# THEORETICAL ANALYSIS OF A MONOPOLE ANTENNA WITH CONDUCTING FLAT DISC ON THE EARTH

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## 1 Introduction

A monopole antenna on the Earth has a long history since Marconi's era and has been used as a medium wave transmitting antenna. It has been known that the efficiency depends on the electric characteristics of the place of installation. Important factors deciding apparent antenna efficiency are the antenna height, electric constant of the Earth and the size of the earth. However, the optimum values are unknown and there are different views on these factors.

Theoretical analyses have been done for the cylindrical antenna located in the medium including the Earth for the following two cases: the dipole antenna located above the Earth or the monopole antenna placed on the Earth and fed directly without the earth structure[1]. This paper presents a full wave analysis of a monopole antenna with conducting flat disc on the Earth. The electromagnetic fields are Fourier transformed, and the 3D real structure is replaced by the 1D equivalent circuit. The moment method analysis is performed to obtain the currents on the monopole and the flat disc, the ground wave and the apparent antenna efficiency. It is possible to know the necessary size of the earth and the optimum antenna height dependent on the electrical Earth characteristics.

## 2 Green's Functions

Consider a monopole antenna with conducting flat disc on the Earth as shown in Fig.1. Let the current elements $\boldsymbol{J}_a$ and $\boldsymbol{J}_g$ on the monopole and the disc, respectively, be expressed as follows, where the common time factor $e^{j\omega t}$ is implicitly assumed.

$$\boldsymbol{J}_a = \hat{z}\,\frac{1}{2\pi a}\,f(z)\,\delta(\rho-a) \qquad [\mathrm{A/m^2}] \tag{1}$$

$$\boldsymbol{J}_g = \hat{\rho}\,\frac{1}{2\pi \rho}\,g(\rho)\,\delta(z) \qquad [\mathrm{A/m^2}] \tag{2}$$

We apply the equivalent circuit method[2] to obtain the electric field from these current elements. This method is based on the TE and TM waves decomposition of the Fourier transformed system of fields and sources. The transformed fields are replaced by the voltage and the current in the equivalent 1D transmission line, which can be easily solved.

The spectrum domain current elements can be written as

$$\overline{J}_a = \iint_{-\infty}^{\infty} \frac{1}{2\pi a}\,f(z)dz\,\delta(\rho-a)\,e^{j(\ell x+my)}\,dx\,dy = f(z)dzJ_0(ap) \tag{3}$$

$$\overline{J}_g = \int_0^{\infty}\!\!\int_0^{2\pi} \frac{1}{2\pi\rho}\,g(\rho)\,e^{jp\rho\cos(\phi-\phi')}\rho\,d\phi\,d\rho = j\int_0^{\infty} g(\rho)J_1(p\rho)\,d\rho \tag{4}$$

$$p = \sqrt{\ell^2+m^2} \tag{5}$$

where $J_0(ap)$ and $J_1(p\rho)$ are Bessel functions of order zero and one, respectively. These current elements radiate E-wave, and the equivalent circuit in this case is as shown in Fig.2 (a) and (b), where an observation point of electric field is located higher and lower than the current on the monopole, $(z > z')$ and $(0 < z < z')$, respectively. The voltage source $V(p)$ and the current source $I(p)$ correspond to current elements in the spectrum domain, and are expressed as

$$V(p) = \frac{p}{\omega\varepsilon_0}\overline{J}_a = \frac{p}{\omega\varepsilon_0}J_0(ap)f(z)dz \tag{6}$$

$$I(p) = -\overline{J}_g = -j\int_0^\infty g(\rho)J_1(p\rho)\,d\rho \tag{7}$$

The electric fields in the spectrum domain can be written as

$$\overline{E}_z = -\frac{p}{\omega\varepsilon_0}I_m \tag{8}$$

$$\overline{E}_\rho = V_\rho \cos(\phi - \phi') \tag{9}$$

The solutions $I_m$ and $V_\rho$ of the circuits give the electric fields by Eqs.(8) and (9) expressed in terms of the Green's functions as follows.

$$\overline{E}_z = \overline{G}_{zz}f(z')dz' + \int_a^\infty g(\rho')\,\overline{G}_{z\rho}\,d\rho' \tag{10}$$

$$\overline{E}_\rho = \overline{G}_{\rho z}f(z')dz' + \int_a^\infty g(\rho')\,\overline{G}_{\rho\rho}\,d\rho' \tag{11}$$

$$\overline{G}_{zz} = -\left(\frac{p}{\omega\varepsilon_0}\right)^2 \frac{Z_0\cos\beta_0 z_< + jZ_1\sin\beta_0 z_<}{Z_0(Z_0+Z_1)} J_0(ap)\,e^{-j\beta_0 z_>} \tag{12}$$

$$\overline{G}_{z\rho} = j\frac{p}{\omega\varepsilon_0}\frac{Z_1}{Z_0+Z_1}J_1(p\rho')\,e^{-j\beta_0 z} \tag{13}$$

$$\overline{G}_{\rho z} = -\frac{p}{\omega\varepsilon_0}\frac{Z_1}{Z_0+Z_1}J_0(ap)\,e^{-j\beta_0 z'}\cos(\phi-\phi') \tag{14}$$

$$\overline{G}_{\rho\rho} = -j\frac{Z_0 Z_1}{Z_0+Z_1}J_1(p\rho')\cos(\phi-\phi') \tag{15}$$

where $z_>$ and $z_<$ represent the larger and the smaller one of $z$ and $z'$, respectively.

