Dynamic Field Behavior near the Edge of a Dielectric Wedge for Plane Electromagnetic Wave Incidence

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1. Introduction

Physical optics approximation for the fields scattered by the dielectric wedge is obtained in analytic forms for the plane wave incidence. This is corrected by calculating two extinction integrals numerically by assuming surface currents expanded in Neumann series along two wedge interfaces. Calculated dynamic fields either for TE and TM modes are compared with those of static fields near the edge tip. It is shown that the dynamic fields approach to the corresponding static fields for the distance between the field point and the wedge tip less than $10^{-2}\lambda$, where λ is the free space wavelength. The dynamic field behavior for this distance larger than $10^{-2}\lambda$ up to 3λ shows the smooth transition to the far field behavior i.e. the ratio of E_ρ versus H_z becomes almost a constant.

2. Surface sources correcting physical optics solution

A plane electromagnetic wave u_i polarized in the z-direction is incident upon a two-dimensional lossless dielectric wedge of arbitrary wedge-angle(ϕ_d) and permittivity(ε_2) with its incidence angle ϕ_i, as shown in Fig.1(a). Geometrical optics (GO) solutions may be obtained by the ray tracing [9,10] and the physical optics (PO) solution is obtained by substituting the boundary fields obtained from GO approximation into the Kirchhoff's integrals inside(S_2) and outside(S_1) of the wedge, respectively, as [10]

$$u_j^{PO} = u_i\delta_{1j} + \sum_{m=1}^{M_j} u_{m,j}^{go} + v_j \qquad \text{in } S_j,\ \ j=1,2 \ , \tag{1}$$

where δ_{1j} is the Kronecker delta, $u_{m,j}^{go}$ is the mth GO field experiencing m reflections inside the wedge confined in the limited angular range bounded by transition angles, its total number of rays M_j in their respective region S_j, $j=1,2$, and v_j represents the edge diffracted field for PO approximation in terms of the Sommerfeld integral along the steepest descent path (SDP) in the complex w-plane in S_j.

Extinction theorem[11] states that the total fields inside the mathematically complementary regions($\overline{S_j}$) are zero if the exact boundary fields are used. That is,

$$u_i\delta_{1j} + \oint_{C_j}\left(G_j\frac{\partial u_j}{\partial n_j} - u_j\frac{\partial G_j}{\partial n_j}\right)dl = 0, \quad \text{in } \overline{S_j} \ , \quad j=1,2 \ , \tag{2}$$

where u_j, G_j and C_j are respectively the boundary fields, the two dimensional Green's function in free space, and the boundaries in the mathematically complementary regions $\overline{S_j}$ filled with homogeneous medium of air for j=1 and dielectric for j=2, respectively.

By substituting the GO boundary fields u^{go} plus the unknown correcting sources s_j into eq.(2), one obtains the equation for s_j as

$$u_i\delta_{1j} + \oint_{C_j}\left(G_j\frac{\partial u_j^{go}}{\partial n_j} - u_j^{go}\frac{\partial G_j}{\partial n_j}\right)dl + \oint_{C_j}\left(G_j\frac{\partial s_j}{\partial n_j} - s_j\frac{\partial G_j}{\partial n_j}\right)dl = 0 \quad \text{in } \overline{S_j},\ j=1,2 \ , \tag{3}$$

where the first and second terms are already obtained as u^{PO} in eqs. from (1) in the physical regions, S_j. Hence, eq.(3) gives

$$\oint_{C_j}\left(G_j\frac{\partial s_j}{\partial n_j} - s_j\frac{\partial G_j}{\partial n_j}\right)dl = -v_j, \quad \text{in } \overline{S_j} \ , \quad j=1,2 \ . \tag{4}$$

In order to obtain s_j from eq.(4), one may expand this unknown source along the infinite boundaries in terms of a complete Neumann's expansion[10] as

$$s_j = \sum_{n=0}^{\infty} a_n J_{\nu+n}(\gamma\rho) + \sum_{n=0}^{\infty} b_n J_n(\gamma\rho) \quad , \tag{5a}$$

$$\frac{\partial s_j}{\partial n} = \sum_{n=0}^{\infty} c_n \frac{1}{\rho} J_{\nu+n}(\gamma\rho) + \sum_{n=0}^{\infty} d_n \frac{1}{\rho} J_{n+1}(\gamma\rho) \quad , \tag{5b}$$

where a_n, b_n, c_n, d_n are unknown expansion coefficients, $J_{\nu+n}(\gamma\rho)$ is the Bessel function of order $\nu+n$, ν is the fractional order related to the static edge condition, and γ is an arbitrarily chosen wave number. Although the first series itself is complete, the second series with the integer order is added to account for the sharp discontinuity of GO solution at the tip of the wedge since $J_0(0)=1$. This series is useful for correcting PO solutions near the edge of the dielectric wedge.

