RECONSTRUCTION OF RADOME PARAMETERS USING A GAUSSIAN BEAM AND A GENETIC ALGORITHM

Zhi Qi MENG*, Takashi TAKENAKA** and Sailing HE***

*   Fukuoka University
Fukuoka 814-0180, Japan
E-mail: meng@fukuoka-u.ac.jp
**   Nagasaki University
Nagasaki 852-8521, Japan
E-mail: takenaka@net.nagasaki-u.ac.jp
***   Zhejiang University
Hangzhou 310027,China
E-mail: sailing@coer.dhs.org

1. Introduction

Planar dielectric multi-layered structures are used as radomes and electromagnetic wave absorbers. In this paper we reconstruct electrical parameters of a multi-layered radome of finite size.

In a practical situation (as considered here), a Gaussian beam is used to illuminate the central area of the radome surface in order to avoid the complicated scattering from the edges of the radome. The inverse problem is formulated as a nonlinear optimization problem, and a binary genetic algorithm (GA) is applied to solve the problem. GA is very convenient for use in optimizing any type of parameters (discrete or continuous), and in general does not have the problem of being trapped in local minima. GA has been used in designs of antennas[1], microwave absorbers[2] and image reconstruction[3], etc.

2. Formulation

Let us consider the reflection of a Gaussian beam incident on a flat dielectric multi-layered structure as shown in Fig.1. For simplicity, the incident beam is assumed to be two-dimensional ($\partial/\partial y = 0$) and polarized perpendicular to the plane of incidence, i.e., the electric field is along the y direction. The half-space regions above and below the multi-layered structure are assumed to be free space. The y component of electric field of the beam with smallest spot size w_s at $z^i = -z_s^i$ propagating along the z^i direction can be represented by

$$E_y^i(x^i, z^i) = \int_{-\infty}^{\infty} G^i(p^i) \exp[-jk_0(p^i x^i + q^i z^i)]\mathrm{d}p^i \ , \tag{1}$$

where k_0 is the wave number in free space and

$$G^i(p^i) \ = \ \frac{k_0 w_s}{2\sqrt{\pi}} \exp\left[-\left(\frac{k_0 W_s}{2}p^i\right)^2 - jk_0 q^i z_s^i\right] \ , \tag{2}$$

$$q^i \ = \ \begin{cases} \sqrt{1-(p^i)^2} & |p^i| \leq 1 \\ -j\sqrt{(p^i)^2-1} & |p^i| > 1 \end{cases} \ . \tag{3}$$

The coordinates (x^i, z^i) are related to (x, z) by

$$\begin{bmatrix} x^i \\ z^i \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} \ . \tag{4}$$

Then the expression of the y component of electric field of the beam with reference to the coordinate frame (x, z) is given by

$$E_y^i(x, z) = \int_{-\infty}^{\infty} G(p) \exp[-jk_0(px + qz)]\mathrm{d}p , \tag{5}$$

$$G(p) = G^i(p\cos\theta - q\sin\theta)(\cos\theta + \frac{p}{q}\sin\theta) , \tag{6}$$

$$\begin{bmatrix} p^i \\ q^i \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} . \tag{7}$$

The reflection coefficient of a TE plane wave $E_y^{\mathrm{plane}} = \exp[-jk_0(px + qz)]$ is given by the following recursive relations for the generalized reflection coefficients(see, e.g., [4]):

$$\tilde{R}_{n,n+1} = \frac{R_{n,n+1} + \tilde{R}_{n+1,n+2}\exp[2jk_{n+1,z}(s_n - s_{n+1})]}{1 + R_{n,n+1}\tilde{R}_{n+1,n+2}\exp[2jk_{n+1,z}(s_n - s_{n+1})]} , \quad n = 0, 1, \cdots, N , \tag{8}$$

$$\tilde{R}_{N+1,N+2} = 0 , \tag{9}$$

where s_n $(n = 1, 2, \cdots, N)$ is the z coordinate for the planar interface between the nth and $(n+1)$th layers(cf. Fig.1) and s_{N+1} is an arbitrary value. $R_{n,n+1}$ is the Fresnel reflection coefficient of a plane wave at the interface:

$$R_{n,n+1} = \frac{\mu_{n+1}k_{n,z} - \mu_n k_{n+1,z}}{\mu_{n+1}k_{n,z} + \mu_n k_{n+1,z}} , \tag{10}$$

and $k_{n,z}$ is the wave number in the z direction in the nth layer and is given by

$$k_{n,z} = k_0 q_n , \tag{11}$$

$$q_n = \sqrt{(k_n/k_0)^2 - p^2} , \tag{12}$$

$$k_n = \omega\sqrt{\left(\varepsilon_n - j\frac{\sigma_n}{\omega}\right)\mu_n} . \tag{13}$$

