HYBRID HIGH-FREQUENCY / INTEGRAL EQUATION METHOD FOR LARGE ARRAYS OF RECTANGULAR OPEN ENDEDWAVEGUIDES

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INTRODUCTION

The electromagnetic modeling of large finite arrays as well as the scattering by finite periodic structures is an important topic for a large variety of engineering applications. Recently a new method, referred to as Truncated Floquet Wave Full-Wave (T(FW)²), has been proposed [1] [2], based on Floquet waves (FWs) diffraction for semi-infinite periodic structures [3]-[6]. This method is based on the solution of an appropriate integral equation in which the unknown function is the difference between the exact current distribution of the finite array and that of the infinite array. This unknown current can be interpreted as due to diffracted fields excited by the FWs pertinent to the infinite periodic array, allowing an efficient representation in terms of a few entire domain basis function shaped as diffracted rays. This method has been applied to the analysis of 2D and 3D arrays of slots [1] [2]. In this paper the analysis of a large array of open-ended waveguides is performed, demonstrating the extreme accuracy and the gain in computational time with respect to a conventional element by element standard analysis. Moreover a uniform asymptotic representation of the array Green's function (AGF) is introduced, which is accurate also at moderate distance from the array edges and vertices.

FORMULATION

Consider a finite planar phased array with rectangular lattice of rectangular waveguides on an infinite ground plane (Fig. 1). Denote by N, M and d_x, d_y the number of elements and the periodicity in the directions x and y, respectively. Without loss of generality, the waveguides are supposed to excited by the dominant transverse electric mode TE_{10}. Each aperture has dimensions a and b in the E and H plane, respectively. The array is globally fed with constant amplitude and linear phase.

The formulation for the approach consists in three subsequent steps. First, the continuity of the magnetic field integral equation (CMFIE) is derived for the array under investigation. To this end, the equivalence theorem is applied, covering the apertures with an electric conductor with two magnetic current distributions $\pm\mathbf{M}(x,y)$ on the two opposite sides. These distributions have equal amplitude and reverse sign in order to ensure the continuity of the tangential component of the electric field. The CMFIE is written as
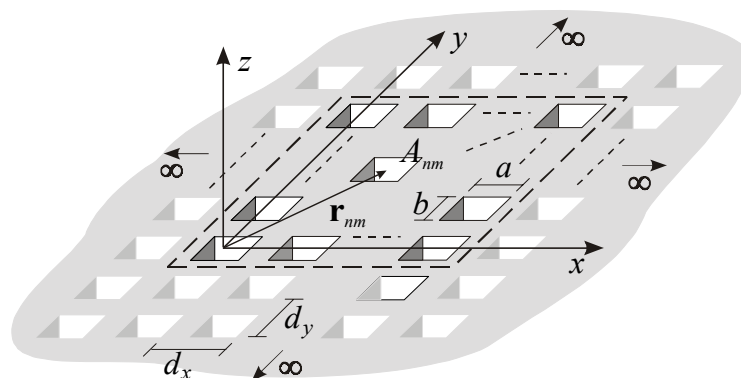

Fig. 1 Finite phased array of rectangular waveguides on an infinite ground-plane (inside the dashed line). The actual array is extended to construct the infinite periodic array through the apertures outside the dashed line. The region composed by the summation of the apertures inside (outside) the dashed line is denoted by A (A).*

$$\chi_A\left[\mathbf{H}_t^{wg}(\chi_A\mathbf{M})+\mathbf{H}_t^{hs}(\chi_A\mathbf{M})\right]=\chi_A\mathbf{H}_t^{imp} \tag{1}$$

where $\mathbf{H}_t^{wg}(\mathbf{M})$ and $\mathbf{H}_t^{hs}(\mathbf{M})$ are the tangential magnetic fields produced by $\mathbf{M}$ in the internal (short-circuited waveguide) and external (grounded half-space) regions, respectively, $\mathbf{H}_t^{imp}$ is the tangential impressed magnetic field and χ_A denotes the characteristic function of the radiating region A: $\chi_A=1$ on A, $\chi_A=0$ elsewhere.

