THE MATHEMATICAL MODEL OF THE MULTILAYER CIRCULAR WAVEGUIDES

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1. Abstract

The special features of dielectrometry monitoring of heterogeneous materials and medium are discussed. An electrodynamical model for primary measuring converter in circular waveguides are considered. The results of investigation of characteristics of regular circular waveguide with discrete layered filler are presented. Presented results may be used for measuring of electrophysical parameters of materials and for monitoring of constituent elements of fluid medium.

2. Introduction

In our paper the problems of the electromagnetic wave propagation in the multilayer waveguides are discussed. A great number of investigators presented the analysis of the problem when the waveguide had two or three layer. We worked out the mathematical model for electromagnetic wave propagation in the waveguide with the arbitrary number (N) of the layers.

3.The mathematical model

For the analysis a multilayered regular circular waveguide has been chosen. The number N of internal dielectric layers is given arbitrary. Let us r_i — the external radius of the layer with number *i*, R_a — radius of the waveguide. The external waveguide wall is perfectly conductive screen.

In general case dielectric and magnetic permeabilities of ε_i and μ_i of the layer with number *i* are complex tensors of the second rank. Let us supposed that external magnetic field is applied along azimuthal φ-axis. In this case the tensors ε_i, μ_i should be written as:

$$\varepsilon_i, \mu_i \Rightarrow \vec{s} = \begin{vmatrix} s & 0 & js_a \\ 0 & s_\varphi & 0 \\ -js_a & 0 & s \end{vmatrix} \qquad (1)$$

The solution of Maxwell equations is carried out by Fourier's method in a cylindrical coordinate system. The dependence of the field's components on time and on coordinates for regular waveguides is chosen as:

$$E, H \sim \exp[j(\omega t + n\varphi + k_0 \Gamma z)], \qquad (2)$$

where ω — circular frequency, n — number of field variations in azimuth direction, Γ — spreading constant normalized on the wave number of free space k_0, $j = \sqrt{-1}$ — imaginary unity.

We analyze the azimuth and longitudinal field components with the help of Maxwell equations in the form of the first order system of the four differential equations.

$$\Psi'_\varphi = \left(-\frac{1}{\rho} - \Gamma\frac{\tilde{s}_a}{s}\right)\Psi_\varphi + \frac{n}{s\rho}\tilde{\Psi}_\varphi + \frac{n}{\rho}\left(\frac{s_a}{s} + \frac{\tilde{s}_a}{\tilde{s}}\right)\Psi_z + \left(\tilde{s}_\perp - \frac{n^2}{s\rho^2}\right)\tilde{\Psi}_z, \qquad (3)$$

$$\Psi'_z = \left(-\frac{\Gamma^2}{s} - \tilde{s}_\varphi\right)\tilde{\Psi}_\varphi + \frac{s_a}{s}\Gamma\Psi_z - \frac{n}{s\rho}\Gamma\tilde{\Psi}_z,$$

where $\vec{\Psi}^m = \rho_0\vec{H}; \quad \vec{\Psi}^e = j\vec{E}; \quad s_\perp = s - s_a^2/s; \quad \rho_0 = 120\pi\ [Ohm].$

The stroke means a derivative along normalized radial coordinate $\rho = k_0 r$. Equation (3) represents a shortened record of wave equation system when indexes *m* and *e* at field components and coefficients are omitted, in this case is supposed that $\widetilde{\Psi}^{e,m} = \Psi^{m,e}$.

The radial components of the electromagnetic field may be written as:

$$j\Psi_\rho = \frac{1}{s}\left(\Gamma\widetilde{\Psi}_\varphi - \frac{n}{\rho}\widetilde{\Psi}_z\right) + \frac{s_a}{s}\Psi_z \tag{4}$$

The relation ship (3) shows that the waveguide under consideration has the natural waves of the hybrid type. When ***n* = 0** the system of the natural waves consists of the independent waves of the electric and magnetic type.

