FEATURES OF MULTIPLE SCATTERED RADIATION IN A RANDOMLY INHOMOGENEOUS MEDIUM CONTAINING PROLATE INHOMOGENEITIES

Vladimir Gavrilenko (1), George Jandieri (2), Andrei Aistov (1)
(1) University of Nizhni Novgorod
Nizhni Novgorod, 603600, Russia
E–mail: vgg@rf.unn.runnet.ru
(2) Georgian Technical University
Tbilisi, 380075, Georgia
E–mail: jandieri@access.sanet.ge

1. Introduction

At the present time the features of light propagation in turbid media such as plasma, atmospheric clouds, ocean, biological tissues and so on, have been well studied enough in recent years [1,2]. The common features of these media are strongly anisotropic scattering and absorption of transference radiation in them. Majority papers concerning to this problem are devoted to medium containing statistically isotropic large-scale inhomogeneities. However, in actually, cross-section of electromagnetic waves scattered on inhomogeneities depends on the direction of the incident wave in many practical applications. In what follows we consider some features of small-angle scattered radiation by prolate inhomogeneities. One such example is the scattering of X-ray emission by molecules of thermotropic liquid crystals. In many cases prolate inhomogeneities are oriented along a certain direction. In particular polymer macromolecules are oriented in a flow of liquid with a velocity gradient [3]. Multiple scattering of short–wavelength radiowaves in the Earth's magnetosphere, which contains random plasma concentration inhomogeneities that are strongly extended along the geomagnetic field have major practical importance. Radiation propagation along the axis of prolate inhomogeneities and at small angles with respect to this axis we restrict our consideration. Formation of the lighting regime of multiple scattered oblique illumination radiation was not considered up to now. The evolution of the angular distribution of ray intensity for light scattering in a random medium along prolate inhomogeneities is studied. The influence of the shape of the scattering indicatrix, the degree of inhomogeneity anisotropy and regular absorption on the angular distribution of the brightness easily measured in practice are analyzed in this paper.

2. Statistical simulation

Let a plane–parallel layer of a chaotically inhomogeneous medium is illuminated by plane monochromatic wave. The layer has a large optical thickness and therefore multiple wave scattering takes place there. Real part of the refractive index is smooth on the wavelength scale. Inhomogeneities are extended along the normal to the layer boundary. Plane z=0 of the Cartesian *XYZ* coordinate system is coincides with layer surface, z-axis is directed along the wave vector of the incident wave. We are interested in angular distribution of the brightness $B(\mathbf{s}, s\ z)$ in the layer, where $\mathbf{s}$ is the unit vector, s is the scattering factor [1,2]. The numerical simulation of radiation transfer solved the problem. The calculations are made by the Monte Carlo method. We used the algorithm of direct simulation [4]. It contains the following stages: the choice of the initial point of the trajectory and free path length of photon motion; the check on departure from the medium or reflection from the layer boundary, the calculation of scattering angles in accordance with the scattering indicatrix, and so on. We supposed that photon weight exponentially decreases with increasing trajectory length [6,7] in absorptive medium. We determined angular distribution of the brightness $B\ (\mathbf{s},\ s\ z)$ at fixed depth s z.

At first we consider normal illumination of the surface. To test the correctness of the computer program, we calculated the ray intensity in particular case of a medium with statistically isotropic inhomogeneities. The scattering indicatrix was specified by the Gaussian function. The results obtained in this way, such as: the first and the second moments of the angular distribution of radiation intensity (ADRI) agreed with the results of calculations by the formulae of small–angle self–similar approximation. The simulation of light propagation in a medium with statistically anisotropic inhomogeneities gave a number of new results. As an example, the results for a layer with thickness $s\ z = 32$ are submitted below. The probability of photon survival in an elementary scattering event being constant inside the layer and it was given by the formula $L = s/(s + k)$ (k is the absorption coefficient). The probability of photon survival ranged from 0.2 to 1. Scattering process in random medium with two-scale inhomogeneities was simulated by the Gaussian indicatrix [6]

$$\chi(\mathbf{s},\mathbf{s}') = \alpha_\perp^2\, \alpha_\parallel^2\, \exp\{-\alpha_\perp^2\, [(s_x - s_x')^2 + (s_y - s_y')^2] - \alpha_\parallel^2\, [(s_z - s_z')^2\, \} / \pi^{3/2}\ , \qquad (1)$$

where $\mathbf{s}' = (s_x', s_y', s_z')$ and $\mathbf{s} = (s_x, s_y, s_z)$ are the unit vectors specifying the directions of the incident and scattered waves

$$\alpha_\perp = \pi \ell_\perp / \lambda\ ,\ \alpha_\parallel = \pi \ell_\parallel / \lambda\ , \qquad (2)$$

