INHOMOGENEOUS STRUCTURE OF THE IONOSPHERIC PLASMA AND ACCURACY OF THE SATELLITE NAVIGATION SYSTEM

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1. Introduction

The satellite navigation systems created only for the military needs of the USA (GPS) and Soviet Union (GLONASS) now are widely used to serve civil purposes: aircraft navigation, traffic control, construction and agriculture, monitoring deformation of large constructions (buildings, bridges), earth surface monitoring to forecast earthquakes. Research on the development of satellite navigation systems and their possible application is carried on in a number of countries all over the world.

Such an interest to the GPS system is determined by rather a low price, high accuracy and accessibility due to the disposition of geodetic labels (satellites) in space.

The main research on developing GPS systems goes in the direction of improvement of positioning accuracy. Since the USA have disabled selective accuracy (an error deliberately brought in to GPS data), accuracy of GPS positioning is mainly determined by an ionospheric delay [1]. It is an inadmissible error for most practical tasks. To eliminate this error there exist some methods.

At present the most exact method (reducing the error up to 99 %) is the two-frequency method. According to this method, within the framework of the first order approximation, the wave propagates along straight line, connecting the receiver and the transmitter, and difference of range brought in by ionospheric plasma from true is inverse proportional to a square of a carrier frequency. Within the framework of the first order approximation it is possible to eliminate the ionospheric error by measuring a phase delay on two frequencies.

However in the two-frequency method the essential peculiarity of the ionosphere, its inhomogeneity is not taken into account. In this method it is assumed that the wave propagates along a straight line. Yet it is known, that in inhomogeneous medium the ray differs from a straight line. The effect of ionospheric inhomogeneities on a signal is known and used in research of these inhomogeneities. In navigation practice the effect of these inhomogeneities is considered as noise. At increase of a level of such "noise" when data is strongly corrupted, they are simply excluded from processing. This increased effect of ionospheric inhomogeneities on GPS data very often occurs on low and high latitudes, on middle latitudes also, especially during magnetic storms.

In the present report it is offered to develop new methods to correct ionospheric effects in GPS receivers in view of presence of ionospheric inhomogeneities of various scales.

2. Accuracy of the two-frequency method

Within the geometrical-optics approximation, the field of the GPS transmitter is of the form

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$$U = A\frac{F(\theta,\varphi)}{D}\exp\left\{i\frac{\omega}{c}\Phi(\vec{r},\omega) - i\omega t\right\} \quad (1)$$

Where the coefficient A is defined by radiated power; $F(\theta,\varphi)$ is a product of directional diagrams of transmitting and receiving antennas; D - a distance between the transmitter and the receiver; $\omega = 2\pi f$ - a frequency; c - a velocity of light; $\Phi(\vec{r},\omega)$ - phase path.

Here we shall be interested in the accuracy of the measurements connected only with the ionosphere. In this case the phase path defined from the eikonal equation, is equal

$$\Phi(\vec{r}) = \int_{z_0}^{z_t} \sqrt{\varepsilon\left(\vec{\rho}(z'), z'\right)} \sqrt{1 + \left(\frac{d\vec{\rho}(z')}{dz'}\right)^2}\, dz', \quad (2)$$

$\vec{r}\,' = \{\vec{\rho}, z'\}, \vec{\rho} = \{x', y'\}$

In (2) integration is carried on a ray connecting points of radiation and receiving. The system axes selected so, that the axis z' passes through a source (GPS satellite) and the receiver (GPS user). Permittivity of the ionospheric plasma is determined by a relationship

$$\varepsilon(\vec{\rho}, z') = 1 - 80.6N(r')f^{-2}, \quad (3)$$

where $N(r')$ is an electronic density.

For frequencies of L - range which are used in GPS, difference e from 1 is small and as a first approximation

$$\Phi(f) = D - \alpha f^{-2} I \quad (4)$$

where $\alpha = 40.3$, $I = \int_{z_0}^{z_t} N(z')dz'$ is the total electron content.

There are a lot of methods of exclusion of the error brought in by the ionosphere (addend in (4)) from phase measurements. The most exact method of correction eliminates the addend in (4), using its frequency dependence. For this measurement of a phase are made on two frequencies. As a result, the following system is obtained:

$$\begin{cases} \Phi_1 = D - \alpha f_1^{-2} I \\ \Phi_2 = D - \alpha f_2^{-2} I \end{cases} \quad (5)$$

$\Phi_i = \Phi(f_i)$, $i = 1,2$.

Two unknown quantities: pseudo-range D for the user position finding and total electron content I for finding of ionospheric parameters are usually found from this system:

$$D^{(1)} = \frac{\Phi_1 f_1^2 - \Phi_2 f_2^2}{f_1^2 - f_2^2}, \quad I^{(1)} = \frac{f_1^2 f_2^2}{\alpha}\frac{\Phi_1 - \Phi_2}{f_1^2 - f_2^2} \quad (6)$$

In (4) the first approximation is used which does not take into account variations of a path. But difference of a path from a straight line in inhomogeneous medium can lead to essential phase change [2]. These variations are taken into account in the following formula of the second approximation

$$\Phi(f) = D - \alpha f^{-2} I - f^{-4}\Delta \quad (7)$$

where

$$f^{-4}\Delta = 0.5\int_{z_0}^{z_t} p_1^{\,2}(z')dz' \tag{8}$$

$$\vec{p}_1 = \frac{-\alpha}{2Df^2}\left\{\int_{z_0}^{z'}\left(z''-z_0\right)\frac{\partial}{\partial\vec{\rho}}N(0,z'')dz'' - \int_{z'}^{z_t}\left(z_t-z''\right)\frac{\partial}{\partial\vec{\rho}}N(0,z'')dz''\right\} \tag{9}$$

is the "pulse" of a ray describing the deviation of a ray from direct, connecting the source and the receiver. Thus the correction of the second order (last addend in the right part (7)) is determined by the degree of ray distortion.

