BOUNDARY DIFFRACTION WAVE AT MULTIPLE KNIFE-EDGE DIFFRACTION

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1. Introduction

At prediction of radiowave attenuation on terrestrial paths with irregular profiles is widely used the model of number successive absorbing knife-edge obstacles[1-6]. In the earlier paper concerning this problem it were suggested heuristic methods. Only in [3] successive using of Fresnel-Kirchoff theory for double diffraction was considered. More recently computation of multiple diffraction integral was introduced [5,7-9].

Fresnel-Kirchoff diffraction theory is used in all this papers. The surface of integration in corresponding integrals consist of Huyghens's sources in the plane of each obstacles.

It is interesting to get diffraction field as boundary wave, successively scattered by the edges of obstacles. Element of edge of each obstacle can be obviously called as Young's source.

The purpose of the present paper is to generalize the Maggi-Rubinowicz technique to the case of multiple diffraction.

2. Theory

Figure 1 shows the geometry associated with the multiple knife-edge diffraction.

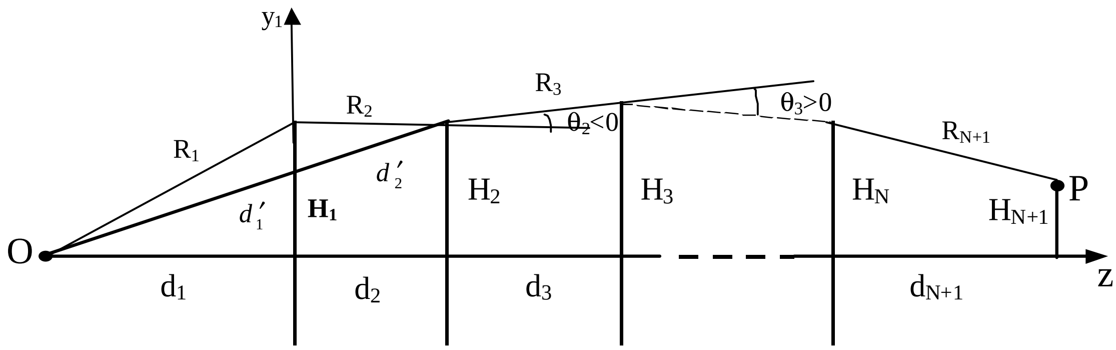

Fig. 1 Geometry of the problem

Between a point source O situated at the origin of cartesian coordinates and point of observation $P(0, H_{N+1},d_1+d_2+...+d_{N+1})$ N perfectly absorbing knife-edge obstacles perpendicularly to z axis are placed (Fig. 1). Edge of obstacles has coordinate $y=H_j$. Assuming $H_j<<d_j,d_{j+1}$ (j=1,...,N) i.e. small angles diffraction is considered. Lets edge's element of jth obstacles to be dl_j. It is necessary to find field from the source on an arbitrary element dl_2. For this purpose connect the point O and dl_2 by straight line. Its intersection with plane x_1y_1 contains the origins of polar (ρ,φ) coordinate system(Fig. 2). Using Fresnel-Kirchoff diffraction formula the field dE_2 on the element dl_2 reradiated by Hyughens's sources of sector $d\varphi$ can be found as

$$dE_2 = -\frac{i}{\lambda}\int_{\rho_1}^{\infty}\frac{e^{ik(r_1'+r_2')}}{r_1'r_2'}\rho\cdot d\rho\cdot d\phi \qquad (1)$$

where $r_1' = \sqrt{d_1'^2+\rho^2}$, $r_2' = \sqrt{d_2'^2+\rho^2}$

By means of usual Fresnel approximation one can find

$$dE_2 = -\frac{i}{\lambda}\frac{e^{ik(d_1+d_2)}}{d_1 d_2} d\varphi \cdot \int\limits_{\rho_1}^{\infty} \exp\left(i\frac{\pi}{b_1^2}\rho^2\right)\rho \cdot d\rho = \frac{e^{ik(d_1'+d_2')}}{d_1'+d_2'}\frac{d\varphi}{2\pi}\exp\left(i\frac{\pi}{b_1^2}\rho_1{}^2\right) \quad (2)$$

