SELF-CONSISTENT THEORY OF TRIGGERED VLF EMISSIONS: AN ANALYTICAL APPROACH

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ABSTRACT

The self-consistent analytical approach to the problem of triggered ELF/VLF emissions is developed. The self-consistency includes calculation of energetic electron beams produced by a quasimonochromatic whistler wave packet from a ground based VLF transmitter and analysis of secondary whistler wave generation by this beam, taking into account the second order cyclotron resonance effects. The theory permits estimation of the maximum amplification of the secondary waves as well as dynamical frequency spectrum of triggered emission. A simple computer program has been developed to find the dynamic spectrogram of triggered VLF signals with spatial and temporal dependences of electron beam velocity and geomagnetic field.

INTRODUCTION

Elaboration of the theory of discrete ELF/VLF emissions, and in particular, ELF/VLF emissions triggered by signals from ground based VLF transmitters, has a long history. *Helliwell* (1967) was the first who formulated some principal questions of this problem in the frame of his phenomenological theory. Important analytical and computational results were obtained by *Nunn* (1986), *Karpman et al.* (1974), and *Matsumoto et al.* (1979). Similar results and some new effects were obtained by *Vomvoridis et al.* (1982) and *Molvig et al.* (1988). The excellent review by *Omura et al.* (1991) describes the history of this question. Further computer simulations, using new sophysticated computer codes, have been developed recently (e.g., *Nunn et al.*, 1997). At the same time the theory of the second order cyclotron resonance (*Trakhtengerts et al.*, 1999) and some common features of computer simulation provided the basis for analytical approach to this problem. This analytical approach is given below.

SELF-CONSISTENT THEORY OF TRIGGERED ELF/VLF EMISSION

The self consistent theory of triggered ELF/VLF emission includes two principal elements:
1) Calculation of electron beams, produced by the pump whistler wave packet.
2)Analysis of the generation of secondary whistler waves (a triggered emission) by this beam.
Calculations of electron beams are based on the use of two integrals of motion for electrons trapped by the initial (pump) whistler wave in an inhomogeneous magnetic field.

$$v_{||} = v_R \equiv \frac{\omega_0 - \omega_H(z)}{k_0} \qquad (1)$$

$$W - \frac{\omega}{\omega_H} W_{||} = \text{const} \qquad (2)$$

where $v_{||}$ is the velocity component along the magnetic field H, $W_{||} = mv_{||}^2/2$, $W = mv_{||}^2/2 + mv_{\perp}^2/2$, $v_{\perp}$ is the component across H, ω_0 and k_0 are the frequency and the wave vector of the initial wave respectively, and $\omega_H(z)$ is the electron gyrofrequency. The relations (1) and (2) should be considered together with the condition for trapping of electrons by this wave. This condition determines the boundary point z_A depending mainly on the wave amplitude and the background electron plasma density. In the case of a dipole magnetic field ($z_A/r_L < 1$), resonance electrons can be trapped by the wave, if $z \leq z_A$, where

$$z_A \approx \pm \frac{k_0 r_L^2}{9\sqrt{2}} h_L \tag{3}$$

where $k_0 \simeq (\omega_{pL}^2 \omega / c^2 \omega_{HL})^{1/2}$, $h_L = b_L/H_L$, $r_L = R_0 L$ (L is the McIlwain parameter), b_L is the magnetic pump wave amplitude, R_0 is the Earth radius, ω_{pL} is the electron plasma frequency, subscript L determines the values in the equatorial cross section, and c is the light velocity. As it follows from (1)-(2) the net acceleration (with the growth of $W_\perp$ and decreasing of $W_{||}$) takes place in the case, when the trapping length of electrons moving in the direction of decreasing magnetic field is longer than that in the direction of increasing magnetic field. We suggest that triggered emission appears only when the net acceleration of the electrons is large enough. On the other hand, the particle trapping is possible if the forward front of the pump wave is going closer to the equator than the boundary for trapping. From those two conditions (the net accerelation and the condition for trapping) it follows that the initial pulse duration τ_0 must be shorter than $(2|z_A|/v_g)$. Taking into account (3) and the expression for the whistler group velocity v_g we obtain

$$\tau_0 \leq \frac{(k_0 r_L)^2 h_L}{9\sqrt{2}\omega(1-\omega/\omega_{HL})^{7/4}} \tag{4}$$

