig. 2

In order to examine the dynamics of the IMAR eigenfrequencies, Eq. 1 was solved numerically by the method Newton. The results are displayed in Fig 2. It follows from Fig. 1 and 2 that the eigenfrequencies of the IMAR modes depend on the angle t in a more complex manner, than the frequencies of the IAR or MAR modes. Let's analyze this in more detail.

The frequency of the first IMAR mode (resonant frequency): at small angles coincides with the frequency of the first MAR mode and increases with increasing angle reaching a maximal at an angle of about 37^0. At larger angles, the IMAR frequency starts monotonically to decrease coming closer to the frequency of the first IAR mode.

The frequency of the second IMAR mode coincides with the frequency of the second MAR mode at small angles and increases up to the angle of about 28^0; then the frequency starts decrease monotonically approaching the frequency of the first IAR mode. As a result of this, two IMAR modes with close frequencies exist at large angles instead of one IAR mode. One IAR mode (which exists only in planar- stratified model of medium), is split in two modes in a case of sphericity stratified model of the medium.

The frequency of the third IMAR mode exhibits very interesting behavior. At small angles, it starts as the third MAR frequency, then it remains nearly constant between 20 and 38 degrees. At larger angles, this frequency comes closer to the first IAR mode, then to the first mode MAR reaching a maximal at the angle of 51^0, and eventually it decreases monotonically while coming closer to the frequency of the second IAR mode. As a whole, its behavior is very different from the behavior of the IAR or MAR modes.

The frequency of fourth IMAR mode starts as the fourth MAR mode at small angles and continuously increases having the angle of 20^0 as a point of inflection. For larger angles, this frequency almost coincides with the frequency of the second MAR mode. Further it reaches a maximal at the angle $\sim 42^0$ and decreases while approaching the frequency of the second IAR mode. At even larger angles, the frequency of this mode becomes very close to the frequency of the second IMAR mode (split of the second IAR mode in a spherical case).

5. Conclusions

As a result of the analysis, we can conclude that the behavior of eigenfrequencies of the IMAR modes as function of latitude differs significantly from the behavior of both the IAR and MAR frequencies of modes.

Only examination of unified object, the ionospheric-magnetospheric Alfven resonator, allows to correctly describe and classify possible natural frequencies in the studied system.

6. References

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2. A.O.Ovchinnikov. An Ionospheric Alfven Resonator for the Spherical Model of the Earth’s Surface. Geomagnetism and Aeronomy, Vol. 39, No. 1, 1999, pp. 64-68.