AMPITUDE FLUCTUATIONS OF MILLIMETER WAVE BEAM PROPAGATING IN TURBULENT AND ABSORPTIVE ATMOSPHERE

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The study of millimeter wave propagation in the near-earth atmosphere has received increasing attention in several decades. This interest is primarily due to the promising large bandwidths at these high frequencies of electromagnetic spectrum. However, the atmosphere irregularities affect on wave parameters. One of the problems of theory of wave propagation in randomly inhomogeneous media is the problem of the development of statistical characteristics of field in the chaotic absorptive medium. The effect of absorption mechanism on scintillation fading has been studied theoretically in several scientific works (see, for example [1-3]). Mainly it was considered the case of plane wave. Only one publication [4] was devoted to wave beam. However, obtained results not shown the good agreement with experimental measurements. Therefore, it is important to more explicitly investigate the amplitude scintillations of millimeter wave beam in the near-earth turbulent and absorptive boundary layer.

One of the most full characteristics of amplitude fluctuations is the two-frequency spatially-temporal correlation function, which is given by

$$\Psi(\vec{r}_1,\vec{r}_2,w_1,w_2,t_1,t_2)=\left\langle c(\vec{r}_1,w_1,t_1)\cdot c(\vec{r}_2,w_2,t_2)\right\rangle, \qquad (1)$$

where $\vec{r}_j$ is the radius-vector of observation point, w_j is the angular frequency of radiation, t_j is the time, $j=1,2$.

The theoretical analysis of fluctuation was carried out by the first approximation of the smooth perturbation method (Rytov method). The scintillations of wave beam in the atmospheric windows were well studied in [5]. In order to take into account the absorption effects the atmosphere must be characterized by the complex refractive index. Assuming the x axis as the direction of the beam propagation, the magnitude of field can be written as

$$E(\vec{r},w,t)=E_0(\vec{r},w,t)\exp(\Phi_1(\vec{r},w,t)), \qquad (2)$$

where $E_0(\vec{r},w,t)$ is the field in the absence of turbulence and $\Phi_1(\vec{r},w,t)$ is complex phase. Let us consider the initial field distribution to be Gaussian. Also, the complex refractive index is statistically homogeneous and isotropic, and its fluctuations are d-correlated along the propagation direction. Using these assumptions and the spectral representation, the complex phase Φ_1 is given by

$$\Phi_1(L,\vec{r},w,t)=\frac{w}{c}\int_0^L dx\int_{-\infty}^{\infty}H(P_*)\exp(i\vec{k}\vec{r}P_*)\left(dn(x,\vec{k},t)+idm(x,\vec{k},t)\right), \qquad (3)$$

where

$$H(P_*)=i\exp\left[-iP_*(L-x)k^2\frac{c}{2w\langle e\rangle^{1/2}}\right];\ P_*=\frac{\langle e\rangle^{1/2}+iP(x)}{\langle e\rangle^{1/2}+iP(L)};\ \langle e\rangle^{1/2}=\langle n\rangle+i\langle m\rangle;$$

$P(x)=\dfrac{xc}{wr_e^2}$ is wave beam parameter, r_e is the initial effective radius of beam, $\vec{r}$ determine the radius-vector of the observation point in the reception plane $x=L$, $dn(x,\vec{k},t)$ and $dm(x,\vec{k},t)$ are the spectral amplitudes of real $n(x,\vec{k},t)$ and imaginary $m(x,\vec{k},t)$ parts of the refractive index fluctuations, $\vec{k}$ is the spatial wave number, $\tilde{n}$ is the light velocity in vacuum. Using the Taylor's "frozen" turbulence hypothesis, after statistical averaging we obtain the following expression for two-frequency spatially-temporal correlation function [6]

$$\Psi(L, \vec{r}_1, \vec{r}_2, w_1, w_2, t) = \frac{2p^2}{c^2} w_1 w_2 \int_0^L dx \int_0^\infty \{(G_n(k) + G_m(k)) \operatorname{Re}[H(P_{1*}) H^*(P_{2*}) J_0(kD)] +$$