where $b_1^2 = \frac{\lambda d_1' d_2'}{d_1' + d_2'}$ and as usual, medium of propagation is considered as slightly absorbing. It is evident from Fig. 2 that

$$d\varphi = \frac{dl_1}{\rho_1}\sin(\rho_1, dl_1), \quad (3)$$

and (2) becomes

$$dE_2 = \frac{1}{2\pi}\frac{e^{ik(r_1+r_2)}}{d_1'+d_2'}\frac{\sin(\rho_1, dl_1)}{\rho_1}\cdot dl_1, \quad (4)$$

where in exponential factor the following relationship is used:

$$r_1 + r_2 = \sqrt{d_1'^2 + \rho_1^2} + \sqrt{d_2'^2 + \rho_1^2} \cong d_1' + d_2' + \frac{\rho_1^2}{2d_1'} + \frac{\rho_1^2}{2d_2'} \quad .$$

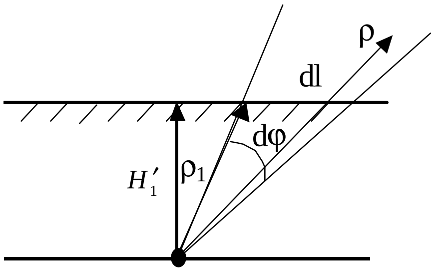

Fig. 2. To calculation of elementary boundary wave

So we obtain the field of Young's source dl_1 due to the incidence wave $E_1=exp(ikr_1)/r_1$ on a diffraction edge. Comparison of expression (4) with known result for boundary wave [10,11] shows agreement between them at small angels of diffraction.

Equation (4) may be written as

$$dE_2 = E_1 dD_1 \frac{e^{ikr_2}}{r_2}. \quad (5)$$

Here dD_1 is elementary coefficient of diffraction. The element dl_2 reradiates the spherical wave and (5) is presented in GTD format with

$$dD_1 = \frac{1}{2\pi}\frac{r_1 r_2}{d_1'+d_2'}\frac{\sin(\rho_1, dl_1)}{\rho_1} dl_1 \cong \frac{1}{2\pi}\frac{d_1 d_2}{d_1+d_2}\frac{\sin(\rho_1, dl_1)}{\rho_1} dl_1 .$$

Repeating the process and performing integration over all dl_j (j=1,2,...,N) one obtains

$$E_{N+1}(P) = \int\limits_{l_1}\ldots\int\limits_{l_N} \frac{\exp\left(ik\sum_{j=1}^{N+1} r_j\right)}{\prod_{j=1}^{N+1} r_j}\prod_{j=1}^{N} dD_j , \quad (6)$$

where $dD_j = \frac{1}{2\pi}\frac{d_j d_{j+1}}{d_j + d_{j+1}}\frac{\sin(\rho_j, dl_j)}{\rho_j} dl_j$. After introducing coordinate system (x_j, y_j) in the plane of jth screen with origin on z axis it can be written

$$\left.\begin{aligned} dl_j = dx_j, \qquad \sin(\rho_j, dl_j)/\rho_j = H_j'/\rho_j^2 \\ H_j' = H_j - \frac{H_{j-1}d_{j+1} + H_{j+1}d_j}{d_j + d_{j+1}} \\ \rho_j^2 = H_j'^2 + \left(x_j - \frac{x_{j-1}d_{j+1} + x_{j+1}d_j}{d_j + d_{j+1}}\right)^2 \\ r_j = \sqrt{R_j^2 + (x_j - x_{j-1})^2} \cong R_j + (x_j - x_{j-1})^2 / 2R_j \end{aligned}\right\}, \quad (7)$$

where R_j is the distance between *(j-1)*th and *j*th edges.

