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A NOVEL SUBBAND-BASED ESPRIT ALGORITHM

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1. Introduction

In many practical signal processing applications, the objective is to estimate the direction-of-arrival (DOA) from measured data. To this end, there have been several approaches to such a problem, including the so-called ESPRIT algorithm [1]. Since its formal derivation in 1985, ESPRIT has been used for DOA estimation, harmonic analysis, frequency estimation, delay estimation, and combinations thereof. ESPRIT has advantages over another widely used multiple signal classification (MUSIC) [2] method for it reduces the computational burden of MUSIC by exploiting the rotational invariance between two subarrays. But the performance of ESPRIT is greatly affected by the signal-to-noise ratio (SNR), especially in low SNR scenario. The estimation success ratio also decreases for correlated signals.

Subband-based signal processing is a technique of fascinating features. S. Rao and W. A. Pealman proved in [3] that with subband decomposition, several superiorities, like lower minimum prediction error, closer entropy rate to the source, whiter signals in the subband than the fullband, could be obtained compared with the direct estimation on the fullband. To conclude those advantages, different modes are isolated in the process of subband decomposition, which inspire us to decorrelate the signals by decomposing them into several subbands. Two key features of subband signals, named SNR amplification and spatial frequency spacing widening, were given by A. Tkacenko and P. P. Vaidyanathan in [4]. Wavelets are non-ideal subband filter banks and wavelet transform (WT) plays the role of binary decomposition of the source. Wavelet packet transform (WPT) is a much detailed WT by decomposing both *approximations* and *details*. Both WT and WTP have their successful applications in harmonic retrieval [5, 6]. Wavelet-based approaches for DOA estimation were suggested in [7] and [8], while both methods failed to decompose the spatial spectrum in the details subband.

In this paper, we suggest a novel subband-based ESPRIT (SB-ESPRIT) algorithm for DOA estimation. SB-ESPRIT begins with a subband decomposition of the measured data matrix. After that, standard ESPRIT is used to estimate the parameters in selected subband. The validity of ESPRIT in subband signals is proven by finding the rotational invariance between two subarrays in each subband. Then the mapping method from subband frequency back to the fullband one is given. The provided computer simulations confirm our proposed SB-ESPRIT approach.

2. Formulation of the problem

In most digital situations, we consider a uniform linear array (ULA) with K isotropic sensors spaced by the distance d, and there are D $(D < K)$ narrowband plane waves centered at frequency $\tilde{\omega}_0$, impinging from the directions $\theta_1, \theta_2, \cdots, \theta_D$. Locating the first sensor at the origin, the received signal sampled at the i-th sensor can be expressed as

$$x_k(n) = \sum_{i=1}^{D} s_i(n) \mathrm{e}^{-j\tilde{\omega}_0 (k-1) \sin\theta_i d/c} + w_k(n) \tag{1}$$

In matrix form, we have

$$\mathbf{x}(n) = \mathbf{A}\mathbf{s}(n) + \mathbf{w}(n) \tag{2}$$

where $\mathbf{x}(n)$, $\mathbf{s}(n)$, and $\mathbf{w}(n)$ denotes respectively the $K \times 1$ received signal vector, $D \times 1$ wavefront vector, and $K \times 1$ additive noise vector. We define $\omega_i = \tilde{\omega}_0 \sin\theta_i d/c$ as the equivalent spatial frequency of the i-th wavefront, the mixing matrix $\mathbf{A} \in \mathcal{C}^{K\times D}$ can be expressed as $\mathbf{A}(\omega) = [\mathbf{a}(\omega_1), \mathbf{a}(\omega_2), \cdots, \mathbf{a}(\omega_D)]$,where $\mathbf{a}(\omega_i) = [1, \mathrm{e}^{-j\omega_i}, \cdots, \mathrm{e}^{-j(K-1)\omega_i}]^{\mathbf{T}}$ denotes the steering vector

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corresponding to the spatial frequency ω_i, and superscript T denotes transpose. If we place the sensors along the x-axis of the coordinate system, with its first sensor located at the origin, the output snapshot of the k-th sensor can written by

$$x(k) = \sum_{i=1}^{D} s_i \mathrm{e}^{-j(k-1)\omega_i} + w(k) \tag{3}$$

where $s_i = |s_i|\mathrm{e}^{-j\phi_i}$ is a factor to scale the complex signal $\mathrm{e}^{-j(k-1)\omega_i}$, the initial phase ϕ_i is uniformly distributed over the interval $[0, 2\pi)$. Suppose the signals are zero mean wide sense stationary (WSS) process, and $w(k)$ is a zero mean white Gauss noise (WGN) uncorrelated with the signals, and have identical variances σ^2 in each sensor. From the above assumptions, the autocorrelation function of the k-th signal is given by

$$R_{xx}(k) = \sum_{i=1}^{D} P_i \mathrm{e}^{-j(k-1)\omega_i} + R_{ww}(k) = \sum_{i=1}^{D} P_i \mathrm{e}^{-j(k-1)\omega_i} + \sigma^2\delta(k) \tag{4}$$

where $P_i := |s_i|^2$ denotes the power of the i-th signal.

