# PERFORMANCE OF HIERARCHICAL BEAMFORMING IN A RAYLEIGH FADING ENVIRONMENT WITH ANGLE SPREAD 

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## 1. Introduction

The use of multiple antennas at the Base Station (BS) is recognized as a key technology to achieve significant improvements in capacity and range for wireless communication systems [1]. In this paper we focus on Code Division Multiple Access (CDMA) since the next generation wireless communication systems are based on CDMA. It is well known that the performance of CDMA systems is limited by both multipath fast fading and Multiple Access Interference (MAI) [2]. The two array architectures commonly considered to combat these adverse phenomena are Diversity and Beamforming arrays.
Beamforming and diversity traditionally have conflicting requirements for optimum performance. Diversity arrays employ widely spaced antenna elements to mitigate the performance degradation due to multipath fading [1], [3]. In contrast, Conventional Beamforming (CBF) arrays employ closely spaced antenna elements, with typical inter-element spacing of half wavelength. They can mitigate MAI by forming a narrow beam towards the desired user and low side-lobes towards the remaining users [4]-[6].
Recently a hybrid scheme of diversity and beamforming called Hierarchical Beamforming (HBF) has been proposed to simultaneously combat both fading and MAI [7]. In HBF, the array elements are divided into groups to form several sub-beamforming arrays. The inter-element spacing within a sub-array is assumed half wavelength, while the distance between the adjacent sub-arrays is large (e.g. $20 \lambda$ or more) to ensure independent fading between sub-arrays. The performance of such an array for downlink of a generic DS-CDMA system in a Rayleigh (Non Line of Sight) fading environment was analysed in [8]. However the analysis assumed zero Angle Spread (AS). This assumption is reasonable in suburban areas where the coverage is from elevated BS antennas as the multipath rays arrive at the BS with a small angle spread (e.g. a few degrees). However when the base stations are located within or near urban clutter, they can consequently experience a much larger AS than the elevated base stations, e.g. $5^{\circ}-15^{\circ}$ depending on the height of the BS [9]. Thus it is important to consider the effect of angle spread.
In this work, we assume perfect channel estimation and extend the work in [8] by comparing the performance of CBF and HBF while taking angle spread into account. It is shown that the consideration of angle spread affects the performance comparison between CBF and HBF. We also study the interaction of beamforming and various spatial and temporal diversity gains by varying the noise level, number of multipaths, number of antennas, number of users and angle spread.
This paper is organised as follows. The system and channel model is described in Section I. The receiver model is presented in Section II. The simulation methodology is discussed in Section III and simulation results are presented in Section IV. Finally conclusions are drawn in Section V.

## 2. System and Channel Model

We consider the situation that the BS is equipped with $F=2$ co-linear sub-beamforming arrays. This choice of the number of sub-arrays is motivated by practical array size considerations and is relevant to a BS serving three sectors, each covering $120^{\circ}$ in azimuth. The number of array elements in each sub-array is $B$. Thus the total number of array elements is $N=F \times B$. Each sub-array has an inter-element spacing of $d=\lambda / 2$, while the spacing between the adjacent sub-arrays $\left(d_{s}\right)$ is assumed large to assume uncorrelated fading. Let $K$ denote the total number of Mobile Stations (MS) in the system, which are randomly distributed in the azimuthal direction, along the arc boundary of a $120^{\circ}$ sector cell, in the far field of the array. The $k=1$ th user is assumed to be the desired user. The location of each MS is characterized by its Angle of Arrival (AOA) $\theta^{k}$, which is conventionally measured from the array broadside. We refer to $\theta=0^{\circ}$ as the broadside direction. As the required
spacing between sub-arrays for space diversity is much smaller than the cell radius, we assume that the AOA is the same at each sub-array.
We focus on the reverse link (from MS to BS) of the CDMA system. The MS transmitter follows specifications of IS-95 CDMA reverse link [2]. In this paper, we consider the un-coded system for simplicity. To simulate the general Rayleigh fading multipath channel, the parameterized vector model is used [10]. The total multiplicative distortion of the $l$ th resolvable multipath of the $k$ th user at the $b$ th antenna in the $f$ th sub-array can thus be given as

$$
\begin{equation*}
h_{f}^{k, l, b}(t)=\sqrt{\frac{\Omega^{k, l}}{S}} \sum_{s=1}^{S} e^{\left[j\left(\phi_{s}^{k, l}+2 \pi \cos \Psi_{s}^{k, l} f_{D} t\right)\right]} e^{\left[-j \mathcal{K} d(b-1) \sin \left(\theta^{k, l}+\vartheta_{s}^{k, l}\right)\right]} \tag{1}
\end{equation*}
$$