Next, we construct an auxiliary infinite array that coincides with the actual finite array on it, and realizes the regular periodic continuation outside (Fig. 1). The CMFIE is formulated for the infinite array as

$$\chi_\infty\left[\mathbf{H}_t^{wg}(\chi_\infty\mathbf{M}^\infty)+\mathbf{H}_t^{hs}(\chi_\infty\mathbf{M}^\infty)\right]=\chi_\infty\mathbf{H}_t^{imp} \tag{2}$$

where $\chi_\infty=\chi_{A+A^*}=\chi_A+\chi_{A^*}$ is the characteristic function of the apertures of the infinite array, and A^* denotes the radiating surface of the complementary array (Fig. 1).

Finally, we introduce a fringe magnetic current distribution $\chi_A\mathbf{M}^d$ as the difference between the magnetic current $\chi_A\mathbf{M}$ of the finite array and the magnetic current $\chi_A\mathbf{M}^\infty$ of the infinite array, windowed on the actual finite array; $i.e.$

$$\chi_A\mathbf{M}=\chi_A\mathbf{M}^d+\chi_A\mathbf{M}^\infty. \tag{3}$$

This term describes the perturbation on the global current of the actual finite array, with respect to the infinite array solution. Using (3) in (1) and subtracting (2) from (1), leads to

$$\chi_A\left[\mathbf{H}_t^{wg}(\chi_A\mathbf{M}^d)+\mathbf{H}_t^{hs}(\chi_A\mathbf{M}^d)\right]=\chi_A\mathbf{H}_t^{hs}(\chi_{A^*}\mathbf{M}^\infty) \tag{4}$$

Equation (4) is referred to as Fringe Integral Equation (FIE). This equation interprets the fringe current $\chi_A\mathbf{M}^d$ as the current deformation, with respect to the infinite array current, that ensures the continuity condition in the region A. This contribution is necessary to compensate for the absence of the field radiated by the current $\chi_{A^*}\mathbf{M}^\infty$ distributed on the complementary array.

SOLUTION SCHEME

The solution procedure is divided into two steps. First, the infinite array equation is solved via a conventional Method of Moment (MoM). Then, the infinite array solution is used to construct the forcing term of the FIE. This latter is solved applying an hybrid high frequency-MoM approach.

In the hypothesis of periodic geometry and excitation, the CMFIE (2) for the infinite array can be solved invoking the Floquet theorem. The magnetic current, except for a phase shift, is a periodic function in x and y, so that the analysis may be reduced to that for a single reference cell by representing both the currents and the associated fields as a summation of Floquet waves. The unknown magnetic current $\mathbf{M}^\infty$ on the reference cell is expanded in terms of modal basis functions and the integral equation (2) is solved applying the Galerkin's method.

The second step of the procedure is the solution of the FIE to calculate the fringe current $\chi_A\mathbf{M}^d$ from which the total current is easily obtained by (3). The equivalent magnetic current of the infinite array is used to construct the forcing term of the FIE, $i.e.$, $\chi_A\mathbf{H}_t^{hs}(\chi_{A^*}\mathbf{M}^\infty)$. The high-frequency representation of this term provides a guideline for the subsequent expansion of $\chi_A\mathbf{M}^d$ in terms of basis functions. The field $\chi_A\mathbf{H}_t^{hs}(\chi_{A^*}\mathbf{M}^\infty)$ can be naturally associated with FW edge and vertex diffraction at the boundary of the radiating surface, that is the complementary infinite array. In order to define rigorously these diffraction contributions, it is necessary to decompose the complementary array in a certain number of arrays of small elements. Approximating each element with an elementary magnetic dipole with the proper equivalent moment, allows one to the calculation of the array Green's function (AGF) of the complementary array. The AGF can be represented as the radiation from equivalent FW current sheets extending continuously all over the complementary surface. The asymptotic treatment of each FW aperture leads to a spatially truncated version of the infinite array FW expansion, plus FW-excited diffracted contribution from the edges and the vertices of the array. In particular, for observation points lying on the array aperture the FW contribution vanishes. Using the locality principle for high frequency phenomena, the edge and corner diffraction coefficients can be derived from canonical problems, such as the semi-infinite array of dipoles [4][5] or a sectoral array of dipoles [6]. Following