With the help of this second order correction it is possible to find the error of the two-frequency method. Instead of (5), we use the following system

$$\begin{cases}\Phi_1 = D - \alpha f_1^{-2} I - f_1^{-4}\Delta \\ \Phi_2 = D - \alpha f_2^{-2} I - f_2^{-4}\Delta\end{cases} \tag{10}$$

Solving the system (10), we obtain instead of (6) the following result for the double-frequency receiving

$$D = D^{(1)} + \Delta D, \quad I = I^{(1)} + \Delta I \tag{11}$$

where errors of the first order for pseudo-range ΔD and for total electron content ΔI are equal

$$\Delta D = -\left(f_1 f_2\right)^{-2}\Delta, \quad \Delta I = -\alpha^{-1}\left(f_1^{-2} + f_2^{-2}\right)\Delta\,. \tag{12}$$

Let us remark, that errors (12) always negative and there is a bias. The errors depend on orientation of a route the GPS satellite - the GPS user and geophysical circumstances. For pseudo-range this error runs up to value about a centimeter.

3. The three-frequency receiving

Recently information [3] on a usage of thirds frequency in systems GPS have appeared.

In presence of measurements at three frequencies, we can have the following system:

$$\begin{cases}\Phi_1 = D - \alpha f_1^{-2} I - f_1^{-4}\Delta \\ \Phi_2 = D - \alpha f_2^{-2} I - f_2^{-4}\Delta \\ \Phi_3 = D - \alpha f_3^{-2} I - f_3^{-4}\Delta\end{cases} \tag{13}$$

Considering (13) as the system of three equations for three unknown quantities, it is possible to find the following expressions of the second approximation

$$D^{(2)} = \frac{\Phi_1 f_1^4}{\left(f_1^2-f_2^2\right)\left(f_1^2-f_3^2\right)} + \frac{\Phi_2 f_2^4}{\left(f_2^2-f_1^2\right)\left(f_2^2-f_3^2\right)} + \frac{\Phi_3 f_3^4}{\left(f_3^2-f_1^2\right)\left(f_3^2-f_2^2\right)}, \tag{14}$$

$$I^{(2)} = \frac{1}{\alpha}\left\{\frac{\left(f_2^2+f_3^2\right)\Phi_1 f_1^4}{\left(f_1^2-f_2^2\right)\left(f_1^2-f_3^2\right)} + \frac{\left(f_1^2+f_3^2\right)\Phi_2 f_2^4}{\left(f_2^2-f_1^2\right)\left(f_2^2-f_3^2\right)} + \frac{\left(f_1^2+f_2^2\right)\Phi_3 f_3^4}{\left(f_3^2-f_1^2\right)\left(f_3^2-f_2^2\right)}\right\}, \tag{15}$$

$$\Delta^{(2)} = -\left(f_1 f_2 f_3\right)^2 \left\{ \frac{\Phi_1 f_1^2}{\left(f_1^2 - f_2^2\right)\left(f_1^2 - f_3^2\right)} + \frac{\Phi_2 f_2^2}{\left(f_2^2 - f_1^2\right)\left(f_2^2 - f_3^2\right)} + \frac{\Phi_3 f_3^2}{\left(f_3^2 - f_1^2\right)\left(f_3^2 - f_2^2\right)} \right\}. \quad (16)$$

Besides more exact results of measurements of pseudo-ranges (14) and a total electron content (15) we have obtained value Δ which characterizes the extent of ionosphere inhomogeneity along the ray connecting the source and the receiver as a bonus.

5. Precise GPS positioning taking into account diffraction effects

The previous results refer to taking into account large-scale (in comparison with the first Fresnel zone) inhomogeneities. However, in the ionosphere there exist various inhomogeneities, including those with sizes of the order or smaller than Fresnel zone. In this case for a field definition it is necessary to use instead of the geometrical-optics approximation (1) one of its generalizations, that take into account diffraction effects.

One of such generalizations is the mixed integral representation obtained in [4-6]. It is convenient, because from it we can obtain the result similar to (1) and a diffraction problem is reduced to that of a ray (the solution is described above). This integral transformation can be carried out with the help of a terrestrial array and a synthesis of the satellite antenna array by a moving satellite [4].

6. The conclusion

In the present report we obtained formulas of the second order for a phase of the signal GPS, which take into account deviations of a ray from a straight line. These formulas are used for an estimation of extreme accuracy of position finding with the help of the two-frequency method. Here is offered an algorithm of correction of the ionospheric second order error with the help of three-frequency receiving. The GPS data processing with the help of the terrestrial and the satellite synthesized arrays is offered for taking into account diffraction effects in presence of inhomogeneities with sizes of the order or smaller than Fresnel size.

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