where $S$ is the number of sub-paths for each resolvable path, $\Omega^{k, l}$ is the mean path power of the $l$ th multipath, $\mathcal{K}=2 \pi / \lambda$ is the wave number, $f_{D}$ is the maximum Doppler frequency, $\phi_{s}^{k, l}$ is random phase of each ray assumed to be uniformly distributed over $[0,2 \pi], \Psi_{s}^{k, l}$ is the Angle of Departure (AOD) for each sub-path relative to the motion of the mobile modelled by a uniform probability density function, $\theta^{k, l}$ is the mean AOA and $\vartheta_{s}^{k, l}$ is a zero mean random angular deviation with standard deviation $\sigma_{A O A}$. The angular distribution of the waves arriving at the Base Station (BS) in azimuth is described by the pdf in the AOA. Measurements have shown that a Gaussian pdf best matches the azimuth pdf in typical suburban environments [9].
Under these assumptions, the total received signal for the $f$ th sub-array can be written in vector notation as

$$
\begin{equation*}
\mathbf{x}_{f}(t)=\sum_{k=1}^{K} \sum_{l=1}^{L} s^{k}\left(t-\tau^{k, l}\right) \mathbf{h}_{f}^{k, l}(t)+\boldsymbol{\eta}_{f}(t) \tag{2}
\end{equation*}
$$

where $\tau^{k, l}$ is the multipath delay, $\boldsymbol{\eta}$ is the noise which is assumed to be Additive White Gaussian Noise (AWGN) and $\mathbf{h}_{f}^{k, l}=\left[h_{f}^{k, l, 1} h_{f}^{k, l, 2} \cdots h_{f}^{k, l, B}\right]^{T}$ is the channel response vector.

## 3. Receiver Model

A block diagram of the receiver incorporating HBF array is shown in Fig. 1. The received signal at each antenna element is first down converted. To detect the $l$ th path, the signal is despread using the PN sequence of the respective MS and synchronized to the delay of the $l$ th path. Next a beamformer is constructed for each resolvable multipath and the signal after PN despreading is combined by the beamforming process. The beamforming output is given by $z_{f}^{k, l}=\left(\mathbf{w}_{f}^{k, l}\right)^{H} \mathbf{y}_{f}^{k, l}$, where $\mathbf{w}_{f}^{k, l}$ is the beamforming weight vector and $\mathbf{y}_{f}^{k, l}$ is the post PN-despread signal vector. We assume the sub-optimal but computationally simpler Maximum Signal to Noise Ratio (MSNR) Beamforming is performed [6], [11]. Thus the weights are set equal to the channel response vector for the desired user i.e. $\mathbf{w}_{f}^{k, l}=\mathbf{h}_{f}^{k, l}$. Further, we assume that these vector channel coefficients are perfectly known. This provides a lower bound on the system performance. Finally all the outputs of the beamformers are Walsh correlated and then processed by the conventional Rake Receiver to combine the various multipath signals and produce the decision variable by equal gain combining for Walsh demodulation for the $f$ th subarray. The overall decision is then made by combining the decision outcomes from the respective sub-beamforming arrays.

## 4. Simulation Methodology

The performance of CBF and HBF array antenna systems is evaluated by means of Monte Carlo simulation runs over the variable of interest $\left(E_{b} / N_{o}\right.$ or $\left.N\right)$. The Figure of merit used in this work is the mean Bit Error Rate (BER). This is the mean taken over the ensemble of the channel Rayleigh fading parameters and the AOA. The performance metric is collected and averaged over $M_{C}=100$ drops [10]. A 'drop' is defined as a simulation run for a given number of MS's and BS over 125 frames, which corresponds to the time required by the desired user to traverse the entire sector azimuth range $\left[-60^{\circ}, 60^{\circ}\right]$. For other users, their AOAs are assumed to be uniformly distributed over $\left[-60^{\circ}, 60^{\circ}\right]$. During a drop, the channel undergoes fast fading according to the motion of the MS's. To simulate MS mobility, we assume that the snapshot rate is equal to the Walsh symbol rate and the angle change between snapshots is $\Delta \theta=0.01^{\circ}$ per snapshot. This value corresponds to a worst case scenario e.g., a MS travelling at $300 \mathrm{~km} / \mathrm{hr}$ at only 100 m from the BS and is widely used in the simulation studies, for instance [5], [6]. Also, we focus on the case of angle spread $\sigma_{A O A}$ in the range $0^{\circ}-15^{\circ}$ [9].