the physical interpretation of the forcing term, the unknown magnetic current $\chi_A \mathbf{M}^d$ is efficiently expanded in terms of a few global domain functions which are shaped as FW-induced diffracted rays. In particular each function has the structure of second order edge diffracted rays, with the pertinent shadow boundaries which truncate their existence domain, plus two vertex diffracted rays that provide a uniform continuity to the whole function. The diffracted ray contains transition functions that ensure the proper ray spreading behavior when the relevant FW is near to cut-off [1]. A few number of diffracted rays is necessary to accurately describe the fringe current, typically two per edge. The global domain functions modulate the current distribution on each aperture, represented in terms of rectangular waveguide modes. Testing the field continuity (4) with proper weight functions, leads to a linear system whose dimensions are completely independent by the number of elements of the array.

NUMERICAL RESULTS

Results are presented for the analysis of a 20×20 elements array in case of broadside and 20° E-plane scan; the element dimensions and periodicity are $a = 0.5714\lambda$, $b = 0.254\lambda$ and $d_x = 0.620\lambda$, $d_y = 0.290\lambda$, respectively. Twenty modes are retained in the basis expansion of the infinite array magnetic current, while the fringe current is represented using two diffracted rays per edge, modulating the TE_{10} mode in the case of broadside beam and TE_{10}, TE_{01} in the case of the E-plane tilted beam. A reference solution has been obtained through a standard full-wave element by element analysis with 6 modal basis function per aperture (TE_{10}, TE_{01}, TE_{20}, TE_{11}, TM_{11}, TE_{30}).

In Fig. 2 and Fig. 3, the magnitude of the reflection coefficients is shown for the apertures on the central column of the array, in the cases of broadside and 20° E-plane tilted beam pointing, respectively. A good agreement is obtained for all the elements in both cases. Following the FW diffraction representation one can interpret the oscillations of the reflection coefficient amplitude as established by the interference between each FW aperture field and its corresponding diffracted ray. In Fig. 4 the magnitude of the co-polar component of the far field is shown for both the previous cases. The fields obtained from the $T(FW)^2$ approach and the element by element approach superimpose. It is worth noting that, for the array analyzed, the computational time with the $T(FW)^2$ method is about 1/50 with respect to that of the conventional element by element approach.

CONCLUSIONS

An hybrid high frequency-MoM method has been proposed for the analysis of large periodic arrays of open ended waveguides. This method is quite general and may be generalized to other types of array elements [2]. Starting from the solution for the infinite array, a suitable fringe integral equation has been formulated, which describes the edge effects on the current of the actual finite array. This perturbation is interpreted in terms of FW-induced diffraction from the edges and the vertices of the array. This allows an efficient representation of the unknown fringe current in terms of a small number of basis functions with domain on the entire array, which are shaped as diffracted rays. Thus, the dimension of linear system to be solved is completely independent from the whole number of elements of the array, with the consequent gain of calculation time.
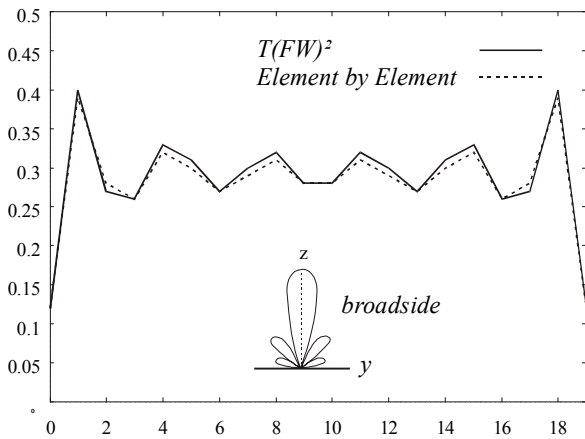

