THE TRANSFORMATION OF PLASMA OSCILLATIONS IN A TRANSIENT MAGNETIZED PLASMA WITH A TIME-VARYING EXTERNAL MAGNETIC FIELD

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1. Introduction

It is well known that the switching on of an external magnetic field in the plasma changes the type of electromagnetic oscillations in it, i.e. changes its properties. In a series of papers, the effects in a magnetized plasma whose density changes in time have been examined. As for background history of the problem, a reader ought to be referred to [1] where one can find rather complete list of previous publications.

The purpose of this paper is to consider the transformation of plasma oscillations caused by both a time variation in external magnetic field in plasma and a plasma density is considered. The time variations of magnetic field and plasma density are approximated by a sequence of step functions.

Let us consider an electromagnetic field in plasma when the external magnetic field and the plasma density vary in time starting from the zero moment. The electric intensity of the electromagnetic field, after the n-th jump in the external magnetic field, can be represented as the Volterra integral equation of the second kind [2]:

$$\boldsymbol{E}^{(n)}(t,\boldsymbol{r}) = \boldsymbol{F}^{(n)}(t,\boldsymbol{r}) + \int_{t_n}^{t} dt' \int_{\infty} d\boldsymbol{r}' \left\langle \boldsymbol{x} \middle| \hat{\Gamma} \middle| \boldsymbol{x}' \right\rangle \frac{\omega_e^2}{4\pi} \int_{t_n}^{t'} \hat{\alpha}^{(n)}(t'-t'') \boldsymbol{E}^{(n)}(t'',\boldsymbol{r}') dt'' \tag{1}$$

where $\left\langle \boldsymbol{x} \middle| \hat{\Gamma} \middle| \boldsymbol{x}' \right\rangle = \left(\nabla\nabla - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right) \frac{\delta\left(t - t' - \frac{1}{c}\left|\boldsymbol{r}-\boldsymbol{r}'\right| \right)}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|}$, t_n is the moment of the n-th jump in the external magnetic field, $\omega_e{}^2 = 4\pi e^2 n/m$ is the plasma frequency and $\boldsymbol{x} = (t, \boldsymbol{r})$ denotes the time-spatial four-vector. In these expressions, c is the speed of light, $\delta(t)$ is a Dirac delta function. The free term of this equation is equal to

$$\boldsymbol{F}^{(n)}(t,\boldsymbol{r}) = \boldsymbol{F}^{(0)}(t,\boldsymbol{r}) + \sum_{k=1}^{n-1} \int_{t_k}^{t_{k+1}} dt' \int_{\infty} d\boldsymbol{r}' \left\langle \boldsymbol{x} \middle| \hat{\Gamma} \middle| \boldsymbol{x}' \right\rangle \boldsymbol{P}_r^{(k)}(t',\boldsymbol{r}') + \int_{t_n}^{t} dt' \int_{\infty} d\boldsymbol{r}' \left\langle \boldsymbol{x} \middle| \hat{\Gamma} \middle| \boldsymbol{x}' \right\rangle \boldsymbol{P}_r^{(n)}(t',\boldsymbol{r}') \,. \tag{2}$$

The first term

$$\boldsymbol{F}^{(0)}(t,\boldsymbol{r}) = \int_{-\infty}^{0} dt' \int_{\infty} d\boldsymbol{r}' \left\langle \boldsymbol{x} \middle| \hat{\Gamma} \middle| \boldsymbol{x}' \right\rangle \frac{\omega_e^2}{4\pi} \int_{-\infty}^{t'} dt''(t'-t'') \boldsymbol{E}(t'',\boldsymbol{r}') \tag{3}$$

takes into account only the prehistory of the electromagnetic field interaction with the plasma until the beginning of the change in the magnetic field, the other terms describe after-effects caused by the external magnetic field jumps. Here, $\boldsymbol{P}^{(n)}$ is the polarization vector.

Let us consider that the external magnetic field is absent till the zero moment and from this moment it varies with time step by step being constant between jumps.

A solution to (1) can be obtained by the resolvent method in the following form $\boldsymbol{E} = \boldsymbol{F} + \hat{R}\boldsymbol{F}$.