λ is the wavelength of the light wave, $\ell_\perp$ and $\ell_\parallel$ are the inhomogeneity correlation lengths in the plane z = const and in the direction of the z–axis, respectively. The correctness of operation of the random number generator, which specified the desired scattering indicatrix, was supported by the numerical comparison of single–scattering intensities obtained by the Monte Carlo method with the results of calculations by formula (1) for various angles of incidence of a plane wave on an elementary scattering volume. The statistical simulation of radiation transfer in the medium under consideration for the case of

$$\alpha_\perp,\ \alpha_\parallel \gg 1,\ \ \alpha_\parallel \geq \alpha_\perp \qquad (3)$$

showed that under conditions where $\ell_\parallel > 2\ell_\perp$ one might observe a local minimum in the ADRI. The minimum in this case for the nonabsorbing medium ($L = 1$) is formed beginning with a certain depth in the region of multiple scattering. The direction of the local minimum coincides with the direction of the greatest inhomogeneity correlation radius. The minimum becomes more pronounced and may be increased at smaller depths in comparison with the case of weakly elongated inhomogeneities. An increase of absorption coefficient k caused only a decrease in the angular width of the ADRI in the simulation provided small-angle scattering. The relative amplitude of the local minimum was virtually unchanged. We analyzed variances of the photon angular spread. For media with prolate inhomogeneities, the increase in angular width of ADRI with increasing depth was found to be slower than the corresponding increase for a medium with statistically isotropic inhomogeneities whose scale coincided with the transverse scale of anisotropic inhomogeneities. Statistical simulations show that ADRI strongly depends on the form of scattered indicatrix in the region of multiple scattering. As an example, we present in Fig. 1 the simulation results for radiation transfer in media with power–law and Gaussian scattering indicatrices. The power–law indicatrix can be used in a wide class of turbulent media by way of changing the exponent index p. It has the following form

$$\chi(\mathbf{s},\mathbf{s}') \sim \exp\{-\alpha_\parallel^2\, [(s_z - s_z')^2\, \} / \{1 + \alpha_\perp^2\, [(s_x - s_x')^2 + (s_y - s_y')^2\,]\,\}^{p/2}\ , \qquad (4)$$

where the parameter $\alpha_\perp$ is determined by the external turbulence scale in the plane z = const. This indicatrix, as well as the Gaussian one satisfied the normalization condition

$$\int_{4p} d\mathbf{s}\, \chi(\mathbf{s},\mathbf{s}') = 4\pi.$$

The inhomogeneity parameters were chosen so that both indicatrix had nearly the same widths at a level of 0,5 for radiation incident along the z–axis. However numerical simulations show that at relatively small depths, the ADRI in a medium with the Gaussian scattering indicatrix was narrower than the distribution in a medium with the power–law indicatrix. This may be attributed to the fact that, in the case of radiation incident along the z–axis, the power indicatrix is characterized by a slower decrease of intensity in the wings than the Gaussian indicatrix. Beginning with a certain depth ($s\ z > 8$) the ADRI in the medium with the Gaussian indicatrix, on the contrary, is wider than the distribution in the medium with power–law scattering indicatrix.

3. Oblique illumination of the surface

The analytical calculations of the in the case of multiple scattering fails even in the small-angle approximation. We shall analyze ADRI for double scattering. For a plane incident wave it has the form

$$\mathrm{B}(\mathbf{s}) \sim \int\int_{-\infty}^{\infty} \mathrm{d}\boldsymbol{s}\, \chi(0,\mathbf{s}') \,\chi(\boldsymbol{s},\boldsymbol{s}') \,.$$

Let investigate oblique illumination of the surface. At first we shall consider light propagation in nonabsorbing media with the Gaussian scattered indicatrix. Double-scale character of inhomogeneities for the given distribution manifest itself more visually if

$$q_0 > \alpha_{\parallel}^{-1/2} \;,\;\; \alpha_{\parallel} >> \alpha_{\perp} \;.$$