$$+ (G_n(k) - G_m(k)) \operatorname{Re}[H(P_{1*}) H(P_{2*}) J_0(kC)] - 2G_{nm}(k) \operatorname{Im}[H(P_{1*}) H(P_{2*}) J_0(kC)]\} k dk \ , \quad (4)$$

where

$$D = \frac{1}{r_1} \sqrt{\left\{ r_1^2 P_{1*} - (\vec{r}_2 \cdot \vec{r}_1) P_{2*}^* + (\vec{V} \cdot \vec{r}_1) t \right\}^2 + \left\{ [\vec{r}_2 \times \vec{r}_1] P_{2*}^* - \left| [\vec{V} \times \vec{r}_1] \right| t \right\}^2} \ ,$$

$$C = \frac{1}{r_1} \sqrt{\left\{ r_1^2 P_{1*} - (\vec{r}_2 \cdot \vec{r}_1) P_{2*} + (\vec{V} \cdot \vec{r}_1) t \right\}^2 + \left\{ [\vec{r}_2 \times \vec{r}_1] P_{2*} - \left| [\vec{V} \times \vec{r}_1] \right| t \right\}^2} \ ,$$

$t = t_2 - t_1$, $G_n(k)$, $G_m(k)$, $G_{nm}(k)$ are the spatial spectral density functions of the real, imaginary parts of the refractive index and their co-spectral density function, respectively, J_0 is the Bessel function of first kind, zeroth order, $(\vec{a} \cdot \vec{b})$ and $[\vec{a} \times \vec{b}]$ are denote the scalar and vector multiplications of vectors, $\vec{V}$ is the transverse displacement velocity of the turbulent irregularities. In the absence of absorption, when $G_m(k) = G_{nm}(k) = 0$, the obtained expression is consisted with the formulae given in [5].

In order to investigate the behaviour of the correlation function (4) it is necessary to make use of the three spatial spectra related to the complex refractive index. Most models refer only to real part of the refractive index $G_n(k)$. A von Karman two-parametric spectrum is usually used to study the chaotic variations of millimeter wave beam propagating in the atmospheric windows [7,8]. This spectrum permits to take into account the saturation effect of the refractive index fluctuations in near-earth boundary layer. Therefore, in contrast to [4] a von Karman two-parametric model will be extended to the spectrum of the imaginary part of the complex refractive index. In this case

$$G_n(k) = 0.033 C_n^2 (k^2 + k_0^2)^{-11/6} \exp\left(-\frac{k^2}{k_+^2} \right),$$

$$G_m(k) = 0.033 C_m^2 (k^2 + k_0^2)^{-11/6} \exp\left(-\frac{k^2}{k_+^2} \right), \quad (5)$$

$$G_{nm}(k) = 0.033 C_{nm}^2 (k^2 + k_0^2)^{-11/6} \exp\left(-\frac{k^2}{k_+^2} \right),$$

where

$$k_0 = \frac{2p}{L_0}, \ k_+ = \frac{5{,}92}{l_0},$$

C_n^2 and C_m^2 are the real and imaginary refractive index structure parameters, respectively, C_{nm}^2 is the joint structure parameter, L_0 and l_0 are the outer and inner scale sizes of the turbulence. It should be noted that the structure parameters in (5) are frequency dependent. In common case the any structure parameter can be obtained by parameters C_{j1}^2 and C_{j2}^2 at two different frequencies w_1 and w_2 as

$$C_j^2 = \sqrt{C_{j1}^2 C_{j2}^2} \ ,$$

where the index j denote the real, imaginary or joint parameter. The value of C_{ji}^2 at the single frequency can be determined by meteorological conditions. They are mainly dependent on the temperature and the water vapor pressure fluctuations (see, for example in [2]).

Equations (4)-(5) permit to investigate simultaneously the turbulent atmosphere effect on amplitude scintillations in the time, space and frequency domain for different beam parameter, path characteristics and propagation conditions.