For example with $L \simeq 4$, $k_0 \sim 1.3$ km^{-1}, $b_L \sim 0.1\gamma$ and $\omega \sim \omega_{HL}/2$, the inequality (4) gives: $\tau_0 < 1$s. Growing or nonlinear damping of a pump wave can change the criterion (4). At the same time, there is a limitation for short pulses. The matter is fact, the acceleration in accordance with (1)-(2) is possible when the pulse duration is much longer than the oscillation period of trapped electrons T_{tr}:

$$\tau_0 \gg T_{tr} = 2\pi(\omega_b k v_\perp)^{-1/2} \tag{5}$$

where $\omega_b = eb/mc$, b is the wave magnetic field amplitude. If $b \simeq 0.1\gamma$ and $L \sim 4$, $kv_\perp \sim \omega_{HL}$, $T_{tr} \sim 5$ms.

THE GENERATION OF THE SECONDARY WHISTLER WAVES

As a result of acceleration by the pump whistler wave, an electron beam with small velocity dispersion appears at the exit from the backward front of the initial wave packet. The beam velocity is changing as a function of z and t in agreement with (1)-(2) and the position of propagating pump pulse. Now we consider the generation of secondary whistler waves by this beam. Its quantitative consideraion is based on the theory of beam-plasma hydromagnetic instability in an inhomogeneous magnetic field developed by *Trakhtengerts et al.* (1999). According to this theory, it is necessary to pass to the new (natural) variables instead of z and t:

$$\xi = t - \int_{z_*}^{z} \frac{dz'}{v_*(z', W_*, z, t)} \qquad \eta = t + \int_{z_0}^{z} \frac{dz'}{v_g(z', \omega)} \tag{6}$$

where $z_*(t)$ is the injection point (the coordinate of the backward front of the initial packet), v_* is the field-aligned component of the electron velocity after its ejection and further propagation along an inhomogeneous magnetic field, and v_g is the group velocity modulus of a whistler wave with frequency ω and wave vector k. The variable ξ is connected with beam (the parallel component of beam energy W_* which depends on ξ only), and η is connected with the generated whistler wavelet (the frequency ω is a function of η in common case). In this case, the full logarithmic amplification of a secondary wavelet can be written in the general form (using the linear approximation relative to the generated whistler wave, see *Trakhtengerts et al*, 1999).

$$\Gamma(\eta, \xi) = \alpha_{\text{eff}}^2 \int_{\xi_0(\eta)}^{\xi} d\xi' \int_{\eta_0(\xi')}^{\eta} d\eta' \cos\left[\int_{\eta''}^{\eta} \varphi(\eta'', \xi')\, d\eta''\right] \tag{7}$$

where $\Gamma = \ln A_{exit}/A_{ent}$, A_{ext} (A_{ent}) are the wave amplitudes at the exit (entrance) of the amplification region, $\alpha^2_{\rm eff}$ is proportional to the electron beam density, and the phase φ is equal to:

$$\varphi = \frac{\Delta v_g}{v_\Sigma}, \quad \Delta = \omega + kv_* - \omega_H, \quad v_\Sigma = v_g + v_* \tag{8}$$

The expression (7) is written for the step-like distribution function; a similar relation can be obtained for the δ-function distribution function. In both cases the dynamical frequency spectra are the same but the magnitude of $\Gamma_{\rm max}$ is different. The phase φ in (8) contains the large parameter,

$$p_0 \equiv \frac{a\omega_{HL}}{v_0} \gg 1 \tag{9}$$

where $a = (\frac{1}{H}\frac{dH}{dz})^{-1}$, v_0 is the characteristic beam velocity, and we can analyze the maximum value of the expresssion (9) for a typical case. The $\Gamma_{\rm max}$ is reached when the 1st order and the 2nd order cyclotron resonance conditions are fulfiled:

$$\Delta(\xi, \eta, \omega) = 0 \ \ \text{and} \ \ (\partial\Delta/\partial\xi) = 0 \tag{10}$$

The maximum value of Γ from (7) is achieved under the conditions in (10) and is equal to

$$\Gamma_{\rm max} \simeq \Gamma_0 \cdot \ P^{1/3} \tag{11}$$

where P is determined by (9), Γ_0 is the cyclotron amplification when the condition for second order CR ($\partial\Delta/\partial\xi = 0$) is not fulfilled [*Trakhtengerts et al*, 1998]. For realistic conditions, $P \sim 10^3 \div 10^4$, and the gain ($\mathrm{P}^{1/3}$) can be very large.