With the aid of the resolvent, the formula for the field intensity in the magnitoactive plasma after the n-th jump in the magnetic field can be expressed as [3]:

$$\boldsymbol{E}^{(n)}(t,\boldsymbol{r}) = \boldsymbol{F}^{(n)}(t,\boldsymbol{r}) + \int_0^t dt' \int_\infty d\boldsymbol{r}' \left\langle \boldsymbol{x} \middle| R^{(n)} \middle| \boldsymbol{x}' \right\rangle \boldsymbol{F}^{(n)}(t',\boldsymbol{r}') \,. \qquad (4)$$

The expression for the resolvent for the sequence of jumps is the same as the resolvent for a single jump in the external magnetic field if the Larmor frequency $\Omega = eB_0/mc$ is changed from step to step.

2. The transformation of the plasma oscillations

Consider the initial field as that of an eigenwave of the plasma, i.e., a plane wave: $\boldsymbol{E}_0(t,\boldsymbol{r}) = \boldsymbol{E}_0 \exp[i(\omega t - \boldsymbol{sr})]$, where $s^2 = c^2(\omega^2 - \omega_e{}^2)$. Suppose that it exists till the external magnetic field in the plasma is switched on. Let us consider that the transformations of the wave are made for a single jump in the magnetic field. For this case a general expression which determines the transformed electromagnetic field for an arbitrary orientation of the external magnetic field is obtained. This expression is a bulky one so we do not write it here.

For the case of external magnetic field $\boldsymbol{B}_0$ orientated along the propagation direction of the primary wave, i.e. $\boldsymbol{b} \parallel \boldsymbol{s} = \{0,0,s\}$, $\boldsymbol{E}_0 = \{E_0,0,0\}$, the electromagnetic field is defined by the following expression [3]:

$$\boldsymbol{E}^{(1)}(t,\boldsymbol{r}) = \int_{-i\infty}^{i\infty} \frac{dp}{2\pi i} \frac{e^{pt - i\boldsymbol{sr}}}{\omega H(p)} \begin{Bmatrix} \omega(p+i\omega)Q^{(1)}(p) - i\omega_e{}^2\Omega^2(p^2+c^2s^2) \\ -i\omega_e{}^2\Omega\, p^2(p+i\omega) \\ 0 \end{Bmatrix}, \qquad (5)$$

where $H(p) = p^2(p^2+\omega^2)^2 + \Omega^2(p^2+c^2s^2)^2$, $Q^{(1)}(p) = p^4 + (\omega^2+\Omega^2)p^2 + \Omega^2c^2s^2$.

By substituting $\boldsymbol{s} = 0$ and $\omega = \omega_e$ into (5), we obtain the transformation for the plasma oscillations after a single jump in the external magnetic field. Substituting $\omega = \omega_e$ in (5) yields the transformation of plasma oscillations when the external magnetic field is switched on at the zero moment and its vector $\boldsymbol{b}$ is normal to the electric field in the plasma oscillations. The latter transforms into two elliptically polarized oscillations:

$$\boldsymbol{E}^{(1)}(t) = \sum_{k=1}^{2} \frac{E_0}{2(\omega_e{}^2 - \omega_k{}^2) + \Omega^2} \left\{ (\omega_e{}^2 - \omega_k{}^2 + \Omega^2)\cos\omega_k t, \quad \frac{\omega_e{}^2\Omega}{\omega_k}\sin\omega_k t, \quad 0 \right\} \qquad (6)$$

where $\omega_k{}^2 = \frac{1}{2}\left(2\omega_e{}^2 + \Omega^2 + (-1)^{k-1}\Omega\sqrt{4\omega_e{}^2 + \Omega^2} \right)$. The time dependence of the plasma oscillations field on the external magnetic field is shown in Fig. 1. The new frequencies of the plasma oscillations are presented in Fig. 2.