Changing variables of integration

$$t_j = \frac{\sqrt{\pi} x_j}{b_j}, \quad \text{where } b_j = \sqrt{\frac{\lambda d_j d_{j+1}}{d_j + d_{j+1}}}$$

and using (7) we get

$$E_{N+1}(P) = C_N \cdot \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \frac{\exp\left(\sum_{j=1}^{N-1} \left[t_j^2 - 2\beta_j t_j t_{j+1} \right] + t_N^2 \right)}{\prod_{j=1}^{N} \left[\upsilon_j^2 + \left(t_j - \beta_{j-1} t_{j-1} - \beta_j t_{j+1} \right)^2 \right]} dt_1 \dots dt_N , \tag{9}$$

where $C_N = \frac{e^{ikR_{N+1}}}{(2\pi)^N} \prod_{j=2}^{N} d_j \cdot \prod_{j=1}^{N} \upsilon_j \exp(R_j) \Big/ \prod_{j=1}^{N} \left(d_j + d_{j+1} \right)$, $\upsilon_j = \frac{\sqrt{\pi} H'_j}{b_j}$ and $\beta_j = \sqrt{\frac{d_j d_{j+2}}{(d_j + d_{j+1}) \cdot (d_{j+1} + d_{j+2})}}$, $\beta_N = 0$.

Expression (9) is obtained from the assumption that edge of each obstacle shadows the edge of next one. When $\theta_j<0$ then $dE_{j+1} \sim 1 - \exp(i\pi\rho_j^2 / b_j^2)$, or in general case

$$dE_j \sim \chi(-\theta_j) + \text{sgn}(\theta_j) \exp\left(i\pi\rho_j^2 / b_j^2 \right),$$

where χ - Heaviside function.

Let $\theta_j<0$, the (9) becomes

$$E'_{N+1}(P) = E^j{}_{N+1}(P) - E_{N+1}(P), \tag{10}$$

where $E^j_{N+1}(P)$ is multiple integral of order N-1 without integration on variable t_j and it must be replaced $d_j \rightarrow d_j + d_{j+1}$, $d_{j+1} \rightarrow d_j + d_{j+1}$.

3. Calculation.

There are various techniques of multiple integrals calculation. For instance in [7,8] amplitude and phase of integrand are approximated by the functions making multiple integral computable. In the [5] multiple integral was expanded in series of terms involving functions known as repeated integral of the error function. Integrals (9) and (10) can be estimated by method of stationary phase for multiples integrals [12].

Following [5] (9) can be represented as

$$E_{N+1}(P) = C_N \sum_{l_1,\dots,l_N=1}^{\infty} (2i)^{\sum_{j=1}^{N} l_j} \prod_{j=1}^{N-1} \frac{\beta_j^{l_j}}{(l_j)!} \prod_{j=1}^{N} I_{l_{j-1}+l_j}(\upsilon_j), \tag{11}$$

where $I_n(x) = \int_{-\infty}^{\infty} \frac{s^n \exp(is^2)}{x^2 + s^2} ds$, $l_0 = l_N = 0$. Obviously when n=2m+1 (m=0,1,2,....), $I_{2m+1}(x)=0$. For n=2m it may be found that

$$I_{2m}(x) = 2\Gamma\left(m + \frac{1}{2} \right) \exp\left(i\pi \frac{2m-1}{4} \right) x^{2m-1} K_{2m}(x) \tag{12}$$

with $K_{2m}(x) = \int_{x}^{\infty} \frac{\exp(is^2)}{s^{2m}} ds$. It may be shown that $K_{2m}(x)$ satisfies recursive relationship:

$$K_{2m}(x) = \frac{1}{2m-1}\left(x^{1-2m}\exp(ix^2) + 2iK_{2m-2}(x)\right), \qquad (13)$$

$K_0(x) = \int_x^{\infty} e^{is^2} ds = F(x)$ - Fresnel integral.

Results of calculation by means of formula (11) for the case of double diffraction shows good agreement (within 0.5 dB) with rigorous results [3].

4. Conclusion

In this paper by introducing elementary diffraction coefficient and using method of GTD generalization of Maggi-Rubinowicz theory for multiple diffraction is performed. Solution is obtained as multiple line integral corresponding to the edges of obstacles. Such integral can be transformed into a series representation which is amenable to computer implementation.

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