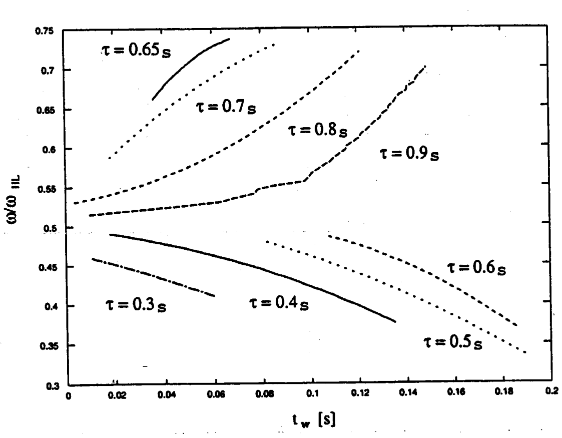
NUMERICAL COMPUTATIONS

The relations (10) determine the dynamical frequency spectrum of the triggered emission. The electron beam energy $W_*(\xi)$ and the first adiabatic invariant $I_\perp(\xi) = v_\perp^2/\omega_H$ which come into (10) are found from the solution of the acceleration problem formulated in the section 1. In our calculations we put $\omega_p^2/\omega_H \approx \text{const} = \omega_{pL}^2/\omega_{HL}$. Actually we have two known surfaces $\Delta = 0$ and $\partial\Delta/\partial\xi = 0$ in the coordinate space (ξ, η, ω_n), the cross section of which gives the curve $\omega_n(\xi, \eta)$. This curve determines, together with the relations (6), the dynamical spectrum of discrete triggered VLF emission.

RESULTS AND CONCLUSION

All the results, given below have been obtained for L =4 (ω_{HL}=8.48$\times$10^{4}s^{-1}), the cold plasma density n_c=100cm^{-3}, the pump wave frequency (ω_0/ω_{HL})=0.5. In this paper we investigate the dynamical frequency spectrum of triggered emission as a function of the pump pulse duration τ_0 and the initial value of the 1st adiabatic invariant at the entrance to the acceleration region (i.e., forward front of the pump wave). It is suggested that the initial field-aligned velocity component is equal to the resonant velocity $v_{R0} = \frac{\omega_0 - \omega_H(z_0)}{k_0}$, where z_0 is the coordinate of the forward front. The results of the computations are shown in Figures 1 and 2 for a pump wave amplitude b=10^{2}mγ. This amplitude is very big, but analysis shows that the beams formed have qualitatively the same kinematic characteristics for weaker pump waves (down to $\sim$ 10mγ). We believe that qualitative results will be valid for smaller amplitudes too.

Our analysis yields the important general conclusion that short pulses ($\tau_0 < z_A/v_{g0}$) excite falling tones and long pulses ($z_A/v_{g0} < \tau_0 < 2z_A/v_{g0}$) excite risers. The z and t dependence of the velocity of electron beam ejected by the pump wave is principal here. This dependence is opposite to the adiabatic change of electron velocity in an inhomeogeneous magnetic field and it permits the second order cyclotron resonance to be realized. Indeed above results are only the part of the problems of triggered VLF/ELF emissions. Phase bunching effects under the beam formation, which have not been taken into account, are very important. But this question is outside of our consideration.



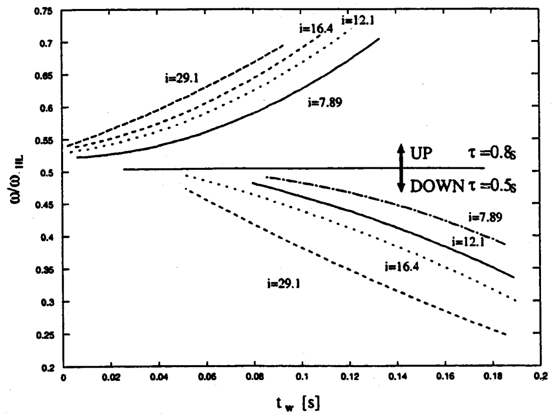


Fig. 1. (left) The dynamic spectrum of the calculated VLF triggered emissions for the different pulse lengths (τ=0.3, 0.4, 0.5, 0.6, 0.65, 0.7, 0.8, 0.9s).

Fig. 2. (right) Same variation for the different initial adiabatic invariants ($I_\perp 0 = i \times 10^{10} \mathrm{m}^2\mathrm{s}^{-1}$($i$=7.89, 12.1, 16.4, 29.1))under two different pulse length τ=0.5 and 0.8s.

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