3. Calculation of the dynamic field behavior near the tip of the dielectric wedge

When a plane wave polarized in the z-direction (TM mode) is incident upon the dielectric wedge, calculated electromagnetic fields, H_ρ are plotted as functions of ϕ and ρ in Fig.1. Dynamic fields and static fields are quite close already at $\rho = 10^{-3}\lambda$ in their ρ and ϕ dependencies.

Fig.2 shows the calculation of H_z and E_ρ for the plane wave incidence (TE mode). Their dynamic and static solutions agree well upto $\rho < 10^{-2}\lambda$ as shown in Fig.2(b), even for the singular ρ dependence for E_ρ along $\phi=\frac{\pi}{4}$ direction, $\nu\rho^{\nu-1}$. Three different singular behaviors of E_ρ corresponding to their static limits of different values of ν as ρ approaches $10^{-6}\lambda$ converge to the dynamic far field behavior as ρ increases to $\rho > 10^{-1}\lambda$, as shown in Fig. 2(b). The magnitude of E_ρ approaches to almost a constant in $\rho > 10^{-1}\lambda$, i.e., the ratio E_ρ/H_z becomes almost the free space intrinsic impedance regardless of the relative permittivity of the wedge, if the phases of the reflected and diffracted fields deviate little from those of the incident field.

3. Conclusion

Dynamic field behavior near the edge of a dielectric wedge is calculated. Two extinction integrals are used to correct the analytic physical optics approximation, numerically. It is shown that the dynamic fields approach to the corresponding static solutions both for their angular as well as distance dependence if the distance between the field point and the edge tip is smaller than 10^{-2} times the free space wavelength either for the singular or the non-singular edge fields. Calculated results also show that the dynamic fields go through a smooth transition to the far scattered fields for the distance larger than 10^{-1} times the wavelength, where the ratio of the transverse electric and magnetic field becomes almost a constant.

References

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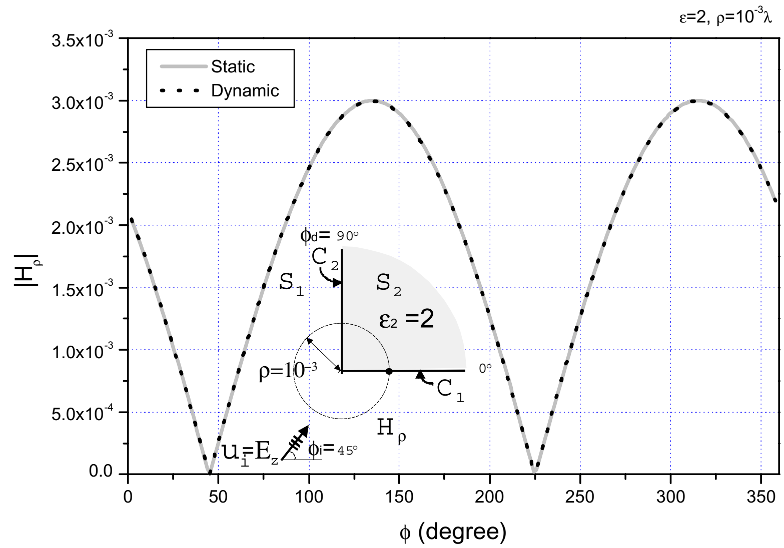

Fig.1 (a) $|H_\rho|$ versus ϕ at $\rho=0.001\lambda$ in case of an E-polarized unit plane wave incidence for $\phi_d=90^o$, $\phi_i=45^o$, $\varepsilon_2=2\varepsilon_1$, $\mu_2=\mu_1$, where λ is the free space wavelength.