Fig. 2: Magnitude of the reflection coefficient for the apertures on the central column of the array, in case of broadside beam pointing
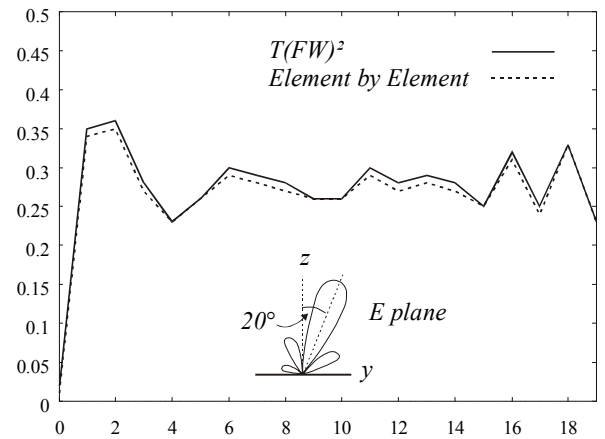

Fig. 3: Magnitude of the reflection coefficient for the apertures on the central column of the array, in case of 20° E-plane scan.
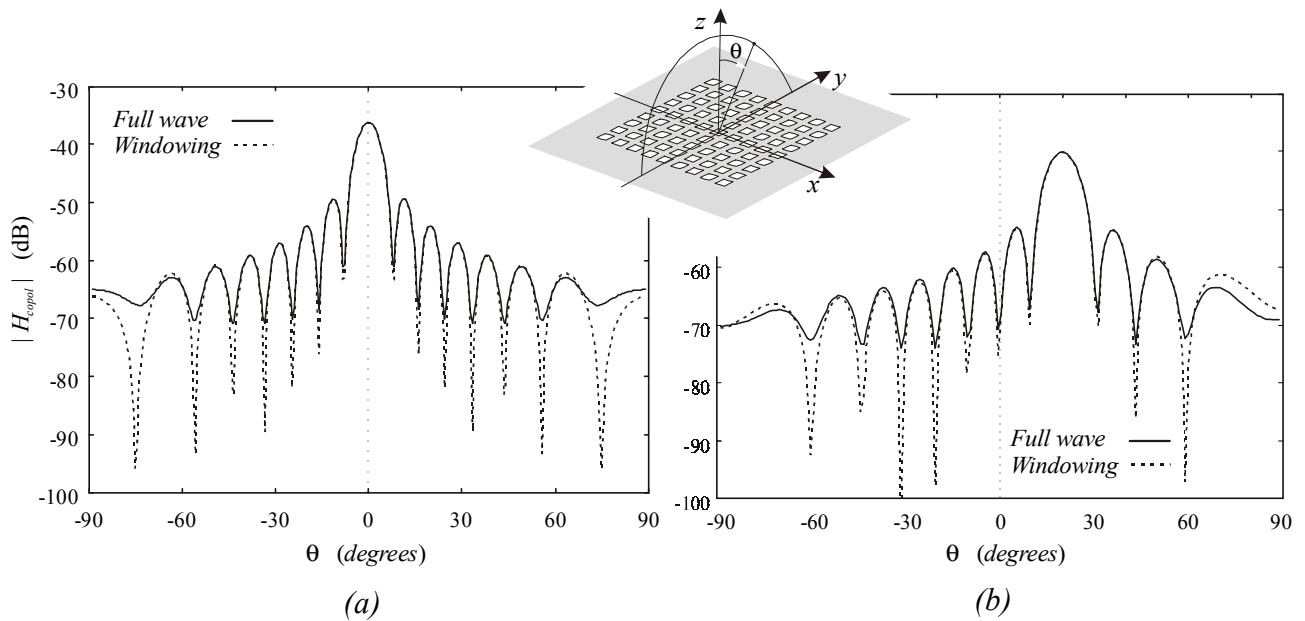
Fig. 4 Magnitude of the E-plane co-polar component of the radiated magnetic field. The full wave solution (continuous line, which identify both the T(FW)² and the element by element standard analysis) is compared to that obtained by windowing the infinite array current to the actual array (dashed line); (a) broadside beam, (b) 20° E-plane tilted beam.

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