ON ESTIMATION ERROR OF TWO-DIMENSIONAL MUSIC ALGORITHM FOR TIME-DELAY AND DIRECTION-OF-ARRIVAL ESTIMATION

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1 Introduction

MUSIC algorithm[1] often requires decorrelation preprocessing such as spatial smoothing preprocessing (SSP)[2], which is mandatory for coherent (completely correlated) signal detection. When we apply the SSP, the effective correlation among signals can be reduced with increasing the number of subarrays ($= M$). However, when we apply the SSP to the two-dimensional (2-D) MUSIC algorithm[3], we must select parameter settings of the algorithms carefully, because the decorrelation efficiency changes by interrelation between number of subarrays and signal parameters of the incident waves (the time-delay (TD) and the direction-of-arrival (DOA)). In the two-dimensional case, there exist regions where the SSP cannot work if the TDs and the DOAs are separated enough.

In this paper, we analyze estimation error on the 2-D MUSIC algorithm for the TD and the DOA in the coherent and the correlated signal environment. The SSP is adopted for the coherent case. Computer simulation results show that the estimation error of the TDs and the DOAs for the coherent signals has a characteristic disturibution due to decorrelation performance of the SSP. On the other hand the error in correlated signals (not coherent) case almost monotonously increases as the time-space difference becames small.

2 2-D MUSIC Algorithm

We assume that d waves impinge on the uniform linear antenna array. Let the position of the receiving antenna be $x_l (l = 1, 2, \cdots, L_a)$. The measured value of the antenna at x_l and frequency f_i are give by

$$r(f_i, x_l) = \sum_{k=1}^{d} s_k e^{-j2\pi f_i (t_k - (x_l/c)\sin\theta_k)} + n(f_i, x_l), \tag{1}$$

where s_k, t_k and θ_k denote the complex amplitude, the TD and the DOA of the k-th incident wave, respectively. $n(\cdot)$ is a noise component of the corresponding antenna at the frequency. In the coherent case, we employ the SSP to the data. The data of L_f uniformly sampled frequency points (sampling frequency period of $\triangle f$) and L_a uniform antenna-array (elements separation of $\triangle x$ is $\lambda/2$ interpolated at each frequency[2]) in m-th subarray can be written by

$$\begin{aligned} \boldsymbol{r}_m = [&r_{m,m}, r_{m+1,m}, \cdots, r_{m+N_f-1,m}, \\ &\cdots \quad \cdots, \\ &r_{m,m+N_a-1}, \cdots, r_{m+N_f-1,m+N_a-1}]^T, \\ N_f = L_f &- M + 1, \qquad N_a = L_a - M + 1, \end{aligned} \tag{2}$$

where ^{T} denotes transpose.

The SSP is defined by the average of M covarience matrix constructed by $\boldsymbol{r}_m$. Using the ensemble average of the snapshot (denoted $E[\cdot]$), it can be expressed by,

$$\boldsymbol{r}_m = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{n}, \tag{3}$$

$$\begin{aligned} \boldsymbol{R}_{SSP} = E[\frac{1}{M}\sum_{i=1}^{M}\{\boldsymbol{r}_m \boldsymbol{r}_m{}^H\}] &= \boldsymbol{A}\boldsymbol{S}_{SSP}\boldsymbol{A}^H + E[\boldsymbol{A}\boldsymbol{s}\boldsymbol{n}^H] + E[\boldsymbol{n}\boldsymbol{s}^H\boldsymbol{A}^H] + E[\boldsymbol{n}\boldsymbol{n}^H] \\ &\simeq \boldsymbol{A}\boldsymbol{S}_{SSP}\boldsymbol{A}^H + \sigma^2 \boldsymbol{I}, \end{aligned} \tag{4}$$

where ^{H} denotes complex conjugate transpose.

In the case of two signals ($d = 2$) with unit power, the signal covariance matrix $\boldsymbol{S}_{SSP}$ can be modeled as

$$\boldsymbol{S}_{SSP} = \begin{bmatrix} 1 & \rho_{SSP} \\ \rho^*_{SSP} & 1 \end{bmatrix}, \tag{5}$$

where ρ_{SSP} is the effective correlation coefficient and * denotes complex conjugate.