In order to investigate the effect of absorption on magnitude of the amplitude fluctuations of the wave beam it is convenient to consider the variance s_c^2. Using the equation (4), the following relationship can be written

$$s_c^2 = \Psi(L, \vec{r}, \vec{r}, w, w, 0) = \frac{2p^2 w^2}{c^2} \int_0^L dx \int_0^\infty \{(G_n(k) + G_m(k)) \operatorname{Re}\left[\left|H(P_*)\right|^2 J_0(kD)\right] + $$
$$+ (G_n(k) - G_m(k)) \operatorname{Re}\left[H^2(P_*)\right] - 2G_{nm}(k) \operatorname{Im}\left[H^2(P_*)\right]\} kdk \qquad (6)$$

where $D = 2ir \operatorname{Im} P_*$. This equation permits to determine the variance of fluctuations in any observation point of the plane $x = L$. For case of plane wave assuming $P = 0$, we obtain the relationship similar to formula studied in [1-3].

Let us consider the fluctuations on the beam axis. We can rewrite the equation (6) as a sum of three integrals

$$I_n = \frac{1.3028 \cdot w^2}{c^2} \int_0^L dx \int_0^\infty \tilde{G}(k) \operatorname{Re}^2\left(H(P_*)\right) kdk \ ,$$
$$I_m = \frac{1.3028 \cdot w^2}{c^2} \int_0^L dx \int_0^\infty \tilde{G}(k) \operatorname{Im}^2\left(H(P_*)\right) kdk \ , \qquad (7)$$
$$I_{nm} = \frac{1.3028 \cdot w^2}{c^2} \int_0^L dx \int_0^\infty \tilde{G}(k) \operatorname{Im}\left(H^2(P_*)\right) kdk \ ,$$

where

$$\tilde{G}(k) = (k^2 + k_0^2)^{-11/6} \exp\left(-\frac{k^2}{k_+^2}\right).$$

Then the equation (6) is convenient to rewrite as [2]

$$s_c^2 = s_0^2 (1 + \Delta) \ , \qquad (8)$$

where

$$s_0^2 = I_n \cdot C_n^2 \ , \ \Delta = \frac{I_m}{I_n} \frac{C_m^2}{C_n^2} - \frac{I_{nm}}{I_n} \frac{C_{nm}^2}{C_n^2} \ . \qquad (9)$$

The parameter Δ is the measure of the magnitude of the absorption-induced fluctuations relative to the pure scattering-induced fluctuations.

The numerical estimations of Δ were carried out at the 60 GHz frequency band, which correspond to oxygen absorption lines. The magnitudes of the structure parameters $C_n^2 = 2.39 \cdot 10^{-13} m^{-2/3}$, $C_{nm}^2 = 5.9 \cdot 10^{-15} m^{-2/3}$, $C_m^2 = 4.57 \cdot 10^{-16} m^{-2/3}$ were taken as the most possible values [4]. The results of calculations for the various values of the dimensionless parameter $Xk = k_0 \sqrt{Lc/w}$ are represented in figure. The analysis of the dependencies has shown that the absorption may influence on the amplitude scintillations. It can be seen from this figure, the enhancement of the variance s_c^2 is increases as the parameter Xk determined by the ratio of the first Fresnel zone to the outer scale size of turbulence decreases. Also, values of Δ for the wave beam are significant more the same values for the plane wave in the case when $Xk \geq 0.2$. This feature was noted in [4], but our results exceed the theoretical estimations obtained in [4] and show better agreement with the experimental data. This is apparently due to using spatial spectrum model. The maximum magnitudes of the enhancement are observed for $P \approx 4$. The exact value of the wave beam parameter depends on the magnitude of Xk. Otherwise when $Xk < 0.2$ the enhancement may even become negative and minimum values of Δ are took place for the wave beam. This suppression of the amplitude fluctuations has been experimentally observed at the near-ground path [9].

It should be noted that as followed by (9) the magnitude of Δ is dependent on the structure parameters, which are determined by the variance of temperature and humidity. Therefore, under the other meteorological conditions the behaviour of Δ shall be different then represented curves at figure. This effect was considered in [2] for the case of the plane wave. It has revealed that when water vapor pressure turbulence begins to dominate, enhancement of the amplitude scintillations is suppressed and can be considered to be effectively nonexistent even for the large scale sizes of turbulence (the little values of the parameter Xk. Doubtless in such meteorological conditions the amplitude fluctuations of the wave beam must be distinctive. This is confirmed by the experimental measurements published in [2]. It has been found that the value of Δ was as positive (enhancement of the scintillations) and so negative (suppression) for the same line-of-sight link.

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