In the case of a weak magnetic field: $\Omega << \omega_e$, the both types of oscillations are almost circularly polarized and have the frequencies $\omega_k{}^2 \approx \omega_e{}^2 \pm \omega_k\Omega$, which are slightly different from each other.

$$\boldsymbol{E}^{(1)}(t) = \sum_{k=1}^{2} \frac{E_0\,\omega_e}{\Omega - (-1)^{k-1}2\omega_e} \left\{ \left[\frac{\Omega}{\omega_e} - (-1)^{k-1} \right]\cos\omega_k t, \ \left[1 - (-1)^{k-1}\frac{\Omega}{2\omega_e} \right]\sin\omega_k t, \ 0 \right\}. \qquad (7)$$

In the case of a strong magnetic field: $\Omega >> \omega_e$, the oscillation with the frequency $\omega_1 \approx \Omega$ will be a circularly polarized cyclotron oscillation with a small amplitude:

$$\boldsymbol{E}_1(t) = -E_0\frac{\omega_e{}^2}{\Omega^2}\{\cos\Omega t, \quad \sin\Omega t, \quad 0\}. \qquad (8)$$

The other oscillation will have the frequency $\omega_2{}^2 \approx \omega_e{}^4/\Omega_1{}^2$ and almost linear polarization:

$$\boldsymbol{E}_2(t) = -E_0\{\cos\omega_2 t, \quad \sin\omega_2 t, \quad 0\}. \tag{9}$$

When the external magnetic field is parallel to the electric field of the plasma oscillations, i.e. $\boldsymbol{b} \parallel \boldsymbol{E}_0 = \{E_0, 0, 0\}$, the latter are not changed

Formula (6) enables to obtain a general expression for transformation of plasma oscillations after n jumps in the external magnetic field.

$$\boldsymbol{E}^{(n)}(t) = \sum_{k=0}^{2^n-1}\left\{E_{nk}^x \cos\omega_{nk}t, \quad E_{nk}^y \sin\omega_{nk}t, \quad 0\right\} \tag{10}$$

where

$$E_{nk}^x = \left(E_{(n-1)[k/2]}^x + E_{(n-1)[k/2]}^y\right)\frac{\omega_{(n-1)[k/2]}^2 - \omega_{nk}^2 + \Omega_n^2}{2\left(\omega_{(n-1)[k/2]}^2 - \omega_{nk}^2\right) + \Omega_n^2},$$

$$E_{nk}^x = \left(E_{(n-1)[k/2]}^x + E_{(n-1)[k/2]}^y\right)\frac{\Omega_n}{\omega_{nk}}\frac{1}{2\left(\omega_{(n-1)[k/2]}^2 - \omega_{nk}^2\right) + \Omega_n^2},$$

$$\omega_{nk}{}^2 = \frac{1}{2}\left(2\omega_{(n-1)[k/2]}^2 + \Omega_n^2 + (-1)^k\,\Omega\sqrt{4\omega_{(n-1)[k/2]}^2 + \Omega_n^2}\right).$$

Here $[\cdot]$ denotes the integer part of the number.

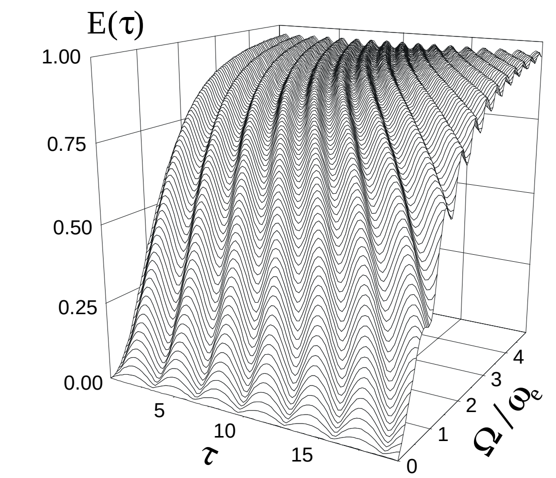

Figure 1. The time dependence of the plasma oscillations field after a single jump in the external magnetic field.