Here q_0 is the refraction angle. In this case wave vectors of scattered waves are close to surface of the cone, apex angle of which is approximately q_0. Deviation of the wave vectors of the order $\alpha_{\parallel}^{-1/2}$ at q_0 <<1 from the surface of this cone is predominantly determined by longitudinal correlation radius $\ell_{\parallel}$. Distribution of radiation along the surface of this cone is determined by transversal correlation radius $\ell_{\perp}$. Numerical simulation show that localization of the wave vectors of scattered waves near the surface of the cone may be observed also at a depth corresponding to multiple scattering of light. Isolines of the angular distribution of outlet multiple scattered radiation from randomly inhomogeneous layer having thickness $s\ z$ = 44 and parameters $\alpha_{\perp}$ = 20, $\alpha_{\parallel}$ = 500 , q_0 = 15^0 as an example are presented in Fig. 2. The probability of photon survival in an elementary scattering in absorptive medium is equal L = 0,2. From these figures follow that foreshortening cone is deformed – it is compressed slightly in the plane of incidence *XZ* of the illuminated wave. Absorption leads to decrease of the apex angle of the cone. Evolution tendency of the width of the ADR in depth regime was obtained using spherical coordinate system and nondimensional parameters

$$\nu = \nu_{\mathrm{z}} / \nu_{\infty} \;, \tag{5}$$

where

$$\nu_{\mathrm{z}} = (\, m_z^{\,2} / m_z^0) - (\, m_z^1 / m_z^0 \,)^2 \;, \tag{6}$$

$$m_z^k = \iint_{\mathrm{s_z} > 0} \mathrm{d}\theta\, \mathrm{d}\varphi\;\; \mathrm{B}(\mathbf{s}, \sigma\, \mathrm{z})\, \theta^{\,k}\;\; \sin\theta \;,$$

k = 0,1,2; θ is the polar angle with respect to z axis and azimuthal one is in the plane *XZ*, ν is normalized dispersion of ADRI, ν_{∞} is the magnitude of the dispersion in depth regime. The results of numerical simulations are illustrated in Fig. 3. Dispersion of the angle θ of straggling photons weakly depends on degree of stretched inhomogeneities (curves 1,2) at normal illumination of the surface. Nonmonotonically growth of this dispersion in proportion to depth is revealed at oblique illumination. Calculations show that qualitative feature of multiple scattered light in absorptive media at stretched and isotropic inhomogeneities sometimes are the same [7]. Relative excess of the dispersion at the own depth regime is increased in proportion to angle of the incident radiation (curves 3’ and 4) and medium absorption coefficient (curves 3 and 3’). At the same time simulation show that this effect strongly increased in proportion to stretched degree of inhomogeneities (curves 1’, 3’ and 2’). We present in Fig. 4 relationship between absolute values of dispersion of the angular distribution. As one would expect the increase of absorption coefficient leads to decrease of dispersion ν_{z} (curves 2 and 1 ; 4, 3 and 3’). The increase of linear size of inhomogeneities leads to same effect (curves 1, 2 and 3’ , 4). Simulation shows that dispersion of ADRI in nonabsorbing medium is increased linearly in depth in the region of small-angle multiple were scattering (curves 2 and 4).

References

1. I.N.Minin, Theory of Radiation Transfer Theory in Atmospheres of Planets. Moscow. 1988 (in Russian).
2 T. Nakai , K. Yamamoto, Proceedings of the 1992 International Symposium on Antennas and Propagation. Sapporo, 1992. vol. 3. pp. 949-952.
3 M. Nakagaki, W. Heller, Journal of Polymer Science. 1959. vol. 38. pp. 117-131.

4 M.A. Nazaraliev, Statistical Simulation of Radiative Processes in the Atmosphere. Novosibirsk, 1990 (in Russian).

5 L.S. Dolin, Dokl. Acad. Nauk SSSR. 1981. vol. 260, # 6. pp. 1344-1347 (in Russian).

6 Ishimaru A. Wave Propagation and Scattering in Random Media V. 2. Multiple scattering, Turbulence, Rough Surfaces and Remote Sensing. (1978, Academic Press. New York - San Francisco - London).