We transpose the equation (3) to the system of the two wave equations for the longitude electric and magnetic components.

$$\rho^2\Psi_z'' + \rho\Psi_z' + \left(a\rho^2 - b\rho - c\right)\Psi_z + d\rho\widetilde{\Psi}_z + e\rho\widetilde{\Psi}_z' = 0, \tag{5}$$

where

$$a = s_\perp\widetilde{s}_\varphi - \Gamma^2;\; b = \frac{s_a}{s}\Gamma; \quad c = \frac{s_\varphi}{s}\frac{\chi^2}{\widetilde{\chi}^2}n^2;$$

$$d = \frac{n}{s}\left(s_a\widetilde{s}_\varphi - \widetilde{s}_a s_\varphi\frac{\chi^2}{\widetilde{\chi}^2}\right);\; e = \frac{n}{s}\left(1 - \frac{\chi^2}{\widetilde{\chi}^2}\right);\; \chi^2 = s\widetilde{s}_\varphi - \Gamma^2.$$

The transverse components are connected with the longitude components and its derivatives:

$$j\chi^2\Psi_\rho = \widetilde{s}_\varphi\left(s_a\Psi_z - \frac{n}{\rho}\widetilde{\Psi}_z\right) - \Gamma\Psi_z', \tag{6}$$

$$\chi^2\Psi_\varphi = \Gamma\left(s_a\Psi_z - \frac{n}{\rho}\widetilde{\Psi}_z\right) - s\Psi_z'.$$

The particular solution of the system (5) can be written as:

$$\Psi_i = \sum_{k=0}^{\infty} A_k\rho^{k+\lambda_i}\ , \tag{10}$$

where A_k, λ_i are the unknown values.

Let us substitute the relationship (10) to the system (5). As a result we obtain the system of the algebraical equations for the coefficients A_k.

$$\left(\nu_k^2 - c\right)A_k + aA_{k-2} - bA_{k-1} + d\widetilde{A}_{k-1} + e\nu_k\widetilde{A}_k = 0, \tag{11}$$

where $\nu_k = \lambda + k; \quad A_k = 0$ for $k < 0$.

Let us suppose that $A_0 \neq 0$. When $k = 0$ we obtain the equation for determining the unknown degrees λ:

$$\lambda^4 - \left(c + \widetilde{c} + e\widetilde{e}\right)\lambda^2 + c\widetilde{c} = 0 \tag{12}$$

The equation (12) has four solutions:

$$\lambda_{1,2} = \pm n\sqrt{s_\varphi^e / s^e}; \quad \lambda_{3,4} = \pm n\sqrt{s_\varphi^m / s^m} \tag{13}$$

It is possible that the values λ_i are equal or its difference is the integer value. In this case the general solutions may be obtained by means of the limit transitions. Now let us consider that the solutions λ_i are different and its difference is not the integer value. In this case the fundamental solutions system consists of the four particular solutions of the wave equation (5). The general solution of system (5) is:

$$\Psi_z(\rho) = \sum_{i=1}^{4} C_i \sum_{k=0}^{\infty} A_k(\lambda_i)\rho^{k+\lambda_i} , \tag{14}$$

where C_i — integration constants. These constants are defined with the help of the boundary conditions.

As a result, we obtain the system of the four algebraical equations:

$$\sum_{i=1}^{4} C_{i,l,\xi} \sum_{k=0}^{\infty} A_{k,l,\xi}\, \rho_l^{k+\lambda_{i,l}} = \sum_{i=1}^{4} C_{i,l+1,\xi} \sum_{k=0}^{\infty} A_{k,l+1,\xi}\, \rho_l^{k+\lambda_{i,l+1}} , \tag{15}$$

where $\xi = z, \varphi$.

The relationship (15) may be simplified with the help of the boundary conditions on the perfectly conductive wall of the waveguide. In this case the field components are presented as a linear combination with two constants $C_{1,N}$ and $C_{2,N}$, where N — the number of the external layer. As a result, we obtain:

$$\sum_{i=1}^{2} C_{i,N,\xi} \Phi_{i,\xi,N}\left(\rho_{N-1}, s_N^e, s_N^m\right) = \sum_{i=1}^{4} C_{i,N-1,\xi} \Psi_{i,\xi,N-1}\left(\rho_{N-1}, s_{N-1}^e, s_{N-1}^m\right), \tag{16}$$

where ρ_{N-1} — is radius of the layer with number N-1;

$\Phi_{i,\xi,N}$ — is the tangential components of electric and magnetic field in the internal layer.