In modeling of the correlated signals, we use the $\boldsymbol{S}_c$ insted of the $\boldsymbol{S}_{SSP}$,

$$\boldsymbol{S}_c = \begin{bmatrix} 1 & \rho_c \\ \rho^*_c & 1 \end{bmatrix}. \tag{6}$$

Covarience matrix $\boldsymbol{R}_c$ of the desired signal correlation can be created by setting ρ_c for the desired value. In this modeling, the cross terms between signal and noise are omitted.

$$\boldsymbol{R}_c = \boldsymbol{A}\boldsymbol{S}_c\boldsymbol{A}^H + E[\boldsymbol{n}\boldsymbol{n}^H] = \boldsymbol{A}\boldsymbol{S}_c\boldsymbol{A}^H + \sigma^2\boldsymbol{I}. \tag{7}$$

For large number of snapshots, this approximate is valid.

These correlation matrices in Eq.(4) and Eq.(7) are used for the 2-D MUSIC algorithm. Difference of the 2-D MUSIC algorithm from the 1-D MUSIC is a structure of N^2-dimensional mode vectors $\boldsymbol{a}(\theta_k, \tau_k)(k = 1, \cdots, d)$ only. Therefore, following scanning function is often used;

$$P_{music}(\theta, \tau) = \frac{\boldsymbol{a}(\theta, \tau)^H \boldsymbol{a}(\theta, \tau)}{\sum_{h=d+1}^{N^2} |\boldsymbol{a}(\theta, \tau)^H \boldsymbol{e}_h|^2}, \tag{8}$$

$$N^2 = N_f N_a,$$

where $\boldsymbol{e}_h(h = d + 1, \cdots, N^2)$ are the eigenvector corresponding to noise.

3 Simulation Result

In this section, we use data covarience matrix $\boldsymbol{R}_{SSP}$ in Eq.(4) for the coherent case, and $\boldsymbol{R}_c$ in Eq.(7) for correlated case. The signal parameter of #1 source is fixed at $(0\ ^\circ, 10\ ns)$, and the signal parameter of the #2 source is varied. We generate Gaussian random numbers for noise whose SNR is 10 dB. Number of snapshots in each trial is 100. The RMSE in this simulation is defined by,

$$RMSE = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\{2\pi|(\triangle f \tau_e - \triangle g \sin\theta_e) - (\triangle f \tau_k - \triangle g \sin\theta_k)|\}^2}, \tag{9}$$

$$\triangle g = f_1 \triangle x / c,$$

where (θ_e, τ_e), (θ_k, τ_k) and c denote the true parameters of #2 source, the estimated parameters of #2 source in each trial and velocity of wave, respectively. K is number of trials.

Figure 1 shows the effective signal correlation reduced by the SSP where number of subarray is 2, the number of frequency points is 11, and the frequency band is from 2.425 GHz to 2.475 GHz. Also,the number of antenna element is 4, and element separations $\lambda/2$ (assuming that the element spacings are ideally interpolated). The correlation efficient between the i-th incident wave and the j-th incident wave reduced by the SSP is calculated by Eq.(10).

$$|\rho_{SSP}(i, j)| = \left| \frac{\sin[\pi M\{\triangle f(\tau_i - \tau_j) - \triangle g(\sin\theta_i - \sin\theta_j)\}]}{M \sin[\pi\{\triangle f(\tau_i - \tau_j) - \triangle g(\sin\theta_i - \sin\theta_j)\}]} \right| \tag{10}$$

There is the line shape region plotted as correlation factor of 1. In this region, decorrelation effect cannot be obtained by the SSP. In Fig.2, RMSE of the same scenario as in Fig.1 is plotted. As can be seen is Figs.1 and 2, the unresolvable region almost coincides with the region of $|\rho_{SSP}(i, j)| = 1$.