Using Fourier's inverse transform, Green's functions in the real domain are obtained. Green's function for $\overline{G}_{zz}$ in the real domain reduces to

$$G_{zz} = -\frac{\eta_0}{4\pi}\left({k_0}^2 + \frac{\partial^2}{\partial z^2}\right)\left[\Phi^-(z-z') + \Phi^+(z+z')\right] \tag{16}$$

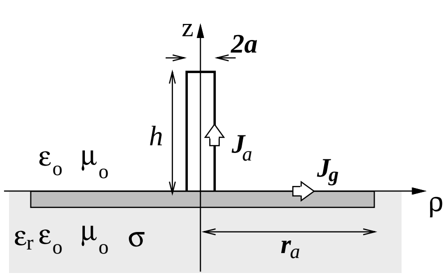
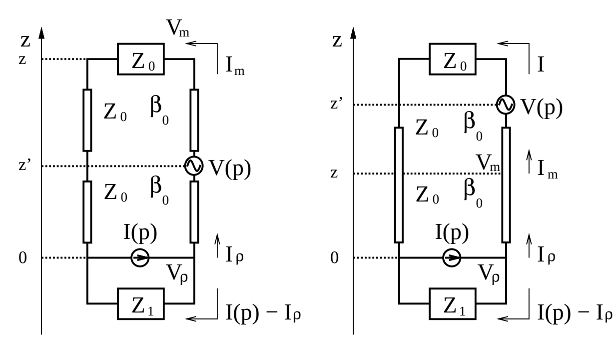



Figure 1: Structure and geometry of a monopole antenna with conducting flat disc on the Earth




(a) $z > z'$ (b) $z < z'$

Figure 2: Equivalent circuit

$$\Phi^{-}(z-z') = \Phi(z-z') + \Phi^{\mathrm{mod}}(z-z') \tag{17}$$

$$\Phi^{+}(z+z') = \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left[\Phi^{-}(z+z') - 2\left\{F_1(z+z') + F_2(z+z')\right\}\right] \tag{18}$$

where $\Phi$ is a factor of directly radiated wave from the monopole current of central axial current approximation in the vacuum, $\Phi^{\mathrm{mod}}$ is a factor of modification for $\Phi$, and $\Phi^{+}$ is a factor of the reflected wave from the Earth and the modifications $F_1$ and $F_2$.

# 3 Moment Method Analysis

The above Green's functions in the real domain are utilized in the moment method analysis. We adopt the Galerkin's procedure with triangular pulses as expansion and weight functions. The monopole and the disc are subdivided as shown in Fig.3. Elements of impedance matrix $Z^{11}_{mn}$, $Z^{12}_{mn}$, $Z^{21}_{mn}$ and $Z^{22}_{mn}$ are obtained correspondingly to $G_{zz}$, $G_{z\rho}$, $G_{\rho z}$ and $G_{\rho\rho}$, respectively.

As each Green's function includes second partial differentiations with respect to source and observation points, the partial integral can be performed twice.

Because current is continuous at feed point, the impedances for segment included feed point can be written as

$$Z^{10}_{m0} = Z^{11}_{m0} + Z^{12}_{m0}\ , \qquad Z^{20}_{m0} = Z^{21}_{m0} + Z^{22}_{m0} \tag{19}$$

$$Z^{01}_{0n} = Z^{11}_{0n} + Z^{21}_{0n}\ , \qquad Z^{02}_{0n} = Z^{12}_{0n} + Z^{22}_{0n} \tag{20}$$

Moreover the impedance of feed point current $J_0$ can be written as

$$Z^{00}_{00} = Z^{11}_{00} + Z^{12}_{00} + Z^{21}_{00} + Z^{22}_{00} \tag{21}$$

The voltage matrix $V$ whose elements are voltage given at each segment can be written as

$$\begin{bmatrix} Z^{11} & Z^{10} & Z^{12} \\ Z^{01} & Z^{00} & Z^{02} \\ Z^{21} & Z^{20} & Z^{22} \end{bmatrix} \begin{bmatrix} I_z \\ I_0 \\ -I_\rho \end{bmatrix} = \begin{bmatrix} V \end{bmatrix} \tag{22}$$

where $I_z$, $I_0$ and $I_\rho$ represent, respectively, the currents $J_a$, $J_0$ and $J_g$ integrated with respect to the circumferential direction. $V$ is the feed point voltage. Solving this matrix equation, the currents, $I_z$ and $I_\rho$, in the $z$ and $\rho$ direction are obtained.