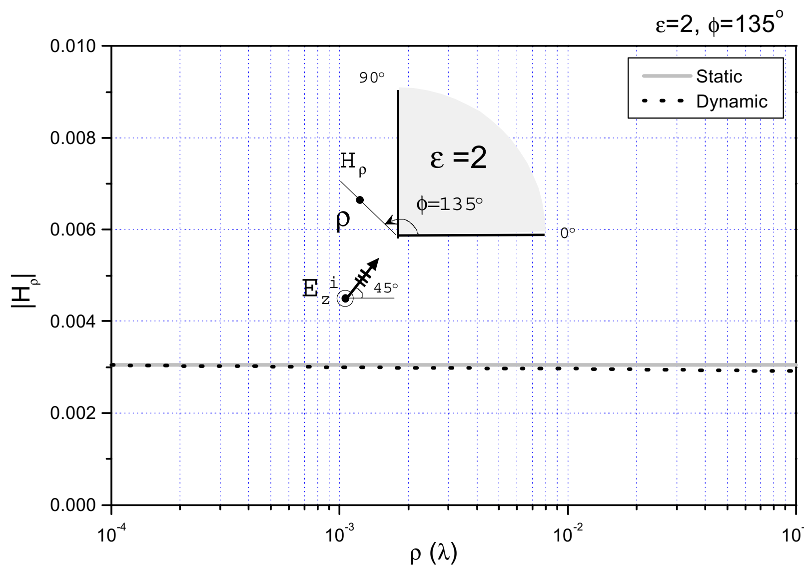

Fig.1 (b) $|H_\rho|$ versus ρ versus ρ along $\phi=135^o$ in case of an E-polarized unit plane wave incidence for $\phi_d=90^o$, $\phi_i=45^o$, $\varepsilon_2=2\varepsilon_1$, $\mu_2=\mu_1$, where λ is the free space wavelength.

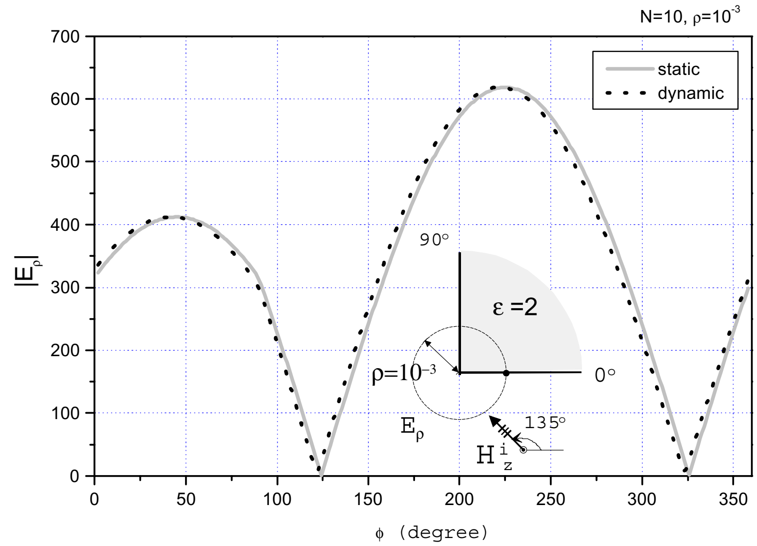

Fig.2 (a) $|E_\rho|$ versus ϕ at $\rho=0.001\lambda$ in case of an H-polarized unit plane wave incidence for $\phi_d=90^o$, $\phi_i=135^o$, $\varepsilon_2=2\varepsilon_1$, $\mu_2=\mu_1$.

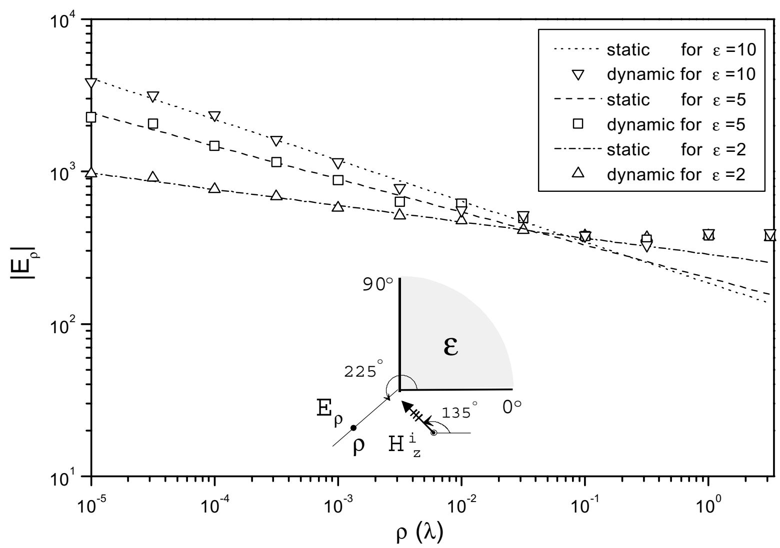

Fig.2 (b) $|E_\rho|$ versus ρ along $\phi=45^o$ for $\varepsilon_d=2\varepsilon_o$, $5\varepsilon_o$, $10\varepsilon_o$ in case of an H-polarized unit plane wave incidence for $\phi_d=90^o$, $\phi_i=135^o$, $\varepsilon_2=2\varepsilon_1$, $\mu_2=\mu_1$.