When the incident waves are not coherent, the MUSIC algorithm can resolve the signals without the SSP. In Figs.3 and 4, the number of frequency points is 10, and the measurement frequency band is from 2.425 GHz to 2.470 GHz. The number of antenna positions is 3. These parameters coincide with those of “subarray” in Fig.2. A contour line of RMSE in Fig.3 encircle around #1 signal, and the RMSE is small in comparison with that in Fig.2. This is because signal correlation is constant and very low. In Fig.4, the signal correlation is 0.9. Therefore, the RMSE increases. However, the contour lines of the RMSE have almost the same shape as these in Fig.3. This is because signal correlation is constant all over the analyzed region.

The RMSE in Fig.4 is smaller than that in Fig.2. This is due to approximation in Eq.(7).

4 Conclusions

In this paper, we reported that the RMSE of the 2-D MUSIC algorithm in the correlated signals (not coherent) is inversely propotional to the time-space distance. Beside, in the coherent case where the SSP is required for signal decorrelation, signal correlation changes by the time-space relation among signals. In such a case, it is important to consider the value of the effective correlation coefficient, in addition to the time-space distance.

References

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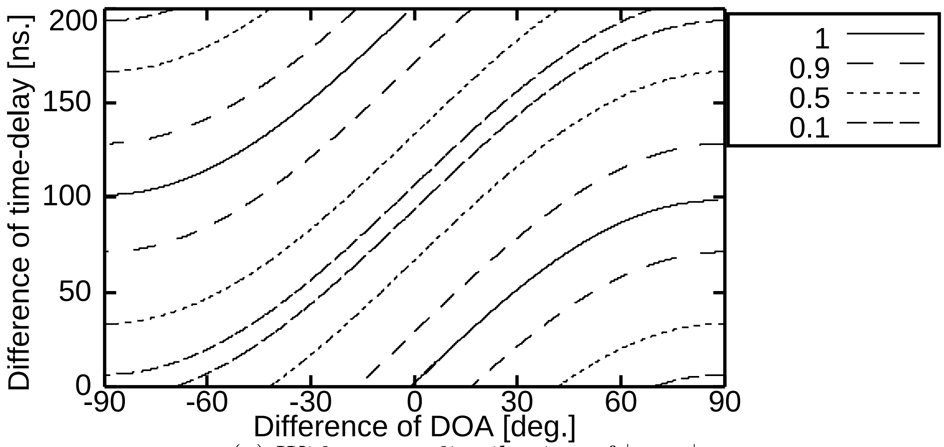
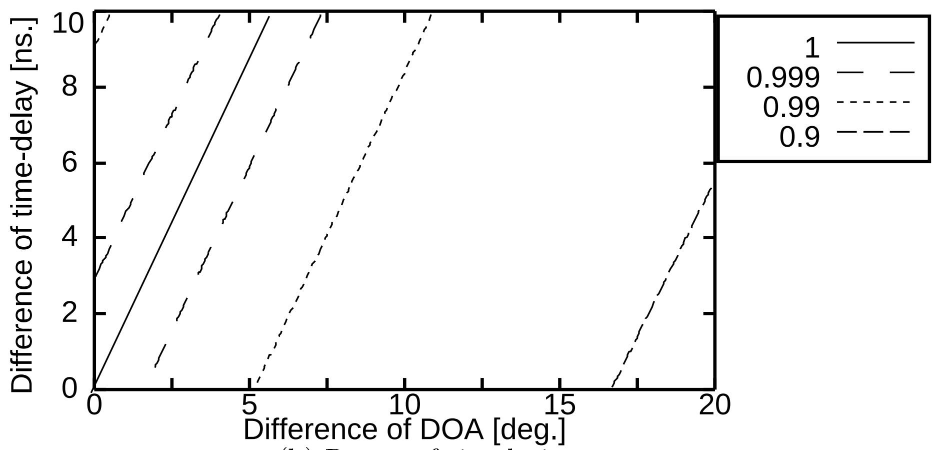


(a) Wide range distribution of $|\rho_{SSP}|$.



(b) Range of simulation area.

Figure 1: Effective signal correlation coefficient of the SSP. #1 signal is located at $(0°, 10ns)$.