7 A.V. Aistov, V.G. Gavrilenko, Optika i Spectroskopia. 1995. vol. 78, # 4. pp. 672-677 (in Russian).

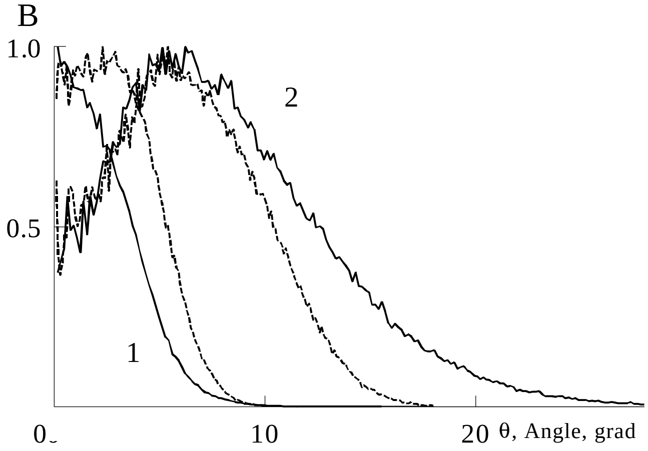

Fig. 1. Angular distributions of ray intensity at different depths 1- *s z* =2, 2- 32; *L*=1. Solid lines –a medium with Gaussian scattering indicatrix, $\alpha_\perp$=20, $\alpha_\parallel$ =200; dashed curves– a medium with the power–law indicatrix $\alpha_\perp$=10, $\alpha_\parallel$ =300, *p*=2.5 .

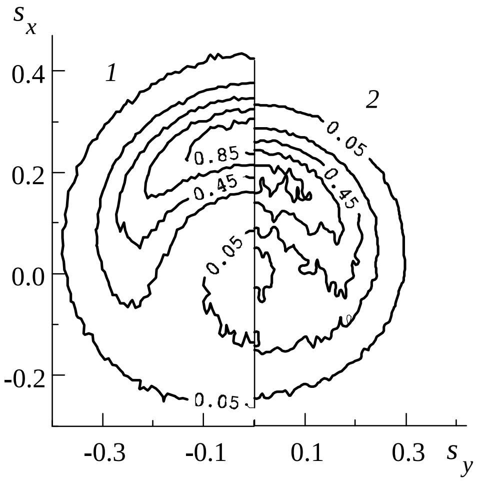

Fig. 2. Angular distribution of radiation intensity in absorptive (1) and nonabsorbing (2) media.

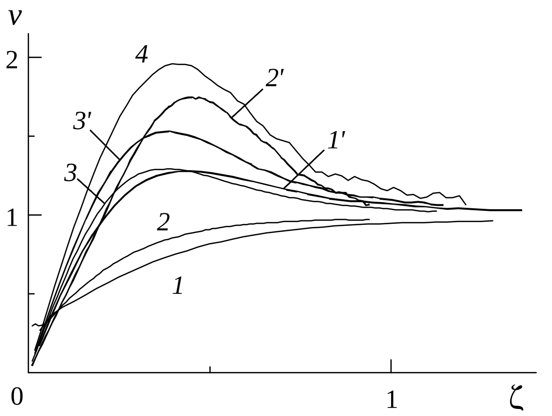

Fig. 3. Illustrating dependence of normalized dispersions of the angular distribution versus parameter $z = v_\infty k\ z$ in medium at $\alpha_\perp$=20. 1-q_0=0, $\alpha_\parallel$ =20, *L*=0.2 ; 2-q_0=0, $\alpha_\parallel$ =500, *L*=0.2; 1’-q_0=15^0, $\alpha_\parallel$ =20, *L*=0.2; 2’-q_0=15^0, $\alpha_\parallel$ =500, *L*=0.2; 3-q_0=15^0, $\alpha_\parallel$ =200, *L*=0.4; 3’-q_0=15^0, $\alpha_\parallel$ =200, *L*=0.2; 4-q_0=25^0, $\alpha_\parallel$ =200, *L*=0.2.

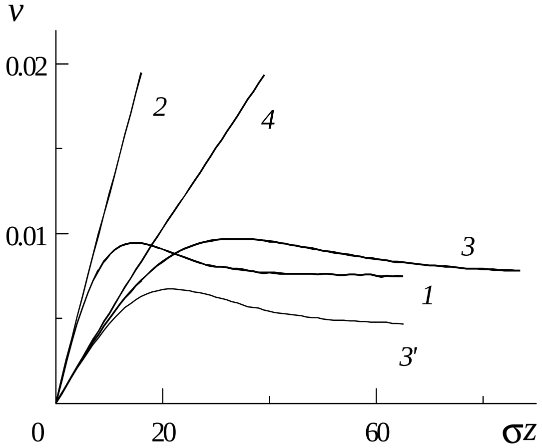

Fig. 4. Dispersions of the angular distribution at q_0=15^0, $\alpha_\perp$=20. 2-$\alpha_\parallel$ =20, *L*=1, 4- $\alpha_\parallel$ =200, *L*=1, designations 1, 3 and 3’ are the same as in figure 3.