To prevent aliasing of the spectrum, Shannon's spatial sampling theorem must be satisfied, *i.e.* $\max\{\omega_i\} = \max\{2\pi \cdot \sin\theta_i \cdot d/\lambda\} \leq \omega_s/2$, here $\omega_s = 2\pi$ is the sampling frequency and λ denotes wavelength of the signal. By insuring $d \leq \lambda/2$ in the array, the measured data from all sensors can be taken as samples of the spatial signals.

3. The SB-ESPRIT

The idea of SB-ESPRIT is to filter the measured data into several subbands and then apply ESPRIT algorithm to each subband. Subband decomposition is based on the ideal bandpass filters, while it is not applicable for the infinite length of the filters. Non-ideal wavelet filters are often used to have subband decomposition though there is overlapping between the highpass filter and the low one. We notice that wavelet decomposition is a binary decomposition of the *approximation* while it keeps the *detail* unprocessed. Inspired greatly by the works of C. B. Lambrecht [5], we choose wavelet packet to decompose both the *approximation* and *detail* in each level. For the convenience of analysis, we define two matrixes H and G to filter the measured data of (2) into a high frequency subband and a low frequency subband, which are formulated by

$$\mathbf{x}_h(n) = H\mathbf{A}\mathbf{s}(n) + \mathbf{w}_h(n), \quad \mathbf{x}_g(n) = G\mathbf{A}\mathbf{s}(n) + \mathbf{w}_g(n) \tag{5}$$

where H and G are $N_f \times K$ filtering matrixes, here $N_f = \mathrm{fix}[(K + N_d)/2] - 1$ with N_d denotes the length of filter and fix$[y]$ means rounds the elements of y to the nearest integers towards zero. The filtered matrixes are named high frequency matrix $\mathbf{x}_h(n) := H\mathbf{x}(n)$ and low frequency matrix $\mathbf{x}_g(n) := G\mathbf{x}(n)$. Thus we have two $N_f \times 1$ matrixes $\mathbf{x}_h(n)$ and $x_g(n)$ and each of them can be used to compose two subarrays. Taking $\mathbf{x}_h(n)$ for example, we choose its rows from 1 to $(N_f - 1)$ to form the 'measured' matrix $\mathbf{x}_h^1(n)$ of the first subarray, and 2 to N_f to form $\mathbf{x}_h^2(n)$ of the second subarray.

To apply ESPRIT algorithm to each subband, it is important to exploit the rotational invariance between these two subarrays. For simplicity, we choose Haar wavelets as the analysis filters and suppose K is even and $K/2$ is integer. The 1-level highpass filtering matrix H is given by

$$H = \begin{bmatrix} J_2 & o_2 & \cdots & o_2 \\ o_2 & J_2 & \cdots & o_2 \\ \vdots & \vdots & \ddots & \vdots \\ o_2 & o_2 & \cdots & J_2 \end{bmatrix} \in \mathcal{R}^{K/2\times K} \tag{6}$$

with $J_2 = [1/2, 1/2]$ and $o_2 = [0, 0]$.

Substituting (6) into the low frequency subband of (5) yields

$$\mathbf{x}_h(n) = \begin{bmatrix} J_2 & o_2 & \cdots & o_2 \\ o_2 & J_2 & \cdots & o_2 \\ \vdots & \vdots & \ddots & \vdots \\ o_2 & o_2 & \cdots & J_2 \end{bmatrix} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \mathrm{e}^{-j\omega_1} & \mathrm{e}^{-j\omega_2} & \cdots & \mathrm{e}^{-j\omega_D} \\ \mathrm{e}^{-j2\omega_1} & \mathrm{e}^{-j2\omega_2} & \cdots & \mathrm{e}^{-j2\omega_D} \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{e}^{-j(K-1)\omega_1} & \mathrm{e}^{-j(K-1)\omega_2} & \cdots & \mathrm{e}^{-j(K-1)\omega_D} \end{bmatrix} \mathbf{s}(n) + \mathbf{w}_h(n) \tag{7}$$

which can be simplified as

$$\mathbf{x}_h(n) = \tilde{\mathbf{A}}\mathbf{s}(n) + \mathbf{w}_h(n) \tag{8}$$

where $\tilde{\mathbf{A}} = H\mathbf{A}$ denotes the $K/2 \times D$ subband mixing matrix

$$\tilde{\mathbf{A}} = \begin{bmatrix} \tilde{\mathbf{a}}_1 & \tilde{\mathbf{a}}_2 & \cdots & \tilde{\mathbf{a}}_D \end{bmatrix} \tag{9}$$

with $\tilde{\mathbf{a}}_i = \left[1 + \mathrm{e}^{-j\omega_i}, \mathrm{e}^{-j2\omega_i} + \mathrm{e}^{-j3\omega_i}, \cdots, \mathrm{e}^{-j(K-2)\omega_1} + \mathrm{e}^{-j(K-1)\omega_1}\right]^{\mathbf{T}}$.