By using the generalized reflection coefficient $\tilde{R}_{0,1}$, the reflected beam wave in the 0-th region, the upper half-space, can be written as

$$E_y^r(x, z) = \int_{-\infty}^{\infty} \tilde{R}_{0,1}(p)G(p) \exp[-jk_0(px - qz)]\mathrm{d}p \tag{14}$$

If we introduce coordinates (x^r, z^r) for the reflected waves, Eq.(14) becomes

$$E_y^r(p^r; \theta) = \int_{-\infty}^{\infty} \tilde{R}_{0,1}(-p^r\cos\theta + q^r\sin\theta)G^i(p^r) \exp[-jk_0(p^r x^r + q^r z^r)]\mathrm{d}p^r . \tag{15}$$

Note that the reflection coefficient $\tilde{R}_{0,1}$ contains the information for the multi-layered structure. Here after, we denotes the reflection coefficient $\tilde{R}_{0,1}(p^r)$ as $\tilde{R}_{0,1}(p^r; \theta; \boldsymbol{t})$ in order to show explicitly the dependence of the coefficient on the incident angle θ and the parameter vector $\boldsymbol{t}$ which consists of medium parameters and the thickness for each layer.

3. Inverse problem

Now we consider the inverse problem of determining the medium parameters and thickness of each layer from the knowledge of the incident waves and the measured reflected waves. The dielectric structure is illuminated by a Gaussian beam with different angles θ_m, $m = 1, 2, \cdots, M$. For each illumination, the reflected wave is measured at $z^r = \mathrm{const.}$. We then obtain the measured reflection coefficient by taking Fourier transform of the reflected wave. Finally, the

inverse problem is formulated as a nonlinear optimization problem of finding the vector $\boldsymbol{t}$, which minimizes the following objective function:

$$\xi(\boldsymbol{t}) = \sum_{m=1}^{M} \int_{-1}^{1} \left| \left[\hat{\tilde{R}}_{0,1}(p^r; \theta_m) - \tilde{R}_{0,1}(p^r; \theta_m; \boldsymbol{t}) \right] G^i(p^r) \exp[-jk_0(p^r x^r + q^r z^r)] \right| dp^r , \qquad (16)$$

where $\hat{\tilde{R}}_{0,1}(p^r; \theta_m)$ is the measured reflection coefficent and $\tilde{R}_{0,1}(p^r; \theta_m; \boldsymbol{t})$ is the calculated reflection coefficient for a structure corresponding to parameter vector $\boldsymbol{t}$. We apply a binary genetic algorithm to this minimization problem.

4. Genetic algorithm

Genetic algorithms(GA), which imitate some of the mechanisms of evolution in nature, are used as optimization procedures. A GA optimizer starts with a randomly selected population of character strings, referred to as chromosomes, and thereafter generates successive populations evolving toward the best adaptation to a particular environment through genetic operations: selection, crossover and mutation. The fitness for the environment of the problem can be defined as

$$F = e^{-\xi(\boldsymbol{t})} , \qquad (17)$$

which means the population should evolve until the fitness for one of the chromosome becomes 1.

For the optimization problem the vector $\boldsymbol{t}$ is a chromosome, i.e., an array of parameters or genes:

$$\boldsymbol{t} = \boldsymbol{t}(\varepsilon_1, \sigma_1, \mu_1, d_1, \varepsilon_2, \sigma_2, \mu_2, d_2, \cdots, \varepsilon_N, \sigma_N, \mu_N, d_N) . \qquad (18)$$

Each parameter of $\boldsymbol{t}$ is encoded in a binary sequence. We assume the thickness D for the whole structure is known, and thus the thickness d_n satisfies the constraint:

$$\sum_{n=1}^{N} d_n = D . \qquad (19)$$

Since a gene should be independent from each other in a GA, we define a thickness rate r_n as follows:

$$s_n = r_n s_{n+1}, \quad n = 1, 2, \cdots, N-1, \quad s_N = D , \qquad (20)$$

where $0 \leq r_n \leq 1$. Instead of layer thickness d_n we will reconstruct r_n.