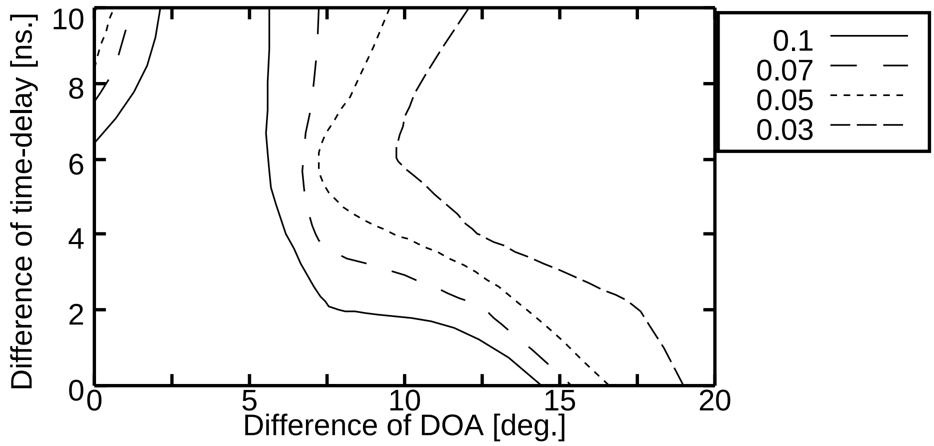


Figure 2: RMSE of parametric index 1. (Using SSP)

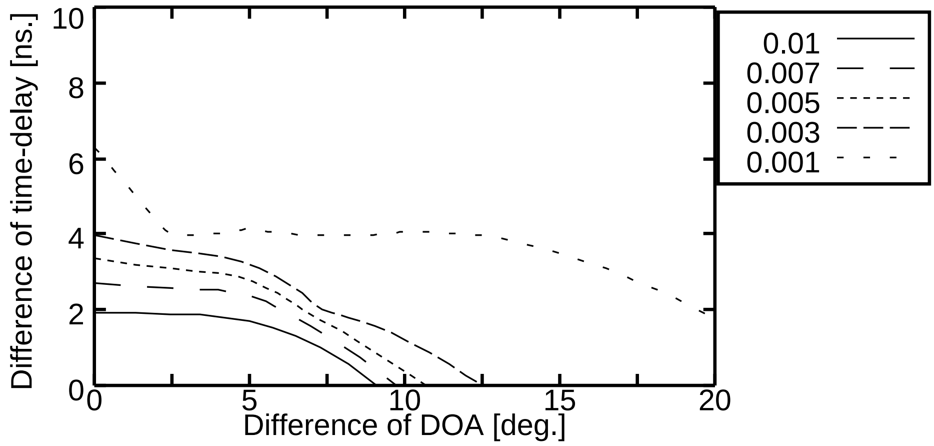


Figure 3: RMSE of parametric index 2. ($|\rho_c| = 0.0$)

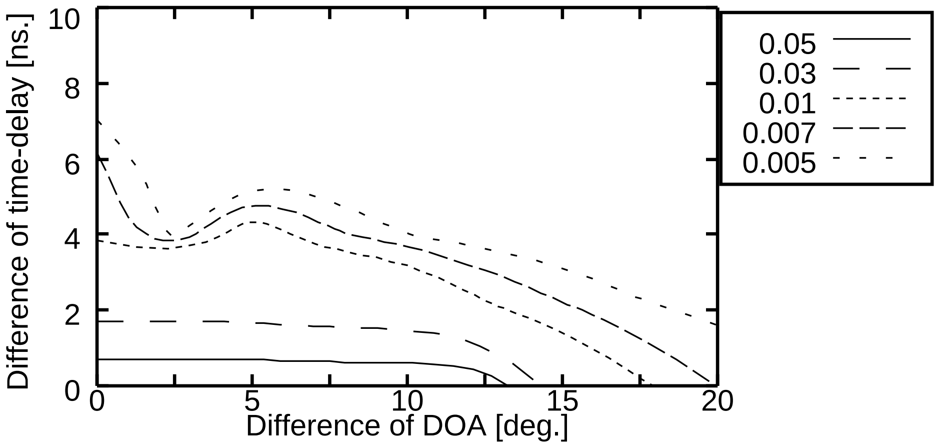


Figure 4: RMSE of parametric index 3. ($|\rho_c| = 0.9$)