Expressions at the coefficients $C_{i,k}$ form a rectangular matrix with the elements $a_{j,k}$ $(j = 1\ldots4; k = 1\ldots6)$, with which iterative transformations are carried out.

$$a_{j,p}^{*} = a_{j,p} - \frac{a_{q,p}}{a_{q,7-q}} \cdot a_{j,7-q} \tag{17}$$

where q=1,2,3,4; p=1,…6-q; j=1,2,3,4.

As a result, field components in the layer with number p = N-1 depends on two constants $C_{1,N}$, $C_{2,N}$. These constants are defined for layer with number N.

The approach discussed combines the algorithm of linear algebra with a rigorous solution of boundary electrodynamics problem for arbitrary layer’s number. This method removes the necessity in analytical presentation of the dispersion equation.

The algorithmic (implicit) way of forming the dispersion equation above was checked on multilayer gyrotropic waveguide structures [1] with the azimuthal magnetization. The results of the simulation are in the good agreement with result of [2].

Numerical solution of the dispersion equation, which is given in the implicit form, gives good results in practical calculations. However, there are a few problems when using this algorithm in a broad frequency range, and in case of considerable changes in material characteristics of waveguide system layers. Solution branches for adjacent types of oscillations are on the complex plane in close proximity to each other or intersect with the inversion of wave types. In this case, in the course of numerical analysis of the dispersion equation, the change is possible from the solution related to one type of oscillations.

To localize solutions related to the type of waves being studied, traditional ways of the numerical analysis of the dispersion equation were complemented with the argument principle.

This method enables to find the number of transcendental equation roots, which get inside the closed loop, and also to obtain algebraic relations for defining them. The combination of the argument principle with the extrapolation prediction of dispersion characteristic enables to determine optimal dimensions of the integration loop, and in doing so to realize the selection of solutions related to the type of oscillations investigated.

4. Obtained results

Developed algorithms of rigorous solution of electrodynamics problem for defining complex propagation constants were used to analyze potentialities of applying complex waveguide structures as converters for measuring electrophysical properties of materials.

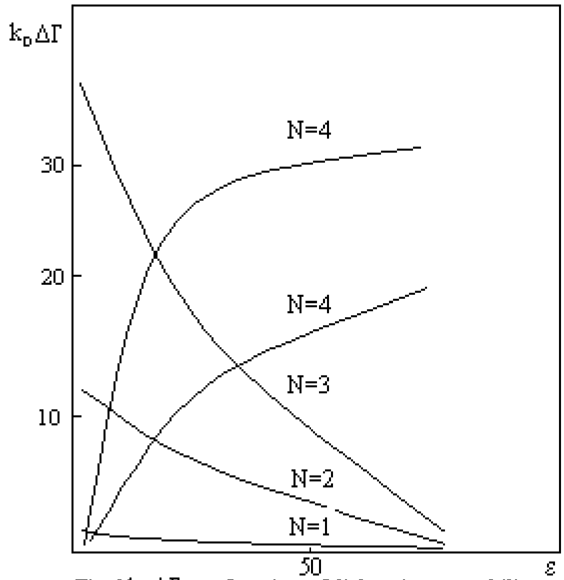

Fig. 1 $k_0\Delta\Gamma$ as a function of dielectric permeability

We construct the measuring converter in multilayer circular waveguide [3,4].

In order to converter does not distort flood's structure, the first layer's diameter has been given as equal to the pipeline diameter.

The converter may be used for measuring in the wide frequency range and different values dielectric permeability (from 1 to 100). Fig. 1 shows imaginary part of dielectric permeability of the investigated material.

The value $k_0\Delta\Gamma = k_0\left(\Gamma''(s) - \Gamma''(\varepsilon)_{\min}\right)$ defines integral converter's sensitivity. One can see that the presented mathematical model may be used as a base for investigation of the electromagnetic field integration with different materials and for the construction of the different microwave devices.

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