At the beginning of the optimization process the range of each parameter is chosen based on a priori knowledge. If we denote the reconstruction tolerance for a parameter α as p_α, where α can be ε, σ, μ or r, the number of possible values for α is given by $N_\alpha = (\alpha_{\max} - \alpha_{\min})/p_\alpha$, where $\alpha_{\max}$ and $\alpha_{\min}$ are upper and lower bounds of the parameter, respectively, determined by a priori knowledge. If the binary representation of N_α needs M_α bits, i.e., the parameter or gene α is encoded with M_α bits, then the chromosome size is

$$N_c = \sum_{n=1}^{N} (M_{\varepsilon_n} + M_{\sigma_n} + M_{\mu_n} + M_{r_n}) . \qquad (21)$$

Figure 2 shows the correspondence between the array of parameters and the chromosome.

5. Numerical results

As a numerical example, we reconstruct a three-layered radome as shown in Fig.3. The incident angles are $\theta = 1°, 2°, \cdots, 15°$. The unknown parameters are $\varepsilon_1, \sigma_1, \varepsilon_2, \sigma_2, d_2$ (the parameters of layer 3 is the same as layer 1 and the total thickness of the radome is known). The true values for these parameters are given in Table 1. On the basis of a priori knowledge, the

searching ranges of the relative dielectric constants ε_1, ε_2, the electric conductivity σ_1, σ_2, and the thickness d_2 are set to be $1 \sim 10$, $0 \sim 0.2$S/m and $1.8 \sim 2.2$mm at the beginning of the optimization process.

A reconstruction result of the parameters is also shown in Table 1. The result agrees well with the true value. Because GAs are stochastic in nature and a single trial of reconstruction does not validate the method, we made fifty runs. All of them are successful.

	ε_1	σ_1 (S/m)	ε_2	σ_2 (S/m)	d_2 (mm)
True value	3.65	0.11680	6.32	0.02338	2.0
Reconstruction result	3.65	0.11677	6.36	0.02185	2.0

Table 1: The true values and reconstruction results

6. Conclusion

Reconstruction of the parameters of a multi-layered radome of finite size has been considered. To make the reconstruction method practical for use in the presence of the radome edges, we used a Gaussian beam as an incident wave. As a numerical example, a three-layered radome parameters were reconstructed. The inverse problem was formulated as a minimization problem, to which a genetic algorithm was applied. The reconstruction result agreed well with the true value.

Acknowledgment

References

[1] E. E. Altshuler and D. S. Linden, *IEEE Trans. Antennas Propagat.*, **45**, 1, pp.1-4, 1997.

[2] E. Michielssen, J. Sajer, S. Ranjithan and R. Mittra, *IEEE Trans.*, **MTT-41**, 6/7, pp.1024-1031, 1993.

[3] Z. Q. Meng, T. Takenaka and T. Tanaka, *Journal of electromagnetic waves and applications*, **13**, 1, pp.95-118, 1999.

[4] W. C. Chew, *Waves and Fields in Inhomogeneous Media*, Van Nostrand Reinhold, New York, 1990.
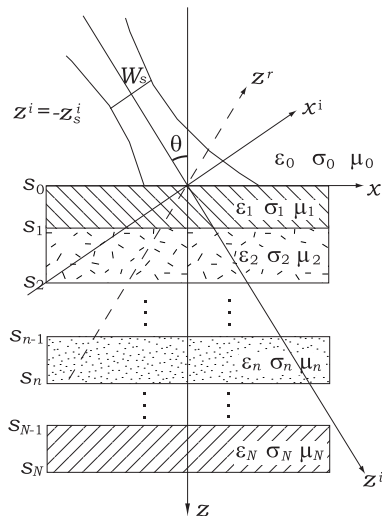

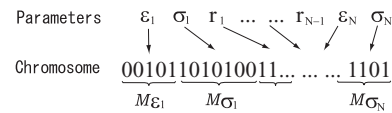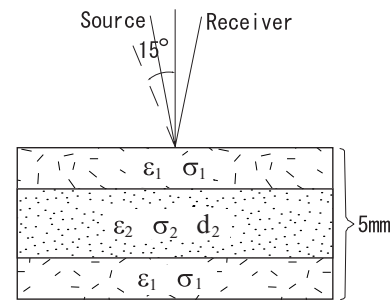Fig.1: Geometry of the scattering problem.

Fig.2: Struture of a chromosome.

Fig.3: Configuration for a three layered radome.