Like the selection of subarrays in the original algorithm of ESPRIT, the first subarray is composed of the sensors from 1 to $(K/2-1)$ and the second subarray is from 2 to $K/2$. The mixing matrixes $\tilde{\mathbf{A}}_1$ and $\tilde{\mathbf{A}}_2$ of two subarrays can then be related by a diagonal matrix Φ to expressed as $\tilde{\mathbf{A}}_2 = \tilde{\mathbf{A}}_1\Phi$. Here Φ is the rotational invariance in the subband signals, given by

$$\Phi = \mathrm{diag}\{\mathrm{e}^{-j2\omega_1}, \mathrm{e}^{-j2\omega_3}, \cdots, \mathrm{e}^{-j2\omega_D}\} \tag{10}$$

By exploiting the diagonal elements of Φ using standard ESPRIT, we can obtain the spatial frequency $\tilde{\omega}_i$ in the subbands without having to know the mixing matrix $\tilde{\mathbf{A}}_1$. So the validity of SB-ESPRIT is shown with a 1-level Haar wavelet decomposition. It can also been easily proven with an l-level *any-type* wavelet decomposition. It is interesting to notice that the subband frequency is amplified in Φ, which accords with the superiority of frequency widening.

As we desire the fullband frequencies, we need to map the frequencies from subbands back to the fullband. To an l-level Haar wavelet packet decomposition, we map the frequencies as follows

$$\omega_{fb} = \begin{cases} \dfrac{\tilde{\omega}_{l,m} + (m-1)\pi\mathrm{sgn}(\tilde{\omega}_{l,m})}{2^l}, & m = 1, 3, 5, \cdots \\ \dfrac{\tilde{\omega}_{l,m} - m\pi\mathrm{sgn}(\tilde{\omega}_{l,m})}{2^l}, & m = 2, 4, 6, \cdots \end{cases} \tag{11}$$

where $\mathrm{sgn}(\tilde{\omega}_{l,m})$ denotes the sign of $\tilde{\omega}_{l,m}$.

Here we give the summary of the SB-ESPRIT algorithm based on TLS criterion

Step 1. Form the matrix $\mathbf{X} = [\mathbf{x}(1), \mathbf{x}(2), \cdots, \mathbf{x}(N)]$ by taking N snapshots of model (2).

Step 2. Filter $\mathbf{X}$ with wavelet packet filters H and G to yield two matrixes $\mathbf{X}_h = H\mathbf{X}$ and $\mathbf{X}_g = G\mathbf{X}$.

Step 3. Determine the number of signals by applying the minimum description length (MDL) criterion to the mother node $\mathbf{X}$ and its two children nodes $\mathbf{X}_h$ and $\mathbf{X}_g$. Accept the children nodes and goto **Step 2** if there are no modes lost. Otherwise stop the decomposition at the mother node.

Step 4. Prune the binary tree using the best bases method to find the optimal leaf nodes.

Step 5. Divide each leaf nodes into two subarrays and apply TLS-ESPRIT to estimate the subband spatial frequency $\tilde{\omega}_{l,m}$.

Step 6. Map the subband frequency $\tilde{\omega}_{l,m}$ back to the fullband frequency $\omega_{i,fb}, i = 1, 2, \cdots, D$ using (11) and then the DOA's from

$$\theta_i = \arcsin\left\{\frac{\omega_{i,fb} \cdot c}{\tilde{\omega}_0 \cdot d}\right\} = \arcsin\left\{\omega_{i,fb} \cdot \frac{\lambda}{d}\right\} \tag{12}$$