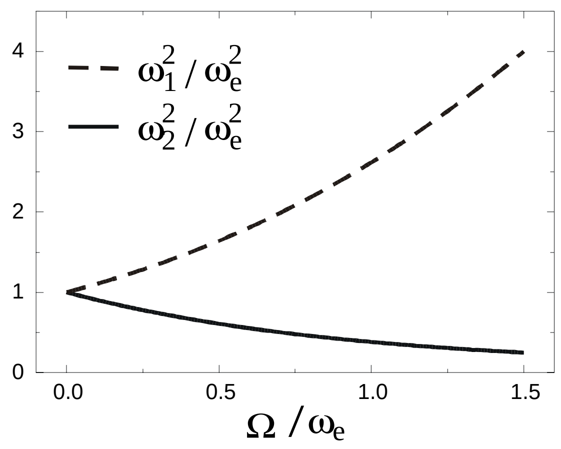

Figure 2. New frequencies of the plasma oscillations after a single jump in the external magnetic field

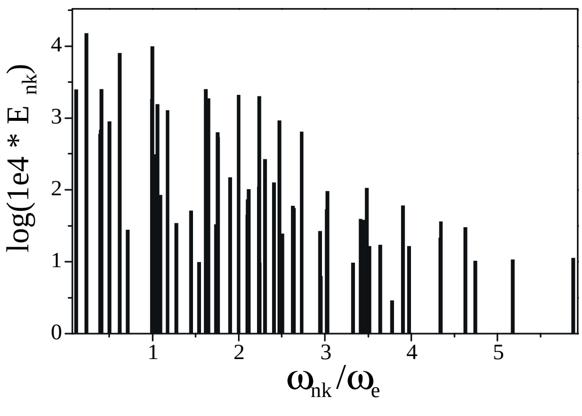

Figure 3. The spectrum of the plasma oscillations after $n = 5$ jumps in the external magnetic field for $\Omega_n / \omega_e = \sqrt{n}$.

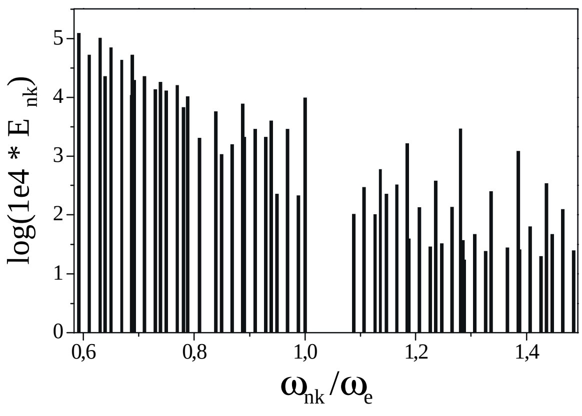

Figure 4. The spectrum of the plasma oscillations after $n = 5$ jumps in the external magnetic field for $\Omega_n / \omega_e = 1 / \left(1 + n^2\right)$.

The spectrums of the plasma oscillations for different laws of external magnetic field change after $n = 5$ jumps in the external magnetic field are presented in Fig. 3, 4. After each jump in the external magnetic field, each oscillation splits into two elliptically polarized oscillations. The distribution of the frequencies of the plasma oscillations has an essentially irregular character. A general feature of the spectrum is that when the frequencies of the plasma oscillations approach to zero their amplitude increases. Such a phenomena does not depend on the law of change of the external magnetic field. For the case of increasing external magnetic field, the frequencies of the plasma oscillations have the spectrum wider than that for the case of decreasing field. The displacement of the spectrum to the high frequency area is also takes place.

3. Conclusion

In this paper, a solution to the problem of electromagnetic wave propagation in a plasma with a time-varying magnetic field has been obtained. The method which is used allows to consider the simultaneous changes in the external magnetic field as well as in the plasma density. The problem is formulated as the Volterra integral equations and its solution is made by the resolvent method.

A general expression for transformation of the plasma oscillations after n jumps in the external magnetic field is obtained. The spectrum of the plasma oscillations for different laws of change of the external magnetic field is presented. It is shown that the switching of the magnetic field that is normal to the electric field in plasma oscillations transforms the oscillations into two elliptically polarized oscillations with different frequencies. In the case of a strong magnetic field, the first of these oscillations has almost a circular polarization and frequency close to the cyclotron one, and the second has almost linear polarization and frequency close to the zero frequency. In the case of a weak magnetic field, the oscillations have the frequencies which are slightly different, and the both are almost circularly polarized. The switching on of the magnetic field parallel to the electric field does not affect the latter.

4. References

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[3] A.G. Nerukh, K.M. Yemelyanov, "Electromagnetic Signal Evolution in a Nonstationary Plasma with a Magnetic Field Jump", *Journal of Applied Electromagnetism*, Vol. 2, No 3, pp. 51-64, 1999.