## 5. Results

The performance of HBF is jointly determined by the interaction of a number of factors. These include beamforming gain via closely spaced antenna elements within each sub-array, space diversity gain via widely separated sub-arrays, additional space diversity gain via angle spread and temporal diversity gain via the multipaths. To investigate the impact of these gains on the performance of HBF and CBF, we study the effect of varying the noise level $E_{b} / N_{o}$, angle spread $\sigma_{A O A}$, number of antennas $N$, and number of multipaths $L$.
A. Effect of Noise Level: To begin with, we take a look at the performance of HBF and CBF for the case of a single user $(K=1)$. It is apparent that there is no MAI for the case of one user. Fig. 2 shows the plot of mean BER vs. $E_{b} / N_{o}(\mathrm{~dB})$ assuming $L=2$ paths and zero angle spread. The reference curve in the figure is the theoretical performance of conventional receiver with a single antenna element [2, pp. 761: Eq. (7.89)]. We see that both CBF and HBF show a big improvement in mean BER compared to the conventional receiver and the improvement increases with increasing $E_{b} / N_{o}$. The performance of HBF is superior to CBF due to space diversity gain offered by the widely separated sub-arrays, e.g. for a BER threshold of $10^{-3}$ and $N=8$, an $E_{b} / N_{o}$ of about 6.2 dB is required for CBF but only 2.5 dB is required for HBF .
B. Effect of Angle Spread: Next we consider the effect of angle spread ( $0^{\circ}-10^{\circ}$ ) on the single user performance. Fig. 3 shows the performance of HBF and CBF for $N=6, L=2, K=1$ and moderate angle spreads. The inclusion of angle spread produces spatial fading across the array which results in additional diversity gain and improves the performance of both CBF and HBF. Consequently, we see that the performance further improves as the angle spread $\sigma_{A O A}$ increases from $5^{\circ}$ to $10^{\circ}$. Comparing CBF and HBF in the presence of angle spread, we see that for low $E_{b} / N_{o} \mathrm{CBF}$ is slightly better, but as $E_{b} / N_{o}$ gets higher diversity gain becomes dominant and HBF becomes better.
C. Effect of Number of Antennas: Figs. 4 and 5 show the plots of mean BER vs. Number of antennas $N$ with $E_{b} / N_{o}=10 \mathrm{~dB}, K=15$ users/sector, $\sigma_{A O A}=0^{\circ}-15^{\circ}$ for $L=1$ and $L=2$ paths respectively. From Fig. 4, we see that for zero angle spread, HBF is again better than CBF due to diversity gain afforded by the array architecture. However for both CBF and HBF, there is not much improvement in performance by doubling the number of antennas $N$ from 4 to 8 . By comparison in angle spread scenario, the performance of both schemes improves by increasing the number of antennas. We see that for $\sigma_{A O A}=5^{\circ}$, HBF is better than CBF and for $\sigma_{A O A}=10^{\circ}$ and $15^{\circ}$, both array architectures show a similar performance for the number of users considered.
D. Effect of Number of Multipaths: From Fig. 5 (assuming path diversity with $L=2$ ) we see that for the case of zero angle spread, HBF is again much better than CBF. The trend of diminishing returns with increasing $N$ for both schemes is also present. For $\sigma_{A O A}=5^{\circ}$, HBF is better than CBF. However for $\sigma_{A O A}=10^{\circ}$ and $15^{\circ}$, we see that CBF outperforms HBF as the additional diversity gain from spatial fading becomes dominant with increasing $N$, in the presence of path diversity.
6. CONCLUSIONS

In this paper, we have reported on the performance of Hierarchical Beamforming (HBF) for the reverse link of IS-95 CDMA, with moderate values of angle spread (valid assumption for urban and suburban scenarios) taken into account. It has been shown that while assuming zero angle spread, the performance of HBF is superior to Conventional Beamforming (CBF) due to space diversity gain afforded by the well separated sub-arrays. The inclusion of angle spread produces spatial fading across the array, which results in additional diversity gain and improves the performance of both CBF and HBF. With small angle spread of $\sigma_{A O A}=5^{\circ}$, the performance of HBF is still better than CBF. However for larger angle spreads of $\sigma_{A O A}=10^{\circ}$ and $15^{\circ}$, when path diversity is exploitable and the system is heavily loaded, CBF yields better mean BER results than HBF. These findings are based on the assumption of perfect channel estimation and provide a lower bound on the actual system performance.

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Fig. 1. Receiver Block Diagram.



Fig. 2. Mean BER vs. $E_{b} / N_{o}(\mathrm{~dB})$ for $K=1, L=2$, Fig. 3. Mean BER vs. $E_{b} / N_{o}(\mathrm{~dB})$ for $N=6$ antennas, $\sigma_{A O A}=0^{\circ}$ and $N=4,6,8$ antennas.

$K=1, L=2$ and $\sigma_{A O A}=0^{\circ}, 5^{\circ}, 10^{\circ}$.


Fig. 4. Mean BER vs. Number of antennas $N$ for Fig. 5. Mean BER vs. Number of antennas $N$ for $E_{b} / N_{o}=10 \mathrm{~dB}, K=15$ users/sector, $L=1$ and $E_{b} / N_{o}=10 \mathrm{~dB}, K=15$ users/sector, $L=2$ and $\sigma_{A O A}=0^{\circ}, 5^{\circ}, 10^{\circ}, 15^{\circ}$ respectively.
$\sigma_{A O A}=0^{\circ}, 5^{\circ}, 10^{\circ}, 15^{\circ}$ respectively.
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