4. Simulation Results

In this section, we give computer simulations to compare the SB-ESPRIT with standard ESPRIT algorithm. Both simulations are carried out for a half wavelength ($d = \lambda/2$) spaced ULA with $K = 32$ isotropic sensors. Four sources emitting narrowband signals with the same power, and propagating in distinct directions with DOA's $10°, 20°, 40°$, and $60°$ are considered. The number of snapshots taken from the array is $N = 100$ and the Haar wavelet packets are chosen as the subband decomposition filters. We use Monte-Carlo simulation method to have 100 runs of each example. In the first example, a very low SNR scenario is chosen to evaluate the performance of SB-ESPRIT and ESPRIT in high power interference scenarios. Fig. 1 and Fig. 2 illustrate the estimated DOA's with the standard ESPRIT and SB-ESPRIT algorithm for $\theta = 10°, 20°, 40°$, and $60°$ with SNR $= -13$ dB and 100 trial runs. We notice that the estimates with SB-ESPRIT are closely distributed along the DOA's, while those with ESPRIT are not. Fig. 3 shows the resulting root mean square error (RMSE) of the estimated DOA's as a function of SNR. SB-ESPRIT algorithm outperforms ESPRIT especially in low SNR for its ability of SNR amplification in the subbands. The second example depicts the decorrelation ability of SB-ESPRIT algorithm compared with the standard ESPRIT, as shown in Fig. 4. The RMSE is greatly decreased with the growth of SNR.

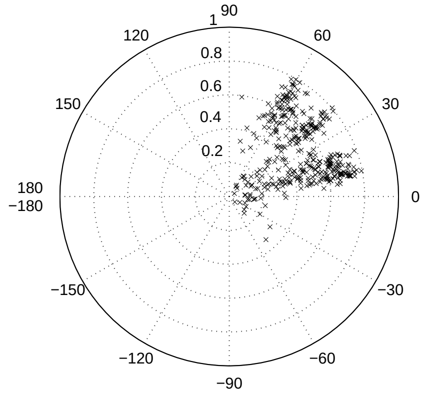

Fig. 1: Estimates with the standard ESPRIT algorithm for $\theta = 10°, 20°, 40°$, and $60°$ with SNR $= -13$ dB and 100 trial runs.

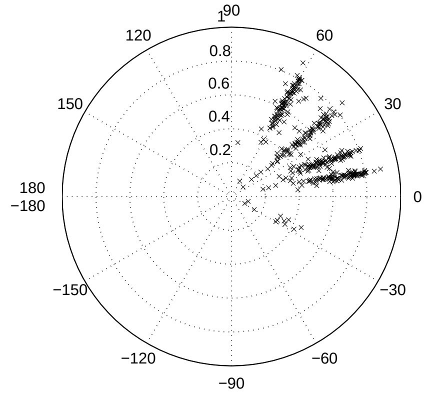

Fig. 2: Estimates with the SB-ESPRIT algorithm for $\theta = 10°, 20°, 40°$, and $60°$ with SNR $= -13$ dB and 100 trial runs.

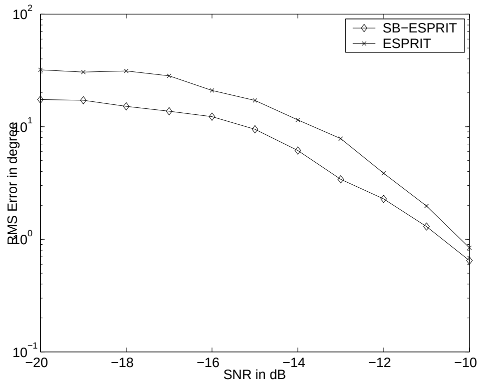

Fig. 3: RMSE of the estimated DOA's as a function of SNR for $\theta = 10°, 20°, 40°$, and $60°$ (cross line - ESPRIT, diamond line - SB-ESPRIT).

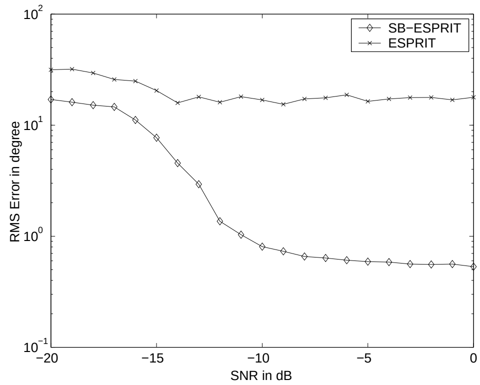

Fig. 4: RMSE of the estimated DOA's as a function of SNR for two uncorrelated DOA's 20° and 60°, and two correlated DOA's 10° and 40° (cross line - ESPRIT, diamond line - SB-ESPRIT)

5. Concluding remarks

A novel subband-based ESPRIT (SB-ESPRIT) algorithm has been proposed in this paper. SB-ESPRIT estimates the spatial frequencies by decomposing the signal into several subbands. Rotational invariance in the subbands is proven and then the subband frequencies are estimated using ESPRIT approach. Due to the frequency spacing widening feature, a mapping method from subband to the fullband is formulated in (11). Simulation results show that SB-ESPRIT outperforms ESPRIT, especially in low SNR scenario. The decorrelation ability of SB-ESPRIT relies on the decomposition level of subband, and is better than ESPRIT to some extent. Simulation results demonstrate the validity of the suggested approach.

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