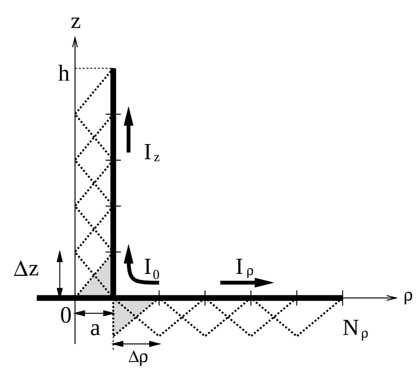



Figure 3: Expansion function

# 4 Ground Wave Intensity and Apparent Antenna Efficiency

Apparent antenna efficiency is proportional to the product of the antenna gain on the Earth and the efficiency, and is a critical factor deciding the service area. When an infinitesimal vertical antenna, whose gain and efficiency are both 1.0, radiates $P$ [kW], the electric field intensity at the distance of 1 [km] is equal to $300\sqrt{P}$[mV/m]. Thus, the apparent antenna efficiency can be defined as follows, where an effective value of the electric field $E_{\mathrm{rms}}$ is in mV/m.

$$\eta_a = \left(\frac{E_{\mathrm{rms}}}{300\sqrt{P}}\right)^2 \times 100 \qquad [\%] \tag{23}$$

## 5 Numerical Results

The current distributions and the ground electric field calculated for the following case are shown in Figs.4 and 5, respectively. The monopole hight and the disc radius are both $0.25\lambda$, the relative permittivity $\varepsilon_s$ and the conductivity $\sigma$ of the Earth are 10 and $10^{-4}$ [S/m], the frequency $f$ is 1[MHz], the antenna thickness is decided by $\Omega = 2\ln(2h/a) = 10$.

The calculated apparent antenna efficiency is shown in Fig.s 6 and 7.

## 6 Conclusion

The full wave analysis is presented for the monopole antenna with conducting flat disc on the Earth, and the apparent antenna efficiency is calculated for various structural parameters. The results will be very useful for the optimum design of the medium wave transmitting antenna.

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## References

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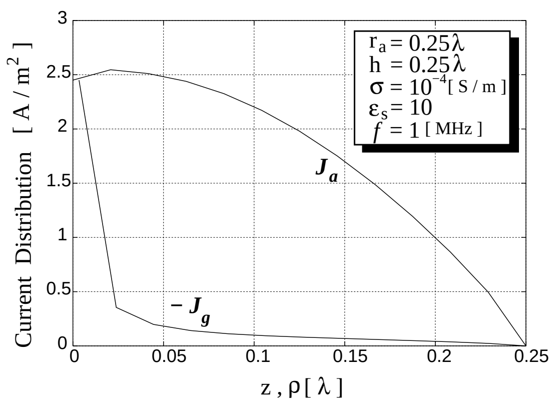


Figure 4: Current distribution

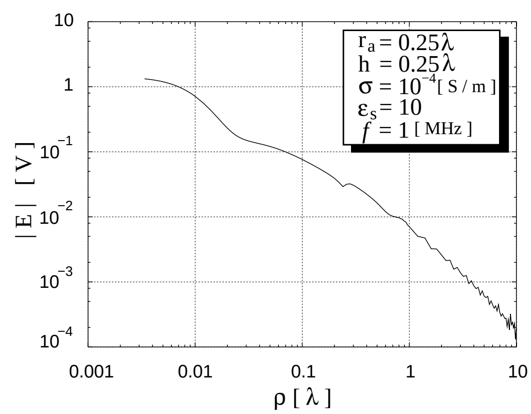


Figure 5: Ground wave intensity

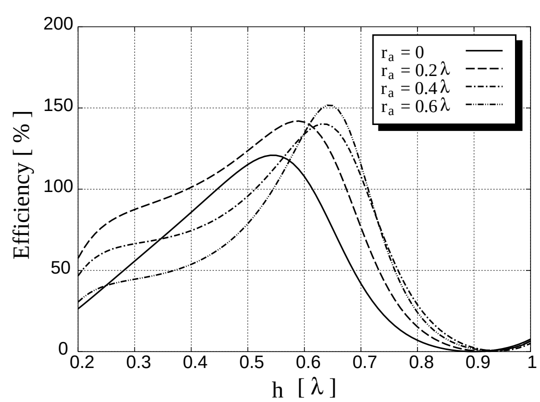


Figure 6: Apparent antenna efficiency vs. antenna height

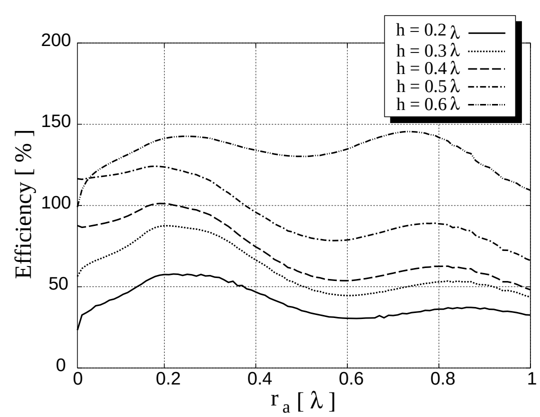


Figure 7: Apparent antenna efficiency vs. radius of conducting flat disc